GREGOR BOEHL

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Institute for Monetary and Financial Stability
GOETHE UNIVERSITY FRANKFURT

WORKING PAPER SERIES NO. 119 (2017)

Institute for Monetary and Financial Stability
Goethe University Frankfurt
House of Finance
Theodor-W.-Adorno-Platz 3
D-60629 Frankfurt am Main
www.imfs-frankfurt.de | info@imfs-frankfurt.de

Monetary Policy and Speculative Stock Markets[☆]

Gregor Boehl[a]

[a] *Institute for Monetary and Financial Stability, Goethe University Frankfurt*

Abstract

Using an estimated model with credit constraints in which excess volatility of stock markets is endogenously amplified through behavioral speculation, I study whether monetary policy can mitigate spillovers. Endogenous speculation and its feedback to the price level are central features to replicate empirical key moments. Standard monetary policy rules are shown to amplify stock price volatility. Numerical analysis suggests that asset price targeting can offset the impact of speculation on either output or inflation (but not on both) and can dampen excess volatility. The dampening effect of this policy is limited due to its undesirable response to non-financial shocks.

Keywords: Monetary policy, asset pricing, nonlinearity, heterogeneous expectations, credit constraints
JEL: E44, E52, E03, C63

1 Introduction

The debate on the interplay between asset prices and the macroeconomy – and the respective implications for monetary policy – has again drawn attention after the events that unleashed after the 2007 collapse of the US real estate market. Can expansionary monetary policy, instead of restoring steady growth, gravitate to fuel financial markets? If so, overheated financial markets might in turn destabilize the economy and comprise further hazard, again calling for relaxed monetary policy. The answer to this question is of particular relevance in the light of the unconventional monetary policy measures conducted by the European Central Bank in response to the great recession. Some economists (Poole, 1970; Cecchetti, 2000; Borio and Lowe, 2002) have suggested to let monetary policy target asset prices to prevent the aforementioned spiral of bubbles, instability and unconventional monetary policy. Behind such suspicions of a feedback between the financial sphere and real aggregates lie at least two postulates. The first

[☆] This work received the Student Award of the Society of Computational Economics and was funded by the Bielefeld Graduate School in Economics and Management. I am grateful to Alex Clymo, Herbert Dawid, Cees Diks, Cars Hommes, Christian Proaño, Christian Stoltenberg, Sander van der Hoog, Christiaan van der Kwaak, Volker Wieland, Fabian Winkler and participants of several conferences and seminars for discussions and helpful comments on the contents of this paper.

Email address: `boehl@econ.uni-frankfurt.de`
URL: `https://gregorboehl.com`

postulate is the existence of a mutual link between asset prices and real activity, i.e. that causality might run in both directions. Recent economic literature stresses the relevance of financial conditions and suggests to also study the macroeconomic impact of equity, and equity prices.[1] Yet there is limited insight on how and when stock prices impact real activity. The second postulate is that asset prices do not always reflect the discounted fundamental value but can entail distortions by themselves, possibly biased by speculation. Such destabilizing speculative process suggests that traders might not be fully rational. If speculation can be profitable, financial markets might be more prone to instability than real markets.

This work considers a macro-economy where both of the above postulates are implemented explicitly. I assume that stock prices are not only driven by economic fundamentals, but also that they feed back on real activity. Additionally, speculative dynamics in the financial market are added and the model is estimated to replicate statistical key-moments of European data. As it turns out, both the linkage as well as a speculative process are necessary to match the data well. Having thus motivated policy intervention, I ask whether Taylor-type interest rate rule that targets asset prices is able to *ex-ante* mitigate the impact of speculation on real activity, hereby reducing excess volatility on stock markets and lowering the risk of financial hazard.

The first contribution of this work is to provide microfoundations of the feedback between stock prices and real macroeconomic aggregates. The channel considered here transmits through the *external finance premium* introduced by (Bernanke et al., 1999, BGG).[2] I extend a DSGE model of a monetary production economy by a sector of financial intermediaries. Firms lever their profits by borrowing from the financial intermediaries and pledge their equity as collateral, while borrowing conditions and finance costs depend on the value of collateral offered. The external finance premium thus depends on firms' net worth. I assume that firms issue equity shares and can choose their net worth by deciding over dividends payed to shareholders. If firms seek to maximize the dividends per share, I show that under quite general assumptions asset prices are linked to the profit rates and then are competitive among firms. This connects asset prices, return on equity, and the external finance premium. Since goods markets are not perfectly competitive, return on equity in turn depends on price-setting which, in aggregate, determines the consumer price level.

The second, methodological contribution is the interaction of speculative and rational agents. Financial markets work fundamentally different than commodity markets. While commodity goods are a direct means to an end (consumption or production) assets are held because of their expected return in the future. If the price of an asset depends on its resale value, beliefs on the future development of asset prices can become self-fulfilling and incentive schemes can differ notably compared to the purchase of commodity goods. To illustrate, imagine a firm chose a production volume according to its cost function and expectation of prices and this expectation turns out to be correct. Under

[1] See e.g. Assenmacher and Gerlach (2008); Abbate et al. (2016) and Miao et al. (2016).

[2] A further potential channel is a wealth effect that works through aggregate demand: increasing stock prices raise the nominal value of assets held by households, and amplifies consumer demand. Unfortunately such effect is ruled out in a representative agent framework where seller and buyer are identical and changes in asset prices level out to zero in aggregate. Since that, an increase in asset prices falls short to increase households' real spending opportunities.

this circumstance profits will be maximal. If prices were overestimated however, the firm would be unable to sell the produced stock profitably and incur a relative loss. Asset markets work differently. If a positive price change has been overestimated by a trader, he will realize a higher profit than expected. Unlike in commodity markets, asset traders can benefit from overoptimistic forecasts at least in the short run. This can be the source of herding behaviour, because instead of focussing on the underlying fundamental, it can be *behaviorally rational* (Hommes, 2013) to follow the majority in their beliefs. Such behavior is not well captured by rational expectations since rational traders would be fully aware that the price does not reflect economic fundamentals. To allow for realistic asset price dynamics I distinct between expectations on real economic aggregates and expectations on stock prices. While the latter are modeled to be perfectly rational, financial traders form boundedly rational expectations that embed speculative dynamics. This restriction addresses the prominent critique of *the wilderness of bounded rationality*[3] and preserves the forward-looking component of the framework but allows for speculative dynamics in the asset market. The speculative process induces a strong degree of nonlinearity into the model which allows for endogenous dynamics.

My policy analysis then focusses on three effects. I show that conventional monetary policy can amplify fluctuations in stock prices. An increase in stock prices decreases firms marginal costs of finance because the prospect of higher dividends will give an incentive to shareholders to invest while at the same time banks are willing to charge lower risk premiums. Through competitive markets firms will pass on lower marginal costs to the consumers, hereby increasing the consumer price level. When monetary policy then responds with a decrease of interest rates, this again boosts stock prices. If stock traders are boundedly rational, a dangerous feedback loop can emerge. Second, a monetary policy that targets asset prices can dampen excess volatility of financial markets and as such mitigate the *source* of spillovers. Lastly, monetary policy can either unlink output from asset price dynamics or to effectively mitigate the direct feedback of expectations on current prices, but not both. However, since such policy would also react to movements in the stock market that are triggered by shocks to economic fundamentals, the overall response of monetary policy would not be optimal.

The next Section 2 reviews the empirical key-features and gives an overview on the related literature. In Section 3 the macroeconomic framework is presented and micro-foundations for the mutual link between asset prices and macroeconomic aggregates are provided. Equilibrium dynamics under rational expectations and to parameters estimates are presented in Section 4. Endogenous fluctuations in stock prices are introduced in Section 5 together with simulation results and policy analysis whereas Section 6 concludes.

2 Empirical findings and connection to the existing Literature

Episodes with booms and busts are a recurrent phenomena. In a 2003 analysis of the housing market and equity prices in industrialized economies during the postwar period, the IMF found that booms in both markets arise frequently (on average every

[3]See Sims (1980). In brief, this critique states that while the concept of rational expectations can be uniquely defined, there are infinitely many possibilities of modelling bounded rationality.

13–20 years) with entailed drops in prices averaging around 30% and 45% respectively. These busts are associated with losses in output that reflect declines in consumption and investment. Table 1 brings together key moments of inflation, output and stock prices in Core-Europe. The data is obtained from the OECD, stock prices are represented by the MSCI-Europe index.[4] Let me briefly summarize the stylized facts that are embedded in the data.

i) The standard deviation of stock prices is roughly one order of magnitude higher than the standard deviations of inflation and output.
ii) Inflation is (weakly) countercyclical.
iii) Stock prices and output are positively correlated.
iv) Stock prices and inflation are negatively correlated.
v) The negative correlation between stock prices and inflation is stronger than the correlation between output and inflation.[5]

	π	y	s
SD	0.0092	0.0104	0.1407
π	1	-0.1734	-0.3867
y	–	1	0.6025
s	–	–	1

Table 1: Standard deviations and cross correlations of inflation, output and real stock prices, Core-Europe from 1976 to 2014 (quarterly). A detailed description of the data can be found in Section 4.

These stylized facts are well in line with empirical work (Campbell, 1999; Shiller, 1981; Fama and French, 1988). Barro (1990) and Sargent (2008) argue that stock prices are a good indicator for investment and hence future output, which implies a lead-lag structure of asset prices and output. Winkler (2014) conducts a vector auto-regression (VAR) on asset price shocks. He finds that the response of total factor productivity is insignificant or even negative, while the asset price shock has significant effects on investment. Accordingly, he concludes that the classical view that stock price changes reflect new information about productivity changes might be controversial. Abbate et al. (2016) report similar findings by using a time-varying FAVAR. To my best knowledge, the relationship between inflation and stock prices has not been subject to detailed studies yet. Assenmacher and Gerlach (2008) show that asset prices react almost instantaneously to the interest rate in support of the conventional role of interest rates as the discount factor. They furthermore find that shocks on asset prices can have impact on both, GDP and credit volume and that fluctuations in stock prices can explain about 10% to 15% of the variance of GDP.

[4] At the time of writing the series on inflation and output is available at `https://data.oecd.org/`. Stock prices are downloaded from `https://www.msci.com/indexes`. The data is quarterly and ranges from 03/1976 to 03/2015, hence a total of 158 observations is being used. Time series are deflated by the price index (prices given in 2005) and the HP-Filter is applied to the log of each series with $\lambda = 1600$.

[5] Note that this also suggests that the link between stock prices and real activity should rather be motivated through the supply side than through the demand side.

Tallarini (2000), Rudebusch and Swanson (2012) and others explore the asset price dynamics implied by Epstein-Zin preferences (Epstein and Zin, 1989), while a different branch of the literature followed the idea of habit-formation specifications (cf. Abel, 1990; Ljungqvist and Uhlig, 2000, 2015). Also see Kliem and Uhlig (2016) for a brief overview of this branch of the literature and the estimation of such type of model. These methods generally report mixed success with fitting both, macroeconomic dynamics and asset price volatility.

This stands in contrast to the approach taken in this paper, where financial markets can be a source of fluctuations on its own This hypothesis is backed by a broad amount of literature. In a study on US stock market trading patterns Shiller (2005) finds evidence for speculative behavior. Greenwood and Shleifer (2014) provide a summary on survey data that documents the failure of rational expectations. According to his data the rational expectations hypothesis is almost always rejected. Adam et al. (2016) show that introducing bounded rationality into standard models of consumption based asset greatly improves empirical performance whereas Cars Hommes and coauthors (Hommes et al., 2005; Hommes, 2011; Assenza et al., 2013) find indication for simple, heterogeneous forecasting mechanisms in laboratory experiments. Boehl (2017) reports that agents will act boundedly rational even if a considerable fraction of traders is fully rational. The behavioral approach to stock pricing has furthermore been shown to perform surprisingly well in endowment economies, without the need to rely on non-separable preferences or habit (Adam et al., 2016, 2017).

Closely related and corresponding to the work on behavioural finance, there exists a growing literature on boundedly rational agents that have been introduced to macroeconomic modelling, specifically with respect to expectations on output and inflation, see for instance Evans and Honkapohja (2003), Anufriev et al. (2008), Brazier et al. (2008), Branch and McGough (2009, 2010), De Grauwe (2011) and De Grauwe and Macchiarelli (2013). From the perspective of this literature, recessions are not due to shocks to fundamentals but rather to massive coordination failure. Mankiw et al. (2003), Branch (2004) and Pfajfar and Santoro (2008, 2010) provide empirical evidence in support of heterogeneous expectations using survey data on inflation expectations.

The model used here adopts from the general New Keynesian literature (Woodford, 2003; Galí, 2008). Financial frictions are, with some modifications, inspired by Bernanke et al. (1999). An early treatment of the question of whether central bank should target asset prices has been undertaken by Poole (1970) who finds that monetary policy should counteract asset price movements if the respective disturbances originate in the financial market. The theoretical benchmark result however is that asset price targeting is rather harmful in terms of welfare and is provided by Bernanke and Gertler (2000) where stock prices are represented as the price for capital and bubbles are exogenous. Winkler (2014) uses BGG-type frictions to combine asset prices and real activity, and empathizes the role of learning-based asset pricing (Adam and Marcet, 2011) to reproduce excess volatility of stock prices. As in my model, under rational expectations a monetary policy that targets asset prices induces a welfare-loss while under learning carefully targeting asset prices might lead to a welfare improvement. However, he uses US data while I am focussing on the case for the Euro area. Miao et al. (2012) and Miao et al. (2016) build a Bayesian model with rational stock price bubbles which affect the economy through endogenous borrowing constraints. They finds that the feedback between asset prices and asset price expectations plays a key role on the formation of a stock price bubble and that roughly 20% of the variance of GDP can be explained through fluctuations in stock prices. A different branch of the literature most prominently represented by Galí (2013) uses the concept of rational asset price bubbles to analyze the role of such policy, with the key finding that monetary policy should – if at all – rather *lower* interest rates when facing asset price bubbles. Also based on the concept of rational asset bubbles, Martin and Ventura (2011) rely on Gertler et al. (2010) to create a link between credit volume, firms' value and real activity to explain empirical irregularities around the great recession. Given the scope of my research question, rational asset price bubbles have the drawback that they require an exogenous both, to emerge and to bust. Further they comprise some counterintuitive implications, for instance on the co-movement on the interest rate and the size of the bubble.

3 Model

The economy is populated by a continuum of identical households, a heterogeneity of firms, a financial intermediary and a monetary authority.

3.1 Households

Households are indexed by i. They face a standard problem of maximizing the expected present value of utility by deciding over consumption of a composite good C_t and time devoted to the labour market H_t. For each unit $H_{i,t}$ of labour supplied they receive the real wage W_t. Furthermore they can deposit monetary savings D_t at the financial intermediary for which they receive the gross real rate R_{t+1} in the next period. The maximization problem for individual agents is then

$$\max_{\{C_{i,t}\},\{H_{i,t}\},\{D_{i,t}\}} E_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{\zeta_{i,t} C_{i,s}^{1-\sigma}}{1-\sigma} - \xi \frac{H_{i,s}^{1+\psi}}{1+\psi} \right) \tag{1}$$

s.t. the budget constraint states (in real terms)

$$C_{i,t} + D_{i,t} \leq W_t H_{i,t} + R_t \frac{P_{t-1}}{P_t} D_{i,t-1} + \mu \int_0^{\bar{\omega}_t} \omega H_t / X_t dF(\omega) \; \forall t = 1, 2, ... \tag{2}$$

where $\mu \int_0^{\bar{\omega}_t} \omega H_t / X_t dF(\omega)$ are the audition costs for defaulting wholesalers which are explained in detail in Appendix A. Via the financial intermediary they are distributed equally among households and hence do not enter optimality conditions. Each household is subject to an idiosyncratic preference shock $\zeta_{i,t}$. The composite consumption good consists of differentiated products from the retail sector and is sold in a market with monopolistic competition. The composite good and the aggregate price index for the consumption good are defined by the CES aggregators

$$C_t = \left(\int_0^1 C_{l,t}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \qquad \text{and} \qquad P_t = \left(\int_0^1 P_{l,t}^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}. \tag{3}$$

Optimization yields the usual Euler equation and a labor supply equation

$$\zeta_t C_t^{-\sigma} = E_t \left\{ \beta R_{t+1} \frac{P_t}{P_{t+1}} C_{t+1}^{-\sigma} \right\} \tag{4}$$

$$\xi H_t^{\psi} = \frac{\zeta_t W_t}{C_t^{\sigma}} \tag{5}$$

where ζ_t denotes the i.i.d. aggregate demand shift that is due to the idiosyncratic preference shocks. Since each *individual* shocks are unobservable for other agents, at time t the aggregate shock is not observable either. Given optimality, the budget constraint needs to hold as an equality and agents obey the transversallity condition

$$\lim_{s\to\infty} \beta^{s-t} E_t C_s^{-\sigma} D_s = 0. \tag{6}$$

3.2 Firms

To maintain analytical tractability, firms are divided into a wholesale and retail sector. Wholesalers borrow money from the financial intermediary to finance production and their shares are traded at the stock exchange. Their (homogeneous) good is sold to the retail sector where diversification takes place and the then heterogeneous goods are sold to the households with monopolistic profits.

3.2.1 Wholesale Sector

Let labor be the only production factor and index wholesalers by j. The CRS production function is

$$Y_{j,t} = \omega_{j,t} H_{j,t}, \tag{7}$$

where $\omega_{j,t}$ is a firm-specific idiosyncratic productivity shock similar to the households' preference shock. To simplify the optimization problem I allow for negative dividends to be payed, which implies that firms can obtain financing resources from their shareholders as well, shareholders are willing to increase firms' equity to seize the opportunity of higher future profits. A similar approach is chosen by Martin and Ventura (2010) for aggregated investment. Wholesalers are price takers. Let X_t be the gross markup of retail goods over wholesale goods or, equivalently, let X_t^{-1} the relative price of wholesale goods. This implies that R_{t+1}^H, the gross return on employing one unit of labor, is in expectations and omitting subscript j given by

$$E_t R_{t+1}^H = (X_t W_t)^{-1}, \tag{8}$$

where the reciprocal definition of X_t ensures $E_t R_{t+1}^H > 1$ which is a necessary condition for positive external finance.

Denote firm j's equity by $N_{j,t}$, its period t equity price in real terms by $S_{j,t}$ and define $E_t R_{j,t+1}^S$ to be the expected return on equity implied by the stock price. Given no arbitrage it needs to hold that $E_t R_{j,t+1}^S = \frac{R_{t+1} S_{j,t}}{N_{j,t}}$. Let me assume the following timing structure: goods are produced and sold in the current period, but returns are realized at the beginning of the next period. Then firms decide upon their equity and distribute the rest as dividends Θ_{t+1}. We can hence write dividends each period as

$\Theta_t = H_{j,t-1}/X_{j,t-1} - N_{j,t}$. Finally, the firm's shares are traded at the stock exchange. Assume that unlike creditors, shareholders can liquidate the firm at any time without costs. Given no-arbitrage the price $S_{j,t}$ of one share of the firm needs to satisfy $S_{j,t} = \max\left\{N_t/R_{t+1}, E_t\frac{\Theta_{j,t+1}+S_{j,t+1}}{R_{j,t+1}}\right\}$ since a firm with a higher equity than discounted profits will be liquidated by its shareholders. Acknowledging this, for any *existing* firm it must hold that

$$S_{j,t} = E_t \sum_{s=t}^{\infty} \left(\prod_{l=t}^{s} R_{l+1}^{-1} \right) \Theta_{j,s+1} \tag{9}$$

where the normalization of the number of shares to unity is implied.

Simple example without external finance. The link between stock prices and return per unit of labor R_{t+1}^H can be explained quite intuitively in a world without external finance. For that purpose let me briefly abstract from external finance. Recall that every period firms choose how much of their returns to retain and how much to distribute. Dropping the j-subscript, the Lagrangian reads

$$\max_{\{H_t\},\{N_t\},\{\lambda_t\}} E_t \sum_{s=t}^{\infty} \left(\prod_{l=t}^{s} R_{l+1}^{-1} \right) [H_{s-1}/X_{s-1} - N_s] - \lambda_s \left(W_t H_s - N_s \right). \tag{10}$$

The first-order condition is $H_t/X_t = N_t R_{t+1}$, which combined with the definition of expected dividends gives $E_t\Theta_{s+1} = R_{t+1}N_t - E_t N_{t+1}$. Inserting this result into Equation (9) implies that stock prices reflect the value of equity perfectly as in

$$S_t = \frac{R_{t+1}N_t - E_t N_{t+1} + \frac{E_t\{R_{t+2}N_{t+1}\}+\ldots}{R_{t+2}}}{R_{t+1}} = N_t. \tag{11}$$

It follows that the optimal labor demand $H_t = R_{t+1}S_t X_t$ is determined by the prices prevailing at the financial market in combination with wholesale prices and the economies interest rate.[6]

Full model. Let me now return to the wholesalers' problem with external finance. The volume of external finance demanded is firms' working capital $W_t H_{j,t}$ minus equity, hence

$$B_{j,t} = W_t H_{j,t} - N_{j,t}. \tag{12}$$

As in BGG, I follow a mechanism using the costly state verification (CSV) approach in Appendix A to integrate the financial accelerator. The rate on loans from the intermediary, R_{t+1}^B, contains a risk-premium on the prevailing interest rate which depends on the individual firm's leverage. Hence

$$R_{j,t+1}^B = z\left(\frac{N_{j,t}}{W_t H_{j,t}}\right) R_{t+1} \tag{13}$$

[6]Note that without external finance $N_t = H_t/X_t$, i.e. equity and working capital are the same. Under rational expectations this implies $\frac{S_t}{N_t} = 1$.

with $\frac{\partial z}{\partial N_t} < 0$. When the leverage ratio decreases, the premium on external finance falls because more collateral is provided and the loan becomes less risky. I show in Appendix A that optimality requires the return on assets to equal the rate paid on external funds, $E_t R^S_{t+1} = E_t R^B_{t+1}$ since otherwise wholesalers would have an incentive to increase/decrease the borrowing volume. Similarly to the example above, an increase in S_t will also have an increasing effect on equity N_t. Hence, once a functional form of $z(\cdot)$ is known, we can use

$$\frac{S_{j,t}}{N_{j,t}} = z\left(\frac{N_{j,t}}{W_t H_{j,t}}\right) \tag{14}$$

to eliminate N_t. Since $E_t R^S_{t+1} = E_t R^B_{t+1}$, these returns also need to be equal to $E_t R^H_{t+1}$. Plugging the result into Equation (8), substituting for W_t and H_t and log-linearising the result gives us an aggregate representation of the price X_t for wholesale goods

$$x_t = \nu s_t - \eta y_t - r_{t+1}, \tag{15}$$

where ν is the price elasticity of the markup with respect to stock market prices and $\eta = \frac{\sigma+\psi+\tilde{\nu}}{1-\tilde{\nu}}$ the sensitivity of the markup with respect to output. Intuitively, a competitive market for wholesale goods implies equal prices. Likewise, all firms have to offer the same $E_t R^S_{t+1}$ and then the stock market evaluation of shares determines the amount of equity and dividends payed. In general equilibrium, to comply to market forces and implicit expectations on future dividends, relative prices have to rise. Combined with the fact that managers can not distinct whether aggregate stock prices are overvalued or not, this mechanism can be summarized by *the pressure to perform.*[7]

3.2.2 Retailers

Retailers buy the homogeneous good $Y_{j,t}$ from entrepreneurs and differentiate to sell it in a monopolistic competitive consumer market. Firms' price setting decisions are subject to nominal rigidities la as it is standard in the literature. For details on the solution given the markup X_t, see Bernanke et al. (1999). Letting resellers be denoted by l, it can be shown that setting the optimal price P^*_t, given the corresponding demand $Y^*_{l,t}$, satisfies

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,k} \left(\frac{P^*_t}{P_{t+k}}\right)^{-\epsilon} Y^*_{l,t+k} \left[\frac{P^*_t}{P_{t+k}} - \left(\frac{\epsilon}{\epsilon-1}\right) X_t^{-1}\right]\right\} = 0, \tag{16}$$

with θ being the fraction of retailers per period that is not allowed to change prices in period and $\Lambda_{t,k} = \beta \frac{C_t}{C_{t+1}}$ each periods discount factor. The aggregated price level then follows

$$P_t = [\theta P^{1-\epsilon}_{t-1} + (1-\theta)(P^*_t)^{1-\epsilon})]^{\frac{1}{1-\epsilon}} \tag{17}$$

[7]This result contains two effects: 1) Firms' only possibility to increase profits per labor unit in an CRS economy is to raise prices. An increase in stock prices puts pressure on the price level. 2) A hike in stock prices decreases the leverage ratio, which lowers the cost for external finance and decreases commodity prices. In this model the second effect prevails.

where P_t^* needs to satisfy Equation (16). Log-linearizing the combination of both equations yields the Phillips Curve (22) depending on the log-deviation of the markup x_t from its steady state value.

3.3 Financial Intermediation

There is a continuum of financial intermediaries indexed by k. Each of them takes the deposits $D_{k,t}$ received from households as given and invests a fraction in the stock market by purchasing a $J_{k,t}$-proportion of all traded shares evaluated at the real stock price S_t and issues the rest as credit volume $B_{k,t}$ to the wholesalers. I assume that investment in the financial market is done by traders that are each associated with a financial intermediary. Furthermore the intermediary has access to central bank money for which he will have to pay the real deposit rate R_{t+1}. Next period's real dividends net of seized collateral are expected to be $E_{k,t}\Theta_{t+1}$. Market clearing requires

$$R_{t+1}^D D_{k,t} = \hat{E}_{k,t}[\Theta_{t+1} + S_{t+1}]J_{k,t} + z^{-1}R_{t+1}^B B_{k,t} \tag{18}$$

subject to the constraint $D_{k,t} \geq S_t J_{k,t} + B_{k,t}$. From the fact that the opportunity costs of finance are given by the central bank interest rate, optimality requires $R_{t+1}^D = z^{-1}R_{t+1}^B = \frac{\hat{E}_t\{\Theta_{t+1}+S_{t+1}\}}{S_t} = R_{t+1}$. In case of homogeneous and rational expectations the pricing equation for stocks can be aggregated straightforwardly as in

$$R_{t+1}S_t = E_t\{\Theta_{t+1} + S_{t+1}\}. \tag{19}$$

Let a capital letter without time subscript denote the respective steady-state value. In equilibrium, Θ depends on the markup X and aggregated output Y. $\frac{\partial\Theta}{\partial Y}$ depends on the labor share and is here set to unity. Note that, when log-linearising Equation (19) the coefficient of $E_t y_{t+1}$ of $(1-\beta)$ of $E_t y_{t+1}$ is very small, so introducing a more realistic labor income share would not be a notable improvement. The log-linear version of the asset pricing equation is thus almost independent of expectations on next periods' output and markup. For the percentage deviation of stock prices from their steady state value this yields

$$s_t = (1-\beta)E_t y_{t+1} + \beta E_t s_{t+1} - r_{t+1}, \tag{20}$$

where r_{t+1} denotes the net real interest rate $r_{t+1} \equiv i_{t+1} - E_t\pi_{t+1}$.

3.4 Central Bank and Government

The central bank follows a standard contemporaneous Taylor Rule and can additionally adjust the nominal interest rate i_{t+1} in response to the real stock prices s_t. Then the linearized monetary policy rule reads as

$$i_{t+1} = \phi_\pi \pi_t + \phi_s s_t. \tag{21}$$

A policy that increases the nominal interest rate when stock market prices increase will here be called *asset price targeting* (APT). Asset price targeting is the only additional policy measure explicitly implemented in the model. A problem that the monetary authority faces when responding to movements in stock prices is that it is not *ex-ante* identifiable whether a deviation in asset prices represents a shift in fundamentals or in beliefs. In order to establish a practicable mechanism, if such policy would be in place the central bank must *always* react to movements in stock prices. This is independent of whether these are identified as bubbles or as a correct anticipation of movements real aggregates. I furthermore abstract from governmental expenditures and assume that the government issues no debt.

4 General Equilibrium and Estimation

To establish a benchmark, until the end of this section it is assumed that all expectations are formed homogeneously and agents act completely rational. The linearized economy is characterised by the following set of equations.

$$\pi_t = \beta E_t \pi_{t+1} - \kappa x_t + v_t^\pi, \tag{22}$$

$$y_t = E_t y_{t+1} - \sigma^{-1} r_{t+1} + v_t^y, \tag{23}$$

$$x_t = \nu s_t - \eta y_t - r_{t+1}, \tag{24}$$

$$s_t = (1-\beta) E_t y_{t+1} + \beta E_t s_{t+1} - r_{t+1}, \tag{25}$$

$$i_{t+1} = \phi_\pi \pi_t + \phi_s s_t. \tag{26}$$

Shock terms and the real interest rate are given by

$$v_t^\pi = \rho_\pi v_{t-1}^\pi + \varepsilon_t^\pi, \qquad \varepsilon_t^\pi \sim N(0, \sigma_\pi) \tag{27}$$

$$v_t^y = \rho_y v_{t-1}^y + \varepsilon_t^y, \qquad \varepsilon_t^y \sim N(0, \sigma_y) \tag{28}$$

$$r_{t+1} = i_{t+1} - E_t \pi_{t+1}. \tag{29}$$

Equation (22) is the New-Keynesian Phillips curve, whereas Equation (23) is normally referred to as the dynamic IS-curve. The connection between the textbook model and the BGG-type credit frictions is established in Equation (24). Equation (25) states the no-arbitrage condition for the stock market and Equation (26) is the Taylor rule with the addition of asset price targeting.

The aggregate of individual preference shocks ζ_t is represented by v_t^y and translates to a demand shock. Since individual preferences are not publicly observable, the realization of the shock is not ex-post observable. Likewise, v_t^π is the aggregate productivity shock that results from idiosyncratic productivity shocks to wholesalers. Similar as to the demand shock, v_t^π is not observable in the aggregate since those shocks affect each producer individually. Once the assumption of rationality is dropped, the non-observability of both shocks is an important ingredient to the model. Both shocks follow a standard AR(1) structure with ρ_π and ρ_y respectively.

Equations (22) to (25) can be represented as a 3-dimensional system of the endogenous variables $\mathbf{x}_t = \{\pi_t, y_t, s_t\}$ as in

$$\mathbf{M}\mathbf{x}_t = \mathbf{P} E_t \mathbf{x}_{t+1} + \boldsymbol{v}_t, \tag{30}$$

whereas the elements of these matrices in more detail in Appendix C.

Estimation and Identification

Except when specified otherwise, the deep parameters are fixed to values that are standard in the literature. Let $\beta = 0.99$ represent the short-term perspective of a quarterly model and set the shocks' autocorrelation to $\rho_\pi = 0.9$ and $\rho_y = 0.7$ respectively. Other values are consistent with the calibration of Woodford (2003) as it is chosen that $\psi = 0.3$ and $\omega = 0.66$, resulting in $\eta = \frac{\sigma+\psi+\tilde{\nu}}{1-\tilde{\nu}} \approx 1.58$ and $\kappa = (1-\omega)(1-\beta\omega)/\omega \approx 0.179$. The elasticity of the external finance premium with respect to net worth which determines the elasticity of marginal costs to changes in stock prices, is defined by $\nu = \frac{\tilde{\nu}}{1-\tilde{\nu}}$. Central banks policy in the baseline setup is described by $\phi_\pi = 1.5$ and ϕ_s, the response in interest rate with respect to stock prices, is set to zero implying that the central bank does not target stock prices when setting the policy rate.

Estimation is done using the method of simulated moments (MSM, McFadden, 1989). Once the model is extend by a nonlinear process of speculative trading the use of standard Bayesian estimation is not feasible. Given the aim to compare estimation results of the models with and without speculation, it is advisable to use the same calibration techniques for both.[8] For this reasons Bayesian estimation is redirected to Appendix B.

The underlying intuition of MSM is to find the parameters that minimize a distance measure between simulated and empirical moments. Correspondingly to the generalized method of moments estimation it is possible to have more moments than estimated parameters by using a weighting matrix which corrects for the quality of the moment estimates. The weighting matrix is estimated using by the 2-step procedure. For MSM as well as for Bayesian estimation the data described in Section 2 is used and the simulated moments are retrieved from a batch of 100 simulated time series each of the length of the original data. For the RE model I consider an additional add-hoc exogenous shock on stock prices s_t to provide an equal number of degrees of freedom as the extended model in the next section. Thus,

$$v_t^s = \rho_s v_{t-1}^s + \varepsilon_t^s, \qquad \varepsilon_t^s \sim N(0, \sigma_s) \tag{31}$$

is added to Equation (24).

	ν	σ	ψ	ρ_x	σ_s	σ_π	σ_y
RE Estimation 1	-0.167	4.45	0.714	–	–	0.002	0.002
RE Estimation 2	0.083	–	–	0.790	0.018	0.001	0.004

Table 2: Parameter estimates of the RE model using the Method of Simulated Moments (MSM) and different sets of parameters. The first row allows for an iid. shock on asset prices.

Table 2 shows the parameter values estimated by MSM. For Estimation 1 the exogenous shocks on stock prices are excluded, i.e. set to zero whereas in Estimation 2 for σ and ψ the values from the calibration section above are used exogenous movements in stock prices are permitted.

A central result of the estimation is the positive value of ν that suggests that stock prices have a considerably strong impact on the price level via marginal costs. Since

[8]Bayesian estimation furthermore also targets higher order statistical moments while this work explicitly focusses on the first two moments of the data.

empirical fluctuations in stock prices are roughly ten times stronger than deviations in inflation, a ν of 0.083 implies that the respective impact is relatively large. This is very useful for the scope of this paper since it provides a strong motivation for policy intervention.

	RE Estimation 1		
	π	y	s
SD	0.011 (.001)	0.010 (.002)	0.044 (.006)
π	1	-0.180 (.179)	-0.930 (.024)
y	–	1	0.514 (.015)
s	–	–	1

	RE Estimation 2		
	π	y	s
SD	0.007 (.001)	0.012 (.002)	0.140 (.016)
π	1	-0.130 (.161)	-0.443 (.102)
y	–	1	0.613 (.118)
s	–	–	1

Table 3: Standard deviations and cross correlations of the estimated Rational Expectation model. Without exogenous noise in the stock market (Estimation 1) the model is unable to correctly capture the covariance between stock prices and inflation.

Table 3 shows the simulated moments for both of the estimations and reveals two key problems of the rational expectations based approach. First, it is hard to properly match the ratio of standard deviations. In the rational expectations model, stock prices are mainly driven by fluctuations in the interest rate, which in turn depends on deviations in inflation from the central banks target. Hence, severe fluctuations in inflation would be necessary to replicate the standard deviation of asset prices. Secondly and for the same reason, without speculation the correlation between inflation and stock prices is extremely high whereas the correlation between stock prices and output is driven by the link from stock prices to output.

5 Endogenous Fluctuations in Asset Prices

As argued in Section 1, bounded rationality is a natural candidate to explain the amplification of exogenous shocks. Accordingly, the assumption that stock market expectations are fully rational is dropped in this section and speculative behavior in the financial market is introduced. While it is also imaginable to let all markets be driven by boundedly rationality as for instance in De Grauwe (2011), I want to avoid that the economy is entirely driven by endogenous fluctuations but rather stay close to the macroeconomic benchmark case which relies on rational expectations. This helps to keep this research paper comparable and addresses the critique of the *wilderness of bounded rationality*. The fact that all markets other than the stock market are inhabited by rational agent also preserves the forward looking nature of the model that comes along with the rational expectations structure.

Let me denote the model consistent rational expectations on inflation and output by $E_t\pi_{t+1}$ and $E_t y_{t+1}$, i.e. by using the rational expectations operator. Let speculative expectations on stock market prices – which are yet undefined – be denoted by $\hat{E}_t s_{t+1}$.[9] Since this demands for a mechanism of how rational agents interact with the existence of agents that form non-rational beliefs, it is here assumed that the distribution of agent types is unobservable. Since aggregate shocks are unobservable, exogenous noise and fluctuations induced by speculation are indistinctable. Then, rational agents are ex-post unaware of the presence of non-rational agents.[10] Let $\mathbf{\tilde{v}_t}$ be the *perceived* exogenous shocks, which, as I will show, depend jointly on the real exogenous shocks and the degree of financial market speculation.

Write $E_t\left[\mathbf{x_{t+1}}|\mathbf{\tilde{v}_t}\right]$ to denote the rational expectations solution of (30) in terms of these perceived shocks. Then the dynamic system is implicitly defined by

$$\mathbf{M}\mathbf{x}_t = \mathbf{P}\begin{bmatrix} E_t\left[\pi_{t+1}|\mathbf{\tilde{v}_t}\right] \\ E_t\left[y_{t+1}|\mathbf{\tilde{v}_t}\right] \\ \hat{E}_t s_{t+1} \end{bmatrix} + \mathbf{v_t}, \tag{32}$$

where $\mathbf{v_t}$ again denotes the *actual* exogenous shocks v_t^π and v_t^y. I assume that rational agents are New-Keynesians and do not think that asset prices play a role, which is not only consistent with the vast majority of the literature but also an hypothesis that cannot be rejected by Bayesian estimation. The perceived law of motion for rational agents is

$$\begin{bmatrix} \mathbf{P} & \mathbf{0_{3\times 2}} \\ \mathbf{0_{2\times 3}} & \mathbf{I_{2\times 2}} \end{bmatrix} E_t \begin{bmatrix} \mathbf{x_{t+1}} \\ \mathbf{\tilde{v}}_{t+1} \end{bmatrix} = \begin{bmatrix} \mathbf{M} & \mathbf{0_{3\times 2}} \\ \mathbf{0_{2\times 3}} & \boldsymbol{\rho} \end{bmatrix} \begin{bmatrix} \mathbf{x_t} \\ \mathbf{\tilde{v}}_t \end{bmatrix}, \tag{33}$$

where $\boldsymbol{\rho}$ is a diagonal matrix containing the autocorrelation parameters and $\mathbf{x_t}$ the vector of endogenous variables at t. This is a different way to express system (30) with perceived exogenous shocks instead of the real exogenous shocks. The rational expectations solution of this system is derived in Appendix C. Let this (linear) solution be denoted by the matrix $\mathbf{\Omega}$. It needs to hold by definition that

$$\begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = \mathbf{\Omega}\begin{bmatrix} \tilde{v}_t^\pi \\ \tilde{v}_t^y \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} E_t[\pi_{t+1}|\mathbf{\tilde{v}_t}] \\ E_t[y_{t+1}|\mathbf{\tilde{v}_t}] \end{bmatrix} = \mathbf{\Omega}\boldsymbol{\rho}\begin{bmatrix} \tilde{v}_t^\pi \\ \tilde{v}_t^y \end{bmatrix}, \tag{34}$$

and it follows directly that we can express the conditional expectations on inflation and output without explicitly solving for the perceived shocks $\mathbf{\tilde{v}_t}$ by

$$\begin{bmatrix} E_t[\pi_{t+1}|\mathbf{\tilde{v}_t}] \\ E_t[y_{t+1}|\mathbf{\tilde{v}_t}] \end{bmatrix} = \mathbf{\Omega}\boldsymbol{\rho}\boldsymbol{\tilde{v}_t} = \mathbf{\Omega}\boldsymbol{\rho}\mathbf{\Omega^{-1}}\begin{bmatrix} \pi_t \\ y_t \end{bmatrix}. \tag{35}$$

Plugging this result back into Equation (32) and rearranging in terms of the actual exogenous states and speculative expectations obtains the actual law of motion. The

[9] Technically, $\hat{E}_t$ is an implicit function rather than a mathematical operator.

[10] Boehl (2017) show that in a system where fully rational and boundedly rational agents coexist, the type of dynamics is even more volatile than systems where rational agents are unaware of the presence of boundedly rational agents.

latter can then be expressed as a mapping $\mathbf{\Psi} : (\boldsymbol{\rho}, \mathbf{\Phi}, \boldsymbol{\phi}) \to \mathbb{R}_{3\times 3}$, where $\mathbf{\Phi}$ is the set of model parameters $(\beta, \sigma, \nu, \eta, \kappa)$ and $\boldsymbol{\phi}$ the two policy parameters. Hence,

$$\begin{bmatrix} \pi_t \\ y_t \\ s_t \end{bmatrix} = \mathbf{\Psi} \begin{bmatrix} v_t^\pi \\ v_t^y \\ \hat{E}_t s_{t+1} \end{bmatrix}, \tag{36}$$

representing a solution for the rational expectations equilibrium in terms of the real shock terms with one degree of freedom, which is used for boundedly rational beliefs $\hat{E}_t s_{t+1}$. Note that this actual law of motion, by definition, is not known to any of the agents.

5.1 Theoretical insights

Which policy implications can be deducted from this model without further specification of an expectation formation mechanism? In the absence of real shocks the law-of-motion in (34) can be reduced to

$$\mathbf{x}_t = \mathbf{\Psi}_{:,3} \hat{E}_t s_{t+1} \qquad \text{and in particular} \qquad s_t = \mathbf{\Psi}_{3,3} \hat{E}_t s_{t+1}.^{11} \tag{38}$$

Consider the calibration from Table 2 and for now disregard any exogenous shocks in order to focus on the economic intuition behind the impact of a one-percent deviation in stock price expectations. Given this calibration $\mathbf{\Psi}_{3,3} \approx 1$ is very close to a unit-root, which entails the threat of explosive expectations feedbacks in the financial market. Learning-to-forecast experiments and theoretical evidence[12] have shown that systems with positive feedback, especially when close to unit roots, can exhibit large swings and bubbles. Following this line of argument, $\mathbf{\Psi}_{3,3}$ represents a key measure for the probability of excess volatility on the stock market. Hence, when stabilizing such system it should *ceterus paribus* be the policy makers' aim to minimize $\mathbf{\Psi}_{3,3}$. Likewise, the second best solution would be to minimize $\mathbf{\Psi}_{1,3}$ and $\mathbf{\Psi}_{2,3}$ and thereby reducing the impact of stock prices on real activity.

Figure 1 shows $\mathbf{\Psi}_{:,3}$ as a function of the feedback coefficient to asset prices in the Taylor rule. The plot can be interpreted as the general equilibrium response to an one-percent increase in stock price expectations for different values of ϕ_s. Even for moderate values of ϕ_s the response varies quite drastically, with values of $\mathbf{\Psi}_{3,3} > 1$ for $\phi_s > \lambda_s \approx -0.01$ but monotonically decreasing whenever the central bank *leans against the wind*. The clear message is that if the central bank reacts moderately to stock prices, the positive feedback loop can be mitigated.

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[11]The model is implemented in `Python`. I want to empathize the excellence of contemporary free and open source software – also and especially in comparison with proprietary software – and want to encourage the reproducibility of research. In the rest of this paper I use *Python-like* notation when referring to certain parts of matrices. For a matrix

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \tag{38}$$

I use $\mathbf{A}_{2:3,1:2}$ to denote the lower-left square matrix (row 2 to 3, column 1 to 2) $\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$ or $\mathbf{A}_{3,:}$ to denote the vector in the third row of $\mathbf{A}$ given by $\begin{bmatrix} a_{31} & a_{32} & a_{33} \end{bmatrix}$.

[12]For a review on laboratory experiments on expectation formation, see Hommes (2011).

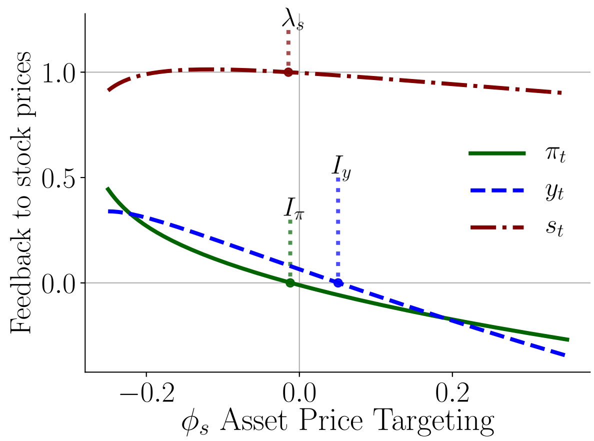

Figure 1: General equilibrium effect of a 1% change in asset price expectations as a function of central bank policy parameters. Responses of output, inflation and asset prices in deviations from steady state. $\mathbf{\Psi_{:,3}}$ as a function of ϕ_s

Two other interesting points deserve attention. At $I_y \approx 0.05$ the impact of speculation on output is exactly offset whereas for higher values of ϕ_s a positive shock on asset prices actually leads to a decrease in output. While the central bank seeks to raise the interest rate with an increase in stock prices, the fall in inflation triggers a decrease in the rate. These two effects balance perfectly in I_y i.e. the net change in interest rates is zero. Since in this model, output dynamics mainly reflect intertemporal substitution effects, output will not deviate from its steady state level. A similar point, although for a negative value of ϕ_s, is $I_\pi \approx -.01$ where the inflation rate is completely unaffected by the immediate impact of a deviation in stock price expectations. Here the effect of *decreasing* rates on marginal costs is perfectly compensated by the effect of an increase in output on wages, hence the net change is zero. Through the Phillips curve, inflation remains at its steady state level. We furthermore learn that fluctuations in asset price expectations can contribute to a high ratio of standard deviations of stock prices and output, σ_s/σ_y. While a shock on expectations would rise stock prices almost by the same magnitude, responses in output and inflation are notable but small.

This result implies that the central bank faces a trade-off. Policy makers can reduce the impact of speculation on either π_t or y_t, but accept the extreme dynamic feedback induced by a high $\mathbf{\Psi_{3,3}}$. Or, they can choose to potentially stabilize the system but increase the impact of stock market expectations on real variables notably. The model further suggests that any policy that *decreases* the interest rate in response to stock price booms might further destabilize both excess volatility and the sensitivity of real aggregates to stock prices. In order to quantitatively asses this trade-off it is necessary to analyse the explicit dynamics under speculation and to provide numerical results for the central bank policy, hence to implement a mechanism on how asset traders form expectations.

5.2 Heterogeneous beliefs and estimation

Assume that traders follow the *Heterogeneous Agent Switching Model* (Brock and Hommes, 1998) and switch endogenously between simple forecasting heuristics. Since beliefs feed back on prices, boundedly rational traders are able to outperform others that believe that the price will return to the rational expectations equilibrium.[13] This model provides a set of empirically relevant properties, most notably a positive correlation between returns and expected returns, and fat tails of the distribution of asset prices. For further discussion of the model dynamics and empirical validation see Hommes (2006).

Traders are heterogeneous in their forecasting rules. Let there be $H > 1$ predictors of future prices and let each predictor $h = 1, 2, \ldots, H$ be of the form $\hat{E}_{t,h}s_{t+1} = g_h s_{t-1} + b_h$. Aggregating over each individual optimality condition (Equation 19) yields the economy wide price for shares S_t.[14] Let $n_{t,h}$ denote the fraction of traders using predictor h at time t, then

$$R_{t+1}S_t = E_t\Theta_{t+1} + \sum_h n_{t,h}\hat{E}_{t,h}S_{t+1}. \tag{40}$$

Let me further assume that traders take the real interest rate r_{t+1} as given.[15] Log-linearization yields

$$s_t = \beta\hat{E}_t s_{t+1} - r_{t+1} \qquad \text{with} \qquad \hat{E}_t s_{t+1} = \sum_h n_h \hat{E}_{h,t}s_{t+1}. \tag{41}$$

Not surprisingly, the first part here is identical to Equation (20) but the second part incorporates the speculative expectations $\hat{E}_t s_{t+1}$. Note that r_{t+1} in turn depends on y_t and π_t, so this equation yet takes the general equilibrium effect of changes in stock market prices into account. Fractions $n_{h,t}$ are updated according to a *performance measure* $U_{h,t}$ of predictor h in period t. As such *realized past profits* are considered, as in

$$U_{h,t} = (\beta s_t - s_{t-1})(\beta\hat{E}_{t-1,h}s_t - s_{t-1}). \tag{42}$$

The choice of the performance measure is an essential ingredient to the model of speculation dynamics, the determining nonlinear properties of the system. Realized profits from trading qualify in several ways for our purpose. As outlined in Section 1, the fundamental difference between macroeconomic real markets and stock markets is that participants can make profits from speculation. Instead of being rewarded for an accurate estimate of the price, it is sufficient to forecast the direction of a price change correctly, hence to decide whether to go short or long. Likewise, a trader A that has a high forecast of next periods' prices will invest more money in the asset than some trader

[13] Cf. footnote 10.

[14] I assume that trader k's demand for shares is a linear function of expected profits with some τ

$$J_{k,t} = \tau\hat{E}_{k,t}\{\Theta_{t+1} + S_{t+1} - R_t S_t\} \tag{39}$$

with $\int_0^1 j_t^k dk = 0$ when expressed as log-deviations from steady state.

[15] Including the nominal interest rate $R_t \frac{P_t}{P_{t+1}}$ in the performance measure does not change the dynamics in a fundamental way, but leads to a slight asymmetry of bifurcations. Then it can not be guaranteed anymore that the mean of the time series of *perceived* shocks equals zero. This, however, is a necessary requirement when solving for rational expectations.

B with a relatively lower forecast. If it then turns out that B was correct in terms of point estimates, A will still realize higher profits since he invested more. This feature is captured by Equation (42).

The probability that predictor h is chosen is given by the *multinomial discrete choice model*

$$n_{h,t} = \frac{e^{U_{h,t-1}}}{Z_{t-1}} \qquad \text{and} \qquad Z_{t-1} = \sum_{h=1}^{H} e^{U_{h,t-1}}. \tag{43}$$

Consider a simple 3-type model where one type of agents are fundamentalists (i.e. traders that take only economic fundamentals into account) and the other two share a trend-following parameter γ and are either negatively or positively biased by α as in

$$\begin{aligned} \hat{E}_{t,1}s_{t+1} &= 0, \\ \hat{E}_{t,2}s_{t+1} &= \gamma s_{t-1} + \alpha, \\ \hat{E}_{t,3}s_{t+1} &= \gamma s_{t-1} - \alpha. \end{aligned} \tag{44}$$

Now that expectation formation mechanisms for both types of agents are given, the model is fully specified. It consists of a linear part associated with the economy and the formation of rational expectations, represented by Equation (34), and a nonlinear mechanism of boundedly rational expectation formation given by $\hat{E}_t s_{t+1}$, the performance measure $U_{h,t}$ (Equations 41 and 42), the fractions $n_{h,t}$ and the normalization factor (Equation 43), and the predictors (Equation 44).

ν	γ	α	σ_π	σ_y
0.090	1.006	1.229	0.001	0.004

Table 4: MSM Parameter estimates for the model with endogenous dynamics in stock prices

As in Section 4 MSM is used to estimate the set of parameters for the boundedly rational expectation formation mechanism, $\{\gamma, \alpha\}$, together with the standard deviations of shocks (σ_y and σ_π) and the elasticity to stock prices ν. A necessary condition to obtain reliable estimates is that the moment function is continuous in the relevant parameter space. As shown in Appendix D, for the range of $\alpha \in [1.224, 1.357]$ and for $\gamma < 1.2$ the dynamics describe limit cycles with amplitude monotonously increasing in α (γ respectively), hence here the condition of continuity is satisfied. For higher values, dynamics become complicated and, accordingly, the moment function would be discontinuous. However, e.g. an α larger than 1.35 would imply that the standard deviation of output is almost entirely driven by endogenous deterministic fluctuations in asset prices. Since this is not a realistic feature, the discontinuity problem for high values of α and γ does not impose a real problem to the credibility of the estimates. Estimated parameter values are summarized in Table 4 whereas the simulated moments can be found in Table 5. Apart from the correlation between output and inflation, all moment estimates are robust and close to the original moments of the data.

The estimate of ν is in line with the corresponding value of RE Estimation 2, which assumes an exogenous component in stock prices. This, contrasting RE Estimation 1,

	π	y	s
SD	0.006 (.001)	0.013 (.002)	0.128 (.039)
π	1	-0.125 (.170)	-0.345 (.090)
y	–	1	0.635 (.140)
s	–	–	1

Table 5: Standard deviations and cross correlation matrix for simulations of the model with endogenous dynamics in stock prices.

suggests that the magnitude of the link is high if stock prices are by themselves a source of fluctuations. A ν of this magnitude furthermore provides a meaningful explanation of the estimates of Assenmacher and Gerlach (2008) and Miao et al. (2012) concerning the degree to which stock price fluctuations explain the variance of GDP. The estimate of γ is almost unity which is well in line with Hommes (2013), implying that traders do extrapolate past trends to a high degree. However, the parameter is only marginally larger than one which satisfies that trend extrapolation itself is not a source of explosive dynamics.[16] Given the high empirical volatility of stock prices, the behavioral bias of $\alpha = 1.229$ can be seen as moderate. Furthermore productivity shocks are less pronounced than preference shocks, which has to be understood in the context that the autocorrelation of productivity shocks with 0.9 is considerably closer to a unit root than the autocorrelation of preference shocks with just 0.7. This result suggests that both, endogenous amplification of exogenous shocks as well as a positive ν are necessary to explain the empirical evidence well.

The endogenous component – the introduction of speculation – of stock markets also naturally reduces the correlation between stock prices and inflation while the high relative standard deviation of stock prices is explained by the matter that fluctuations in expectations in stock prices reflect strongly (almost one-to-one) on stock prices, but the spillover on inflation and output is modest in comparison. Further, an increase in stock prices dampens inflation trough the marginal cost channel and the central bank lowers the interest rate which in turn stimulates demand. This ensures that the correlation between output and stock prices is positive and relatively strong, which would not be the case in a model without a feedback from stock prices to real activity. To summarize the findings from the estimation procedure, the property of excess volatility in combination with a mutual linkage between stocks and real activity is crucial to replicate key-moments of the data.

5.3 Deterministic Simulations

The above nonlinear model of speculation embeds fluctuations (and spillovers) that are not driven by exogenous shocks but which are completely endogenous. In general this type of models can cover a wide range of dynamics, from limit cycles to strange attractors or chaotic behavior, and for different parameters different steady states can (co-) exist and (inter-) change stability. To study the associated macroeconomic dynamics – to identify relevant types and quantitative properties – bifurcation theory is used. For this purpose exogenous (stochastic) shocks are set to zero and from a nonzero initial value

[16]Fixing γ to a value in the integral between .95 and 1 does in fact not reduce the goodness of fit.

11.000 iterations are run for each value in the parameter space, of which a transition phase of 10.000 periods is omitted in the analysis. The result can be summarized by a so-called bifurcation diagram which depicts the long-run dynamics as a function of the policy parameter.[17]

Details and in-depth analysis with respect to the behavioral parameters $\{\alpha, \gamma\}$ are redirected to Appendix D. Qualitatively, that is, in terms of the types of dynamics, a change in each of the behavioral parameters has similar effects. For low values of α (γ) the zero-steady-state is stable and unique. Modest values up to 1.357 (1.219) induce limit cycles whereas higher values first produce more complicated dynamics (so-called *homoclinic orbits*) and then explosive behavior.

Let me focus on changes in the dynamics with respect to the policy parameter ϕ_s, which are shown in Figure 2. For the values proposed by the simulated moments estimation the deterministic system entails limit cycles to a very moderate extend. Comparing these fluctuations to the standard deviations of the data, this suggests that the data is not driven by endogenous dynamics completely, but that speculative agents react sensitively to exogenous shocks and induce excess volatility. Agents might observe change in stock prices that is triggered triggered by a series of exogenous shocks and extrapolate it. Since beliefs are partly self-fulfilling prices might rise even though fundamentals return to the steady state. Caused by another exogenous shock, the bubble might burst unexpectedly while preserving the mean-reverting property of a rational expectations model.

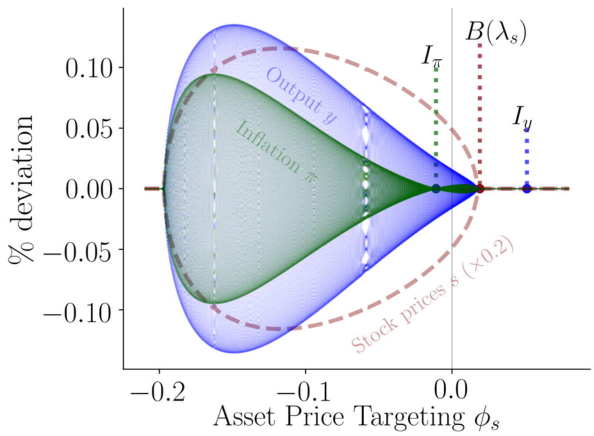

Figure 2: Bifurcation diagram of y_t (blue/light) and π_t (green/dark) with respect to ϕ_s. Deterministic dynamics for ϕ_s. All parameters as in Table 4. The diagram shows the long-run endogenous dynamics as a function of the parameter space.

As shown in Subsection 5.1, an increase in ϕ_s is able to mitigate the impact of stock prices on output and decreases the positive feedback within the speculative process. The

[17]As a technical note, it is clear that the first-order Taylor approximation around the steady state, that is embedded in the baseline DSGE model, will not hold once stock market prices deviate too much from their steady state value. I accept this inconvenience for the sake of simplicity and argue that introducing further nonlinearities would further complicate the interpretation of results while the added value of such undertaking would be unclear.

bifurcation diagram confirms that an increase in ϕ_s dampens the speculative dynamics. Since the dynamic process for s_t depends crucially on $\mathbf{\Psi_{3,3}}$, the amplitude decreases with the magnitude of policy ϕ_s and the graph exhibits an inverse *supercritical Hopf-Bifurcation* in $B(\lambda_s)$, after which the steady state is unique and stable.[18] If the central bank raises the interest rate when stock prices increment, this counteracts the effect falling interest rates and dampens the positive feedback of stock price expectation to stock prices. This is the same general equilibrium effect that is captured by $\mathbf{\Psi_{3,3}}$. The weaker the feedback between expectations and prices, to a lesser extend are speculative dynamics self-fulfilling. Given the values suggested by the estimation deterministic cycles can be completely switched off at $B(\lambda_s)$ already for very moderate values of ϕ_s. The diagram further confirms that at point I_π from Figure 1 the direct impact of speculation on inflation is offset completely.

5.4 Stochastic Simulations

Let me now turn to stochastic simulations on the policy parameter ϕ_s. In Figure 3 standard deviations of simulations are shown where the exogenous shocks v^π and v^y are added. The dynamics emerge as a combination of the iid. noise and endogenous responses of financial market speculation to these shocks. The results from the deterministic case still hold.

a) An increase of ϕ_s reduces stock market volatility,
b) an increase of ϕ_s up to $y_{min} \approx 0.046 \approx I_y$ diminishes the fluctuations in output, but increases volatility of inflation, and
c) an increase of ϕ_s to a value higher than y_{min} leads to additional fluctuations in both, inflation and output.

The *collateral damage* effect in (b) and (c) stems from the fact that an asset pricing targeting policy also reacts to movements in stock prices that are not induced by speculation but by the stochastic process. For a productivity shock this means that when inflation increases, the Taylor rule mechanically increases the interest rate. This in turn will deflate stock prices. But if the central bank also targets stock prices, the equilibrium rate will be lower than the optimal response to the productivity shock, inducing unnecessarily strong responses in inflation and output. Since the response to a demand shock works similarly, in an economy in which stock prices are not a source of fluctuations asset price targeting will always increase volatility in real aggregates. The indicated effect runs in the opposite direction of the stabilizing effect of asset price targeting on speculative dynamics and the respective reduction of spillovers to inflation and output. This suggests that the optimal sensitivity of monetary policy to asset prices is bounded by $\pi_{min} \approx -0.01 = I_\pi$ and I_y.

The same model setup can also be used to analyze the policy implications given by the literature on rational bubbles, as proposed most prominently by Galí (2013). Since a rational bubble would presumably grow proportional to the interest rate, he suggests to actually lower the policy rate when facing asset price bubbles. If such policy is non-discretionary, in the model presented here it would lead to a considerable increase in

[18] A bifurcation generally occurs when one ore more eigenvalues of the *nonlinear* system crosses the unit circle.

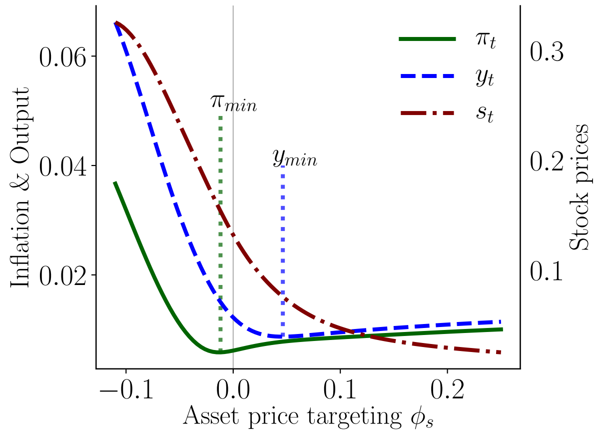

Figure 3: Standard deviations of stochastic simulations for given the asset price targeting policy parameter (ϕ_s) and the model with endogenous dynamics in stock prices. The economy is driven by exogenous shocks, but the nonlinear process in the stock market leads to endogenous amplification.

output and stock price volatility and, even for small values of the policy's sensitivity, would amplify fluctuations in stock prices.

Decision makers of monetary policy face a trade-off that can be summarized by *"fragility versus volatility"*. The model presented does not motivate the inclusion of asset prices themselves into welfare considerations, relevant for the welfare of households is only the impact of stock prices on output and inflation. Every welfare or central bank loss function that embeds a relative ranking of alternative central bank policies can be expressed as a convex combination of these two variances. For values of ϕ_s higher than y_{min} both the variance of output and inflation are monotonously increasing in ϕ_s. It can therefore be concluded that only a very moderate asset price targeting has the potential to increase dynamic stability, decrease volatility and to mitigate the coordination failure induced by speculation in the financial market. A concrete advice for policy however depends on how much a central bank weights fluctuations of output when conducting monetary policy.

6 Conclusion

This work emphasizes that a causal feedback between stock prices and real activity in combination with speculation in the stock market can help to replicate key-moments of the European data on inflation, output and stock prices. In particular the covariances between stock prices and inflation and stocks and output, as well as the relatively high standard deviation of stock prices are well explained by financial market speculation.

Arguing that financial markets entail a list of idiosyncrasies that are not well captured by rational expectations, I introduce speculative behavior in the stock market. Given a small number of consistency assumptions this work shows that it is possible to find the rational expectation solution on all other markets even though expectation formation in the asset market is of a boundedly rational type. Simulation results suggest that any kind of speculation or herding, induced by bounded rationality and/or speculative profits, destabilizes the economy. Depending on the parametrisation, financial market interactions can lead to large and persistent booms and recessions. I thus find that instability is an inherent threat to economies with speculative financial markets.

I show theoretically that the central bank's interest rate setting can amplify the expectation feedback in the financial market, and that this can lead to unstable dynamics and excess volatility. The method of simulated moments identifies the link from asset prices to real aggregates to be small but of macroeconomic significance, implying a potential role for macroprudential policy to mitigate the negative externalities running though this mechanism.

If stock prices impact macroeconomic aggregates, a monetary policy rule that also targets asset prices can mitigate the excess volatility of stock prices but at the cost of intensifying real shocks. The model however suggests that such policy is bounded very narrowly by the unwanted side effects ("collateral damage") of asset price targeting. This result does not only hold for a policy that raises the interest rate when facing stock price bubbles but also for a policy that lowers the interest rate. The latter can easily amplify the speculative process and destabilize the economy furthermore.

Apart from the theoretical results provided in this paper, policy institutions may be well-advised to handle tools like asset price targeting with care since such instruments might add a structural link between asset prices and macroeconomic aggregates. Such additional link embeds the risk of other unforeseeable complications, independently of how tight the actual link from asset prices to real activity is. This is particularly true because stock prices impact solely through signaling effects. An artificial inflation of stock prices in recessions might hence be counterproductive. This however leaves room for other macroprudential policies that potentially restricts the degree of speculation or reduces profits from speculation in financial markets (i.e. policies such as short-selling constraints or leverage requirements) and indicates that such policy would contribute to overall economic stability. This work also suggests that neither stock prices nor indices on stock prices are a good indicator to base decisions on (e.g. credibility, evaluation of competitors). Practitioners should be aware that, regarding stock prices and real activity, causality might run in both directions.

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Appendix A Entrepreneurs' optimization problem

This section follows Bernanke et al. (1999) closely, but instead of assuming risk in the productivity of capital I assume idiosyncratic risk in labor productivity. Firm j's ex post gross return on one unit of labor, ω_j, is i.i.d. across time with a continuous and once-differentiable CDF $F(\omega)$ over a non-negative support and with an expected value of 1. I assume that the hazard rate $h(\omega) = \frac{dF(\omega)}{1-F(\omega)}$ is restricted to $h(\omega) = \frac{\partial(\omega h(\omega))}{\partial\omega} > 0$. The optimal loan contract between firms and financial intermediaries is then defined by a gross non-default loan rate, $Z_{j,t+1}$, and a threshold value $\bar{\omega}_{j,t}$ on the idiosyncratic shock $\omega_{j,t}$. For values of the idiosyncratic shock greater or equal than $\omega_{j,t}$ the entrepreneur will be able to repay the loan, otherwise he will default. $\bar{\omega}_{j,t}$ is then defined by

$$\bar{\omega}_{j,t} R^H_{t+1} H_{j,t} = Z_{j,t+1} B_{j,t}.$$

Dropping firms' subscripts, as in Bernanke et al. (1999) the optimal contract loan contract must then satisfy

$$\left\{ [1 - F(\bar{\omega}_t)]\, \bar{\omega}_t + (1-\mu) \int_0^{\bar{\omega}_t} \omega dF(\omega) \right\} H_t/X_t = R_{t+1}(W_t H_t - N_t),$$

and the expected return to the wholesaler is (dropping time-subscript of ω_t for better readability)

$$E\left\{ \int_{\bar{\omega}}^{\infty} \omega dF(\omega) - (1 - F(\bar{\omega}))\bar{\omega} \right\} H_t/X_t.$$

Given constant returns to scale, the cutoff $\bar{\omega}$ determines the division of expected gross profits H_t/X_t between borrower and lender. Let me define

$$\mathfrak{F}(\bar{\omega}) = \int_0^{\bar{\omega}} \omega f(\omega) d\omega - \bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega) d\omega$$

to be the expected gross share of profits going to the lender with $\mathfrak{F}'(\bar{\omega}) = 1 - F(\bar{\omega})$ and $\mathfrak{F}''(\bar{\omega}) = -f(\bar{\omega})$. This implies strict concavity in the cutoff value. I define similarly the expected monitoring costs as

$$\mu\mathfrak{G}(\bar{\omega}) = \mu \int_0^{\bar{\omega}} f(\omega) d\omega,$$

with $\mu\mathfrak{G}'(\bar{\omega}) = \mu\omega f(\omega)$. See BGG for the proof that the following result is a non-rationing outcome. The resellers problem of choosing the optimal equity can be solved by maximizing discounted profits over equity, or maximizing return on investment and including investment as part of the optimization problem.[19] Thus,

$$\max_{\{H_t\},\{\bar{\omega}_t\},\{N_t\},\{\lambda_t\}} E_t \sum_{s=t}^{\infty} N_t^{-1} \prod_{l=t}^{s} R_{l+1}^{-1} \left[(1 - \mathfrak{F}(\bar{\omega}_s)) H_s/X_s - N_{s+1} \right] \\ -\lambda_s \left(\left[\mathfrak{F}(\bar{\omega}_t) - \mu \mathfrak{G}(\bar{\omega}) \right] H_t/X_t - R_{t+1}(W_t H_t - N_t) \right). \tag{A.1}$$

The first-order conditions for this problem can be written as

$$\begin{aligned}
&H : (1 - \mathfrak{F}(\bar{\omega}_t)) (X_t N_t R_{t+1})^{-1} - \lambda_t \left(\left[\mathfrak{F}(\bar{\omega}_t) - \mu \mathfrak{G}(\bar{\omega}) \right] / X_t - R_{t+1} W_t \right) &&= 0 \\
&\bar{\omega} : \mathfrak{F}'(\bar{\omega}_s)(N_t R_{t+1})^{-1} - \lambda_t \left[\mathfrak{F}'(\bar{\omega}_t) - \mu \mathfrak{G}'(\bar{\omega}) \right] &&= 0 \\
&N : -\frac{S_t}{R_{t+1} N_t^2} - R_{t+1} \lambda_t &&= 0 \\
&\lambda : \left[\mathfrak{F}(\bar{\omega}_t) - \mu \mathfrak{G}(\bar{\omega}) \right] H_t/X_t - R_{t+1}(W_t H_t - N_t) &&= 0
\end{aligned}$$

Combining the first three conditions implies a connection between the optimal choice of labor, prices and stock prices. Using the optimality condition for the cutoff value $\bar{\omega}_t$ and rearranging yields

$$\frac{\mathfrak{F}'(\bar{\omega}_t)}{\mathfrak{F}'(\bar{\omega}_t) - \mu \mathfrak{G}'(\bar{\omega}_t)} = \frac{S_t}{R_{t+1} N_t}$$

where the LHS can be written as a function $\rho(\bar{\omega})$. BGG show that under reasonable assumptions $\rho(\bar{\omega})$ is a mapping from $\bar{\omega}$ to $\mathbb{R}^+$. The inverse of $\rho(\cdot)$ can be used to establish that the premium payed on external funds depends on the return payed on internal funds. As noted in the main body, this is intuitive since the marginal costs of external and internal finance need to be equal. Likewise the risk premium on external funds can be defined to be a function of the leverage ratio (if $N_t = W_t H_t$, the premium is obviously one), which establishes the relationship in the main body.

Appendix B Bayesian estimation

Similar to the MSM estimation in the main body, here an exogenous shock on stock prices v_s is considered which follows an AR(1) structure and hits the economy in the zero profit condition. Note that such a third shock is also necessary to avoid stochastic indeterminacy. For the Bayesian estimation I make use of the standard routines implemented in `Dynare` (Adjemian et al., 2011).

Priors and the result of the estimation can be found in Table B.6. As in Section 4, most priors are taken from Smets and Wouters (2003) while the value of the prior for $\psi = 0.3$ is consistent with the calibration in BGG. In order to remain agnostic about the value of ν, its prior is chosen quite broadly with a mean equal zero, while this value is set to 0.5 in BGG.

The model's key parameter ν, the marginal costs' elasticity to stock prices, is again estimated to be relatively small but positive. It however is not well identified with a

[19] In fact the linkage between stock and wholesale prices would be clearer if I would allow wholesalers to have some monopolistic power and hence scope to adjust prices in response to pressure from the financial market.

	Prior distribution			Posterior distribution		
	Type	Mean	St. error	Mean	5%	95%
σ_π	Uniform	[0,1]	–	0.0016	0.0013	0.0019
σ_y	Uniform	[0,1]	–	0.0054	0.0031	0.0086
σ_s	Uniform	[0,1]	–	0.0348	0.0179	0.0510
ρ_s	Beta	0.85	0.1	0.6141	0.4599	0.7686
σ	Normal	1	0.35	1.0245	0.4629	1.5763
ψ	Normal	0.3	0.2	0.2897	-0.0654	0.6103
ω	Beta	0.66	0.05	0.6604	0.5687	0.7449
$\tilde{\nu}$	Normal	0.0	1	0.0641	-1.3625	1.6922
	Log data density:			1315		

Table B.6: Priors and parameter estimates of the Bayesian estimation

quite large standard deviation of the posterior, reflecting the value of its prior. Note also that the Bayesian estimation targets more moments than the MSM estimation, which is challenging with a relatively small model. Comparing these estimations to the results from the estimation using MSM, most parameter estimates are confirmed. A strong exogenous component in stock price fluctuations as estimated here lacks economic intuition. With exogenous fluctuations in asset prices the vast majority of the asset price dynamics are due to the stock price shock. This does not link well to explanations frequently found in the literature, including news shocks, and lacks economic intuition. While news shocks could explain exogenous movements to some extend, the fact that stock prices appear to be driven by highly persistent news shocks in combination with interest rate setting does not seem likely.

Appendix C Solving for the rational expectations equilibrium

The System in 30 reads as

$$\underbrace{\begin{bmatrix}1-\phi_\pi\kappa & -\kappa\eta & \kappa\nu \\ \phi_\pi\sigma^{-1} & 1 & \phi_s\sigma^{-1} \\ \phi_\pi & 0 & 1+\phi_s\end{bmatrix}}_{\mathbf{M}}\underbrace{\begin{bmatrix}\pi_t \\ y_t \\ s_t\end{bmatrix}}_{\mathbf{x}_t} = \underbrace{\begin{bmatrix}\beta-\kappa & 0 & 0 \\ \sigma^{-1} & 1 & 0 \\ 1 & 1-\beta & \beta\end{bmatrix}}_{\mathbf{P}}\underbrace{\begin{bmatrix}E_t\pi_{t+1} \\ E_t y_{t+1} \\ E_t s_{t+1}\end{bmatrix}}_{E_t\mathbf{x}_{t+1}} + \underbrace{\begin{bmatrix}v_t^\pi \\ v_t^y \\ 0\end{bmatrix}}_{\boldsymbol{v}_t}. \tag{C.1}$$

For a sensible range of parameter values the Blanchard-Kahn-Conditions are satisfied. The closest bound for which eigenvalues cross the unit circle is if $\phi_s < -0.305$. Exchanging shocks $\mathbf{v}_t$ by perceived shocks $\tilde{\mathbf{v}}_t$, in expectations it has to hold that

$$E_t\tilde{v}_{t+1}^\pi = \rho_\pi\tilde{v}_t^\pi \tag{C.2}$$
$$E_t\tilde{v}_{t+1}^y = \rho_y\tilde{v}_t^y. \tag{C.3}$$

Using this form, the PLM can be written by using the system of equations (22) – (26) and by bringing all expectations to the LHS:

$$\beta E_t \pi_{t+1} = \pi_t - \kappa x_t - \tilde{v}_t^{\pi} \tag{C.4}$$

$$E_t y_{t+1} = \sigma^{-1} r_{t+1} + y_t - \tilde{v}_t^{y} \tag{C.5}$$

$$\beta E_t s_{t+1} = s_t + r_{t+1} \tag{C.6}$$

$$E_t \pi_{t+1} = x_t + \eta y_t + i_t - \nu s_t \tag{C.7}$$

$$0 = -i_t + \phi_\pi \pi_t + \phi_s s_t \tag{C.8}$$

$$E_t \tilde{v}_{t+1}^{\pi} = \rho_\pi \tilde{v}_t^{\pi} \tag{C.9}$$

$$E_t \tilde{v}_{t+1}^{y} = \rho_y \tilde{v}_t^{y} \tag{C.10}$$

Using (C.7) and (C.8) to substitute out i_t and x_t and rewriting as a matrix yields the System (33). Rewrite this system as

$$\mathbf{\tilde{P}} E_t \mathbf{\tilde{x}_{t+1}} = \mathbf{M} \mathbf{\tilde{x}_t}.$$

$\mathbf{\tilde{N}} = \mathbf{\tilde{M}^{-1}} \mathbf{\tilde{P}}$ is the 5×5 matrix which summarizes the dynamics of the perceived law of motion of rational agents. I use eigenvector/eigenvalue decomposition to obtain $\mathbf{\Gamma \Lambda \Gamma^{-1}} = \mathbf{\tilde{N}^{-1}}$, where $\mathbf{\Lambda}$ is the diagonal matrix $\text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_5)$ of the eigenvalues of $\mathbf{\tilde{N}^{-1}}$ ordered by size (smallest in modulus first) and $\mathbf{\Gamma}$ the associated eigenvectors, columns ordered in the same fashion. The expectation system can then be rewritten as

$$\mathbf{\Gamma^{-1}} E_t \mathbf{x_{t+1}} = \mathbf{\Lambda \Gamma^{-1}} \mathbf{x_t}.$$

Denote the sub-matrix of $\mathbf{\Lambda}$ that only contains unstable eigenvalues as $\mathbf{\Lambda_u}$, and the associated eigenvectors as $\mathbf{\Gamma_u}^{-1}$. In order to be consistent with the transversallity condition it must hold that $\mathbf{\Gamma_u^{-1}} E_t \mathbf{x_{t+1}} = \mathbf{0}$. Using this fact I can solve for $E_t \mathbf{x_{t+1}}$ by

$$E_t \mathbf{x_{t+1}} = \mathbf{\Gamma_{u,1:3}^{-1} \Gamma_{u,4:5}} E_t \mathbf{\tilde{v}_{t+1}} = \mathbf{\Gamma_{u,1:3}^{-1} \Gamma_{u,4:5}} \boldsymbol{\rho} \mathbf{\tilde{v}_t}.$$

Note that the requirement that $\mathbf{\Gamma_{u,1:3}}$ is invertible implies the Kuhn-Tucker condition, imposing that $\mathbf{\Gamma_{u,1:3}}$ is a square matrix with full rank. This means that the number of forward looking variables has to equal the number of unstable eigenvalues $\lambda > 1$ of $\mathbf{\tilde{N}^{-1}}$. Let me define $\mathbf{\bar{\Omega}} = \mathbf{\Gamma_{u,1:3}^{-1} \Gamma_{u,4:5}}$. The solution from the main body is then $\mathbf{\bar{\Omega}_{1:2,1:2}}$.[20]

Appendix D Bifurcation analysis

The bifurcation diagram for α is shown in Figure D.4 and depicts the system's long-run dynamics as a function of the parameter. Generally, an increase in the behavioral bias α implies two effects. First the *quantitative* aspects of the dynamics change, i.e. the standard deviation increases. Secondly, the *type* of dynamics changes, each which is

[20] This implies that rational agents do not take asset prices into account when forming expectations. However, a more general approach including the adjustment for measurement errors of projecting three endogenous variables on two shock terms (stochastic indeterminacy) approximately lead to the same Ω. Assuming that agents use OLS to regress $\mathbf{x_t}$ on $\mathbf{\tilde{v}_t}$, $\mathbf{\tilde{\Omega}} = \left(\mathbf{\bar{\Omega}^T \bar{\Omega}}\right) \mathbf{\bar{\Omega}^T} \in \mathbb{R}^{3\times 2}$ and $\mathbf{\tilde{\Omega}_{1:2,1:2}} \approx \mathbf{\Omega}$.

indicated by the vertical grey lines. For low values of $\alpha < 1.22$ the fundamental steady state is stable and unique, implying that the speculative forces are not strong enough to have an impact on stock prices without exogenous shocks. A stable steady state implies that there is always a fraction of fundamentalists in the market that outweighs the beliefs of biased agents. Exogenous shocks could lead to a temporal increase in the fraction of belief-biased agents, but their belief is yet not strong enough to prevent the price from returning to its steady state. When α increases limit cycles arise with the amplitude increasing with the parameter value. A small deviation from the steady state is now, by increasing the fraction of belief-biased agents, strong enough to ignite deterministic dynamics. Since beliefs are self-fulfilling to a large extent, the fraction of belief-biased agents is increasing each period, which in turn lets the price rise even further.

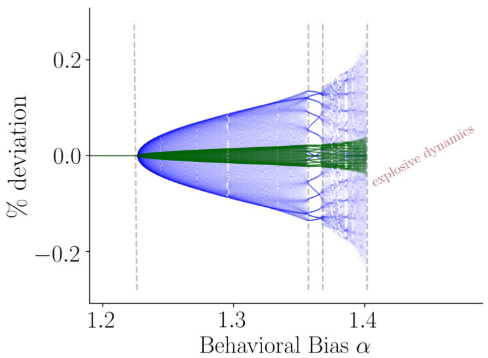

Figure D.4: Bifurcation diagram of inflation and output w.r.t. α

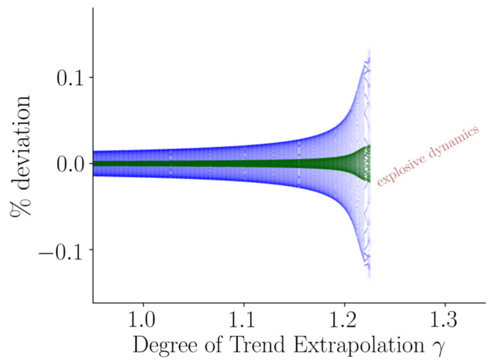

Figure D.5: Bifurcation diagram of inflation and output w.r.t. γ

When the price approaches the value predicted by positively (negatively) biased traders, they reduce their long (short) position. This then reduces their profit and other strategies become more attractive. Once alternative beliefs are more and more enforced, the fraction of positively (negatively) biased traders is too small to maintain the high price level. These financial cycles now have impact on output and inflation, which adapt the cyclic movement of stock prices as shown in the graphic. After $\alpha \approx 1.35$ cycles become unstable and for values below $\alpha \approx 1.37$ the simulations suggest that the system is close to a homoclinic orbit: the zero steady state is globally stable but locally unstable (Hommes, 2013). Long periods of stability can then be interrupted by busts in asset prices that are hard to predict, and, through the credit-collateral channel, can be followed by severe recessions. When α increases even further cycles collapse and dynamics become explosive since the fraction of fundamentalists is not sufficiently large to stabilizes the system (α higher than 1.4). Since agents continue to assume price increases in the next period, this process is self-fulfilling and prices explode to infinity in the long run. The bifurcation diagram for γ in Figure D.5 displays different quantitative aspect but very similar qualitative properties.

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