Endogenous Money, Excess Reserves and Unconventional Monetary Policy

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preliminary and incomplete

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Abstract

Despite massive holdings of excess reserves in the European banking sector and policy rates below zero, interest rates on households' deposits remained elevated long after 2009 and inflation remained subdue. Using an industrial organization model of the banking sector, I show that the general equilibrium effects of unconventional monetary policy can be ambiguous. In the model, loans create deposits, but holding more deposits is associated with higher liquidity risk. Asset purchases create additional reserves which are effective to cut lending rates, thereby stimulating lending but creating additional deposits. In genreal equilibrium, the associated relative increase in banks' liquidity risk can move deposit rates – and hence, household spending – in either direction. The overall effect on inflation remains ambiguous because decreasing lending rates ease firms financing costs. To quantify these channels, I embed this model into a medium-scale DSGE model which is estimated using nonlinear Bayesian methods. Counterfactual analysis amounts the effects of the ECBs post-2010 unconventional monetary policy measures to 0.25 percent of quarterly GDP, and their effect on inflation to be negligible.

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JEL: E63, C63, E58, E32, C62

1 Introduction

Excess reserves holdings by European banks have increased tremendously from virtually zero until 2012 up to 10 times the necessary reserve requirement in 2019. As a consequence of the ECB's additional unconventional monetary policy measures during

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the coronavirus pandemic, reserves holding escalated up even further to 4.6 billion Euros, which corresponds to 25 times the minimal reseres. However, a large spread between deposit rates and the interest rate on reserves suggests that banks still seem to be willing to pay a premium on deposits, although policy rates have dropped below zero and a large fraction of their assets are held as reserves at the ECB. This paper asks a simple question: what is the macroeconomic impact of the unconventional monetary policy measures that led to this scenario? To answer this question, it seems crucial to understand why are banks willing to hold that many excess reserves, especially in the light of negative interest rates on reserves.

Endogenous money. This paper approaches this question by borrowing from the theory of *endogenous money*.[1] Proponents argue that money is created endogenously by commercial banks. Banks first decide on their desired lending volume and will thereby automatically extend the amount of demand deposits they hold. While this statement is trivially true from a macro perspective – in a closed economy either because investment must equal savings in real terms or simply because of Walras's law – it has important implications for the banking sector: Banks are never deposit-constrained and their focus hence lies entirely on the lending market. I take this as a primary building block. When issuing credit, banks cash a deposit for the creditor, implying that loans extend the banks' balance sheet on both sides. Loans then indeed immediately create deposits and banks create money endogenously (as measured in sight deposits) by granting loans. At the same time, the amount of money is procyclical but, in the absence of assumptions like money-in-the-utility, features long-run neutrality. A bank can then exchange claims to loans ("bonds") against reserves held at the central bank, which can be used to settle interbank imbalances or to satisfy the minimal reserve requirement (MRR).[2] This grants a positive role to the central bank by either restricting the amount of reserves, or controlling the relative price for holding them.

Model. I develop an IO model of the banking sector that includes the idea of endogenous money, but where banks face a liquidity problem similar to Poole (1968): they can create loans at will, but the deposits that are thereby created may be withdrawn or wired to another bank. The corresponding transfers must be settled in reserves at the central bank. Banks hence demand reserves to hedge the liquidity risk associated with lending. In ordinary times, when the MRR is binding, banks do not value reserves per-se but only for regulatory reasons as additional reserves allow for additional deposits which are a necessary side effect of newly granted loans. Inside money (as measured in the volume of deposits) increases one-to-one with the volume of reserves that the central bank is willing to provide.[3] Banks are only willing to hold excess reserves if the central bank offers to exchange reserves for bonds at a competitive price. The MRR becomes slack and a central bank policy that supplies additional reserves has only limited impact on borrowing and lending rates. In this regime, the money "created" by commercial banks is indeed *endogenous* and not linked directly to the amount of reserves provided

[1] See e.g. Beneš and Kumhof (2012); McLeay et al. (2014); Werner (2014); Bundesbank (2017). I am here using the terms *inside money* and *endogenous money* interchangeably.

[2] Proponents of the endogenous money perspective argue further that neither reserve nor capital requirements actually impose a limit to the ability of commercial banks to create inside money.

[3] Equivalently, and more in line with the argument made by proponents of the theory of endogenous money, the central bank sets the opportunity costs of holding reserves such that banks demand a specific amount of reserves, which is then always granted.

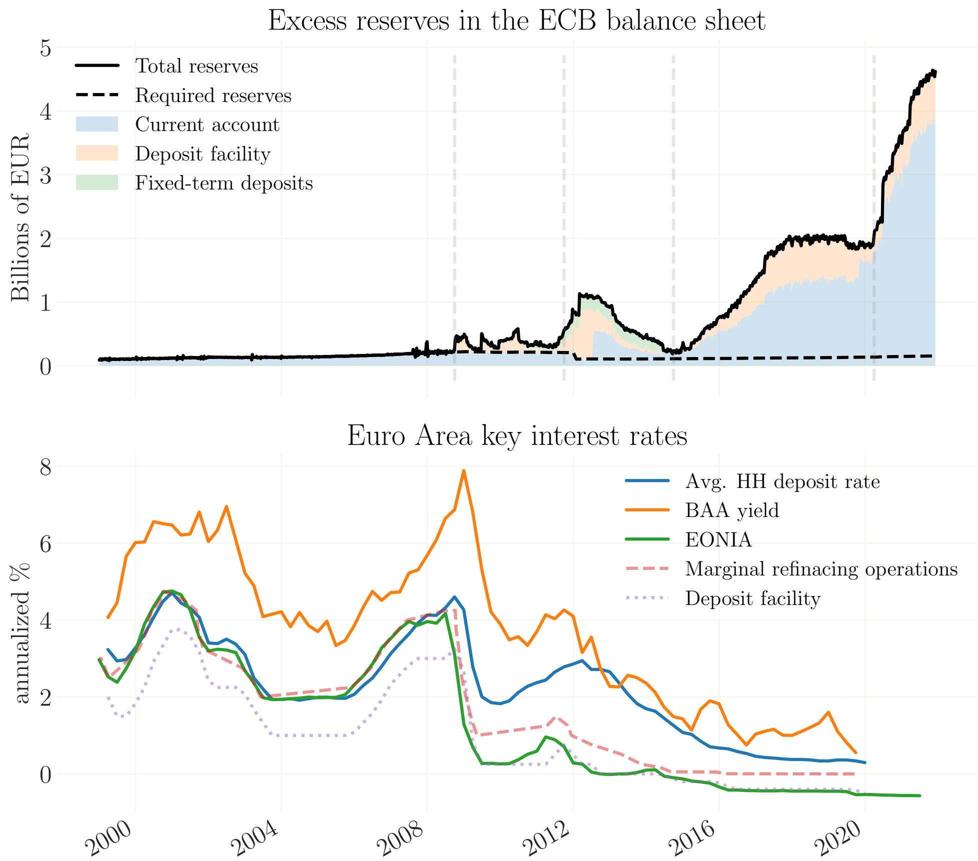

Figure 1: Top panel: reserves stored at the ECB, decomposed into the type of liability. Vertical dashed lines are the GFC and the announcements of LTROs, QE and the PEPP programme. Bottom panel: key interest rates (annualized) in the Euro Area. The MRO (marginal refinancing operations) rate is the rate payed on required reserves, DFR (deposit facility rate) is the rate payed on reserves in excess to required reserves. The EONIA rate stands for the interbank market rate and the BAA yield is a measure for refinancing costs faced by firms. The household deposit rate is a weighted average over different liquidity classes of bank deposits. Source: Calculations based on ECB SDW data.

by the central bank. In other words, once banks hold excess reserves, the monetary multiplier collapses.

Data. Figure 1 shows a decomposition of reserve holdings of commercial banks at the ECB into its main components, together with key interest rates in the Euro Area from 1999 to 2021. The top panel separates the data in two regimes: the regime with a binding minimal reserve requirement (MRR) until the Great Financial crisis in 2008:IV (first dashed vertical line), and the time thereafter where banks are holding excess reserves. Reserve holdings are clearly policy-driven: while from 2009 until late 2011 (second vertical line) banks where still close to the reserve requirement, excess reserve holdings picked up in 2012 with the long term refinancing operations (LTROs, second vertical line). After gradually returning towards the MMR, the QE measures announced in the end of 2014 and again in 2019 (third and fourth vertical lines) catapulted reserve holdings to its current all-time high.

The bottom panel illustrates how interest rates evolved alongside of reserves policy. The deposit facility rate (DFR) and marginal refinancing operations (MRO) rate are directly controlled by the ECB, the latter not only being the rate at which banks can refinance but also the rate which banks receive on required reserves. The interbank lending rate (EONIA), the BAA yield and the household deposit (HHD) rate are market outcomes. The HHD rate, as the risk-free savings rate of households, is the crucial bridge between the banking system and the macroeconomy, lying at the core of any modern New Keynesian macroeconomic model. Until 2008, the close co-movement of MRO, HHD and Eonia rate illustrates the textbook case: reserves are scarce, and bank that are willing to supply reserves at the interbank market do at least demand the same price as they would have to pay at the ECB. Likewise, if the rate on reserves would be above the HHD rate, banks would have incentive to increase the HHD rate relative to their competitors to attract more deposits which could be invested in reserves.

However, this logic collapses after 2008: once reserves become abundant, the Eonia rate quickly converges towards the DFR, which is the rate banks have to pay when parking excess reserves at the ECB. Most notably, the HHD rate detached from MRO and Eonia rate at the same time, and remained elevated thereafter. Given the macroeconomic relevance of the HHD rate, it must also be taken as the ultimate measure of success of any unconventional monetary policy measures. Conventional intuition would suggest that banks are only willing to hold large share of excess reserves if the central bank pays a premium on reserves relative to deposits. How can we reconcile this time series evidence with to the post-2010 European macroeconomic dynamics, and the massive measures of unconventional monetary policy?

Theory. My model provides closed form solutions for borrowing and lending rates and their spread to the interest rate on reserves. Only these rates are relevant for the investment-savings decisions of households and firms while monetary aggregates are not (the *cashless limit*, Woodford, 2011). I develop an expression for the marginal profit from holding reserves, which is a central measure for banks to determine whether to hold excess reserves or not. The marginal profit from reserves must be exactly zero for banks to hold excess reserves and negative while at the MRR. Yet, zero profit by no means imply that banks hold excess reserves for their liquidity value. Rather, banks are willing to hold massive amounts of reserves if the central bank is willing to offer attractive prices for assets in exchange to reserves. In a first step, I use this model to provide a row of theoretical insights into the transmission of monetary policy impulses. In a second step, I implement this banking sector into a New-Keynesian medium-scale DSGE model that can be brought to the data. I then use the estimated model to quantify the effects of the ECB's unconventional monetary policy measures.

Unconventional monetary policy. I show that any sort of central bank asset purchases (in exchange against reserves) is always effective in lowering lending rates, but has ambiguous effects on deposit rates. Asset purchases are much more effective when the MRR binds. This is because at the MRR, an increase in the supply of reserves immediately allows banks to expand lending activity and thereby to extend their deposit holdings. This mechanism collapses when the MRR is not binding, and the provision of additional excess reserves then merely affects the liquidity (convenience) spread between borrowing, lending and interest-on-reserves rate (IOR rate, henceforth). Secondly, I show that impulses to the IOR rate *can* be effective (and will be most of the time), but is much more efficient when the MRR is not binding. The reason is that the fraction of required reserves is generally relatively small (i.e. less than 5%) and hence does not affect the banks investment decision by much if the MRR is binding. When banks are holding excess reserves, the pass-trough of the IOR rate to borrowing and lending rates is close to one-to-one, but finally limited by the zero lower bound on deposit rates.

Deposit channel. Additional liquidity provisions or asset purchases are, hence, always an efficient tool to stimulate lending, but their macroeconomic effectiveness depends on two factors. First, on how much liquidity risk banks are facing, and second, on the quantitative general equilibrium responses of the demand for funds, that is, of aggregate investment. If lending activity increases sufficiently strongly, the larger deposit holdings cause a relative rise in liquidity risk that in turn dampens the fall of the lending rate. Since the spread between borrowing and lending rate is decreasing nevertheless, this may cause an actual increase in the deposit rate. An increasing deposit rate, in turn, may cause a fall in consumption via the dynamic IS curve and thereby open up for overall contractionary effects of unconventional monetary policy. I term this effect the *deposit channel of unconventional monetary policy.* I further show that unless the lower bound on the deposit rate is binding, the central bank can use both tools independently to perfectly target the lending rate and thereby reproduce equivalent macro dynamics independently of whether the MRR is binding or not.

Estimation To assess the macroeconomic impact of these channels empirically, I embed the model of the banking sector outlined above into a fully-fledged medium-scale DSGE model of the Euro Area. Next to the classic macroeconmic time series I feed several important interest rates as well as the ECB balance sheet into the estimation to identify the effects of the unconventional monetary policy measures undertaken since 2010. I apply nonlinear estimation tools to be able to incorporate a perceived effective lower bound on nominal interest rates, which helps to explain the economic dynamics between 2009 and 2015, and further enables the quantification of the negative interest rate policies conducted post-2015. I provide estimates of the reversal IOR rate threshold. Although the reversal rate was not yet reached in 2021, there was only limited leeway for further decreases of the IOR rate into negative territory.

Counterfactual analysis. I show that the measures of liquidity provision and quantitative easing had only a small effect on output – about one-quarter of quarterly GDP – and almost no effect on inflation. The reason for this very limited inflation response is that once banks are holding excess reserves, the additional liquidity through any reserves-related policy stimulates borrowing and lending rates in a similar fashion, thereby effectively fostering demand and supply alike. In contrast to this finding, I document that the ECB's negative interest rate policy was quite successful in stimulating the economy with an effect of about one percent (quarterly) on GDP and a quarter percent on inflation. This is due to – relatively – stronger effects on consumption than on investment, which feed back to a lower real interest rate that in turn further stimulated consumption.

The rest of this paper is structured as follows. In the reminder of this section I will discuss the related literature. Section 2 presents the banking modelling extension and section 3 discusses the theoretical implications of this model. In section 4 this model in embedded into a medium-scale DSGE model and I present the setup for Bayesian estimation. Simulations and empirical results are presented in section 5 whereas section 6 concludes.

Related literature

This paper is most closely related to Bianchi and Bigio (2014), who also build on a banking model based on deposit in- and outflows as a motivation to hold reserves in the spirit of Poole (1968) and Frost (1971). Their banking model goes into greater detail and features an OTC interbank market, which allows them to closely match the empirical dynamics of the US interbank market. The US market for federal funds is treated in a similar fashion, but rather from the finance perspective, in Afonso and Lagos (2015). The models from those two papers do not allow for analytic solutions and are too complex to be included into a medium-scale DSGE model. Still, and although arguably much simpler, my model is able to reproduce the key mechanism discussed in both of the above papers. Also rather connected to the finance literature, the work of Acharya and Rajan (2022) studies the effects the massive excess reserves holdings on the banking sector in the US. They conclude that the effects of central bank liquidity provisions may go in either direction because as a response, banks may provide less for potential episodes of stress. Drechsler et al. (2017) especially focus on the deposit side of the banking market and show that market concentration can importantly impact the pass-through of monetary policy.

One of the first papers that approach the connection of the central bank balance sheet with macroeconomic dynamics is Cúrdia and Woodford (2011). Another related paper is Becard and Gauthier (2020), who extend the benchmark medium-scale NK model with (supply sided) financial frictions with borrowing frictions at the households side. Without explicitly modelling the banking sector, they show that a shock to households' borrowing conditions can replicate the co-movement of consumption, investigated and employment that is lacking in the standard model. I confirm this result for the liquidity shock in my model, which allows for similar dynamics.

Piazzesi et al. (2019) also incorporate a banking model into a NK framework to study the macroeconomic dynamics of a floor vs. a corridor system. Notably, some of their effects run through the assumption of money-in-the-utility, which can be seen as a shortcut. A different road is taken by, Benigno and Nisticò (2020) who especially focus on the interaction of the central bank with the fiscal authority. Diba and Loisel (2021) show that incorporating reduced-form model of the banking sector in a New Keynesian (NK) framework allows to address some of the puzzles associated with the standard NK framework. These findings translate nicely to the model proposed here, which brings in a profound microfoundation through its IO approach.

A young but rather large literature has investigated into the effects of negative interest policies. To only mention a few, Brunnermeier and Koby (2018) develops the concept of a *reversal rate* at which further decreases of the policy rate will have different (and often unwanted) macroeconomic effects than before. Heider et al. (2019) find that empirically, negative interest rates lead to less lending activity. In contrast, Demiralp et al. (2017); Altavilla et al. (2021), find no or rather positive effects of negative rates on lending and on firm activity. Eggertsson et al. (2019) also study the pass-through of negative interest rates to lending activity in a theoretical framework.

2 An IO Model of the Banking Sector

Banks lend funds to firms and the government in the form of (claims to) physical capital K_t and government bonds B_t. When granting a loan, the bank credits the debtors deposit at the bank, thereby extending the bank balance sheet on both sides. Loans hence create deposits D_t, which are hold by households who own the firms.[4] Banks can further exchange government bonds against interest-bearing reserves J_t at the central bank. The balance sheet of bank i then reads

$$Q_t^b B_{i,t} + Q_t K_{i,t} + J_{i,t} = D_{i,t}, \tag{1}$$

where Q_t^b is the price for government bonds and Q_t the price of one unit of capital. Note that in the absence of any additional friction the Modigliani-Miller theorem holds and I hence abstract from any sort of banks net-worth.[5]

Households use deposits as a medium of exchange for their expenditures which means that from the perspective of bank i, deposits may be subject to wire transfers to other banks. The liquidity risk induced by lending activity gives rise to the demand for reserves: since assets $B_{i,t}$ and $K_{i,t}$ are assumed to be illiquid during period t, reserves are used to settle cross-bank transfers.

Denote the net outflow of deposits in period t through transfers by $\Delta D_{i,t}$. For each transfered unit of $\Delta D_{i,t}$ that is larger than the current stock of reserves $J_{i,t}$ the banker has to pay a cost γ. Depending on the nature of the monetary regime, γ could for example be the interbank lending spread, or the penalty rate for overshooting the discount window. Let χ be the probability for one unit of deposits to be transfered (which is time invariant).[6] The probability that any unit of deposits that is transfered from any bank ends up at bank i is given by the fraction $\frac{D_{i,t}}{D_t}$ of deposits that bank i already holds. Proposition 1 states that the distribution of $\Delta D_{i,t}$ approximately follows a normal distribution.

Proposition 1 (Liquidity risk)**.** *Given the probability χ that any unit of deposits get withdrawn, and the probability $\frac{D_{i,t}}{D_t}$ that any withdrawn unit (from any bank) is transfered to bank i, the probability for the event that $\Delta D_{i,t} = x$ for any $x \in \mathbb{R}$ is approximately normally distributed with*

$$Pr\left(\Delta D_{i,t} = x\right) = f\left(x|0, \frac{D_{i,t} D_{-i,t}}{D_t}(2\chi - \chi^2)\right). \tag{2}$$

Proof. See Appendix A.1. ■

Under the simplifying assumption that bankers are risk-neutral, this result allows to obtain an analytical expression for the expected costs of settling excess withdrawals, $\gamma g(J_{i,t}, D_{i,t}) = \gamma E[Z|Z > J_{i,t}] Pr(Z > J_{i,t})$.

[4]The ownership structure is only assumed for simplicity and not necessary for the mechanism to work. So will loans to the government either be hold as deposits at the bank, or, either via transfers or spending, in general equilibrium also end up as households' deposits.

[5]See section 3 for a more detailed discussion.

[6]A somewhat more involved specification would link the *total number of transactions* in the economy to the volume of consumption expenditures. While this results in a rather complicated mathematical representation of the variance of in- and outflows, the model implications would remain similar.

Proposition 2 (Liquidity costs)**.** *If bankers are risk-neutral, the expected volume of withdrawals in excess of reserves holdings is*

$$g(J_{i,t}, D_{i,t}) = h(D_{i,t}) f\left(J_{i,t}|0, h(D_{i,t})\right) - J_{i,t}\left[1 - F\left(J_{i,t}|0, h(D_{i,t})\right)\right], \tag{3}$$

with $h(D_{i,t}) = \frac{D_{i,t}D_{-i,t}}{D_t}(2\chi - \chi^2)$.

Proof. See Appendix A.2. ■

Denote by $R_{i,t}$ the (gross) nominal rate bank i pays on households' deposits. Households can chose to hold cash instead of deposits, but, since deposits are perfectly safe and liquid for households, only have incentive to do so if $R_{i,t} < 1$. This gives rise to a zero lower bound on deposit rates (deposit lower bound, DLB). Additionally, the banks' deposit services are heterogeneous (e.g. through diversification of services) and banks have some degree of market power (similar to Ulate (2021)). The aggregator takes the form

$$D_t = N^{1-\epsilon_D}\left(\sum_i^N D_{i,t}^{1/\epsilon_D}\right)^{\epsilon_D}, \tag{4}$$

where N is the number of banks. Then, bank i faces an inverse supply function of the form

$$\frac{R_{i,t}}{R_t} = N^{\frac{1-\epsilon_D}{\epsilon_D}}\left(\frac{D_{i,t}}{D_t}\right)^{\frac{1-\epsilon_D}{\epsilon_D}}, \tag{5}$$

$$R_{i,t} \geq 1, \tag{6}$$

with $\epsilon_D \in (0, 1]$. For a symmetric equilibrium it follows that

$$D_t = ND_{i,t}, \tag{7}$$

$$R_t = R_{i,t}. \tag{8}$$

Similarly, loan services to firms, $Q_tK_{i,t}$, are heterogeneous and bank i is facing the inverse demand function

$$E_t\left\{\frac{R^k_{i,t+1}}{R^k_{t+1}}\right\} = N^{\frac{1-\epsilon}{\epsilon}}\left(\frac{K_{i,t}}{K^b_t}\right)^{\frac{1-\epsilon}{\epsilon}}. \tag{9}$$

with $\epsilon \geq 1$. R^j_t is the nominal interest rate on reserves (IOR rate). Following Woodford (2001), government bonds are modeled as perpetuities with decaying coupon payments. Let $\kappa \in [0, 1]$ denote the decay parameter for coupon payments. The expected per-monetary-unit return on government bond holdings is then given by[7]

$$E_tR^b_{t+t} = E_t\left\{\frac{1 + \kappa Q^b_{t+1}}{Q^b_t}\right\}. \tag{10}$$

The bond market clears with $\sum_i B_{i,t} = B^b_t$ (government bonds held by commerical

[7]Variables with subscript t are those set in period t. All interest rates are given in nominal terms.

banks) and $B_t = B_t^b + B_t^{cb}$, that is, commerical banks and the central bank together hold all bonds.

A regulatory authority enforces an (occasionally binding) minimal reserve requirement (MRR) of

$$\psi D_{i,t} \leq J_{i,t}, \tag{11}$$

and excess reserves are hence, if any, given by $J_{i,t} - \psi D_{i,t}$. The necessary conditions for an equilibrium of the banking sector are given by proposition 3.

Proposition 3 (Equilibrium of the banking sector). *Under the assumptions that*

1. *each bank i takes aggregate variables $\{K_t, D_t, E_t R^k_{t+1}, E_t R^b_{t+1}, R_t, R^j_t\}$ as given,*
2. *each bank i takes the cumulated deposit choice of competitors $D_{-i,t}$ as given,*
3. *the equilibrium is symmetric,*
4. *no entry and exit,*

and with $\nu = (N-1)(2\chi - \chi^2)$, $\hat{D}_t = D_t/\nu$ and $\hat{J}_t = J_t/\nu$, a competitive equilibrium in the banking sector is given by

$$R^k_{t+1} = \epsilon R^b_{t+1}, \tag{12}$$

$$Q^b_t B^b_t + Q_t K^b_t + J_t = D_t, \tag{13}$$

$$R_t/\epsilon_D = \max\left\{1/\epsilon_D,\ (1-\psi)R^b_{t+1} + \psi R^j_t + \gamma\left(\psi\left[1-\hat{F}\right] - 0.5\hat{f}\right)\right\}, \tag{14}$$

$$R^j_t - R^b_{t+1} + \gamma\left[1-\hat{F}\right] = \min\left\{0,\ R^j_t - R^b_{t+1} + \gamma\left[1-\hat{F}_\psi\right]\right\}, \tag{15}$$

where $\hat{f} = f(\hat{J}_t|0,\hat{D}_t)$, $\hat{F} = F(\hat{J}_t|0,\hat{D}_t)$, $\hat{f}_\psi = f(\psi\hat{D}|0,\hat{D})$ and $\hat{F}_\psi = F(\psi\hat{D}|0,\hat{D})$ are shorthand for the PDF and CDF of the normal distribution.

Proof. See Appendix A.3. ■

Note that the expression at the LHS of (15) corresponds to the marginal profit of reserve holdings, which is given by

$$MPJ\left(R^j_t, R^b_{t+1}, J_t, D_t\right) = R^j_t - R^b_{t+1} + \gamma\left[1 - F\left(\hat{J}_t|0,\hat{D}_t\right)\right]. \tag{16}$$

The MRR is binding whenever $MPJ_\psi = MPJ(\cdot,\cdot,\psi D_t, D_t) < 0$, i.e. when marginal profits of reserves *at the MRR* are negative. In this case equation (15) simply collapses to $J_t = \psi D_t$. If however $MPJ_\psi \geq 0$, banks have an incentive to hold excess reserves and an interior solution with $J_t > \psi D_t$ exists. Equation (15) then reads $MPJ_t = 0$. Thus, if banks are willing to hold excess reserves, these are determined by a conventional optimality condition where marginal profits are zero. Respectively, the DLB translates directly to a constraint on the optimality condition for deposits, which becomes inactive once the DLB binds. Note that the fact that excess reserve holdings follow a conventional zero-profit-condition does not mean that they hold all these excess reserves to cover potential in and outflows of deposits. Rather, when conducting asset purchases the central bank must offer a competitive price such that banks are willing to switch assets against reserves.

Proposition 3 also reveals that the parameters N and χ can be summarized by the composite parameter for liquidity risk, ν, which scales J_t and D_t. Intuitively, a larger probability χ that deposits being withdrawn has a similar effect as a larger number of banks N, because a high N decreases the probability that deposits that are withdrawn from bank i will return to i. In the following $\hat{J} = \frac{J}{\nu}$ and $\hat{D} = \frac{D}{\nu}$ will be called *effective* reserves and deposits.

In this system banks are price-takers and equilibrium rates are equal to marginal costs in addition to monopolistic markups. The structure of the cost function gives rise to spreads between borrowing, lending, and the IOR rate. The next section analyses how these spreads and other monetary aggregates respond to monetary impulses in terms of variations in the IOR rate and the supply of central bank reserves.

3 Theoretical Insights: Transmission of (unconventional) Monetary Policy

The central bank can independently control the supply of reserves J_t and the interest rate on reserves (IOR) R_t^j of the economy. This section provides analytical insights into the how the choice of the policy tools $\{J_t, R_t^j\}$ translate through the banking sector,

a) to control the household deposit rate R_t, which drives the households' consumption-savings decision,
b) to control the lending rate R_t^b, which determines the firms investment in capital and the cost of government debt, and
c) to control the borrowing-lending spread $s_t^b = R_t^b - R_t$, which drives a wedge between those two.

The results from proposition 3 readily provide closed-form solutions for these three objects of interest for when the MRR is not binding. Therefore I first derive general results for the case in which banks hold excess reserves, and then focus on the equilibrium where the MRR binds.

For the sake of simplicity in this section I abstract from monopsonistic power in the deposit market and, further, to define the composite asset $A_t = Q_t^b B_t + Q_t K_t$. The investment demand for A_t is given by the demand function $d_A(R_t^b)$ with $\frac{\partial d_A}{\partial R^b} \leq 0$. Denote the demand elasticity (with respect to a one-percentage change in R^b) as $E_A = \frac{\partial d_A / A}{\partial R^b}$. The bank balance now reads $A_t^b + \hat{J}_t = \hat{D}_t$ while the central bank balance sheet is given by $A_t^{cb} = \hat{J}_t$ and the asset market clears with $d_A(R_t^b) = A^b + A^{cb}$. Dropping time subscripts throughout this section, the equilibrium in the banking sector is given by

$$d_A\left(R^b\right) = \hat{D}, \tag{17}$$

$$R = \max\left\{1,\ (1-\psi)R^b + \psi R^j + \gamma\left(\psi\left[1-\hat{F}\right] - 0.5\hat{f}\right)\right\}, \tag{18}$$

$$R^j - R^b + \gamma\left[1-\hat{F}\right] = \min\left\{0,\ R^j - R^b + \gamma\left[1-\hat{F}_\psi\right]\right\}. \tag{19}$$

Note that in order to solve for an equilibrium it is not necessary to determine the households' supply of deposits. It also becomes clear why the introduction of banks net worth would not fundamentally change the implications of our model: in a competitive equilibrium where banks accumulate net worth and maximize the expected stream of dividends, the optimality condition for net-worth accumulation is that R^b equals the inverse of the

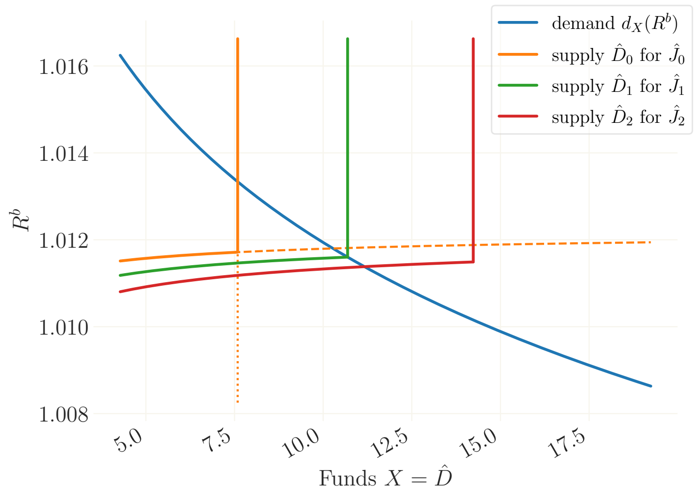

Figure 2: Banking equilibrium given isoelastic demand for funds. Reserves increase from $\hat{J}_0 = 0.171$ (orange curve) to $\hat{J}_1 = 0.241$ (green) to $\hat{J}_2 = 0.321$ (red) and shift the loan supply curve out and downwards. For $\hat{J}_0$ the minimal reserves requirement is binding in equilibrium and slack for $\hat{J}_2$. The dashed line depicts the supply of funds without MRR, the dotted line shows the counterfactual supply of funds if banks would always wish obey the MRR, i.e. without endogenous selection. The loan demand function is $A = 0.977 {R^b}^{0.005}$, $R^j = 1$ and $\phi = 2.26\%$.

discount factor times a liquidity premium. Under conventional assumptions the relative demand for net-worth is then increasing in R^b. Concurrently, a rise in R^b causes the volume of assets A_t to fall since $E_A < 0$, which causes liabilities to decrease. If the (relative) net-worth holdings increase, then the decrease in deposits must be disproportionately high.

Figure 2 illustrates the equilibrium in the loan market for different levels $J_0 < J_1 < J_2$ of reserves. The demand function (blue line) is conventional by assumption. The MRR lends a hockey-stick-shape to the supply function: the horizontal orange line (dashed whenever $\psi D > J$) represents the supply of funds in the absence of the MRR, i.e. all points where R^b equals the marginal costs of lending. This function is increasing in A_t because, as loans create deposit, they increase liquidity risk and thereby raise the associated marginal costs of lending. For the same reason (loans create deposits), the total volume of loans is restricted by the level of reserves supplied by the central bank, J, which leads to the vertical orange line (dotted wherever $MPJ > 0$) where $J = \psi D$. In the banking equilibrium for J_0 the MRR is binding. If the central bank expands its supply of reserves from J_0 to J_1, this shifts the vertical line outwards (more reserves enable more deposits if the MRR binds) and moves the horizontal line downwards because more reserves also mitigate the liquidity risk. The new equilibrium is exactly at the kink where the MRR starts binding. Finally, an additional increase in the supply of reserves $J_1 \to J_2$ will again shift both curves outwards/downwards and banks decide to hold some of the newly supplied reserves as excess reserves, i.e. the MRR is non-binding. Below, I discuss the underlying mechanism in detail.

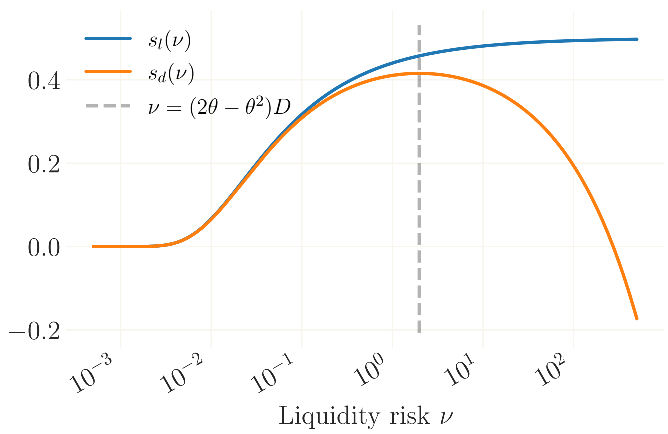

Figure 3: Deposit and lending spreads s_d and s_l as a function of ν taking the reserve ratio $J/D = 2.26\%$ as given. Reserves are set to unity and the graphs scales proportionally to the chosen values of J and D. Note that the maximum of s_d lies at $\nu = 2J - J^2/D$.

3.1 The economy with excess reserves

When neither the MRR nor the DLB binds we have that

$$R^b = R^j + \gamma\left(1 - \hat{F}\right), \tag{20}$$

$$R^b = R + 0.5\gamma \hat{f}, \tag{21}$$

additional to the equilibrium in the funds market given by (17). Inserting (21) into (20) yields

$$R = R^j + \gamma\left(1 - \hat{F} - 0.5\hat{f}\right). \tag{22}$$

Define $s_l = (R^b - R^j)/\gamma$ to be the lending spread, $s_b = (R^b - R)/\gamma$ be the borrowing-lending spread, and $s_d = (R - R^j)/\gamma$ the deposit spread. Figure 3 illustrates the findings from proposition 4 about the ceteris paribus responses of these spreads to changes in liquidity risk ν.

Proposition 4 (Liquidity regimes)**.** *Ceteris paribus, if MRR and DLB are slack and for a given ratio of D/J the economy knows two liquidity regimes:*

1. *The deposit spread s_d is*
   (a) *increasing in liquidity risk ν if $\nu < (2J - J^2/D)$,*
   (b) *decreasing in liquidity risk ν if $\nu > (2J - J^2/D)$, and*
   (c) *goes from $\lim_{\nu\to 0} s_d(\nu) = 0$ to $\lim_{\nu\to\infty} s_d(\nu) = -\infty$.*
2. *The lending spread s_l is continuously increasing in ν, bounded below by 0, and converges to 0.5.*

Proof. See Appendix A. ∎

For low levels of liquidity risk ν banks directly pass-on the level of the IOR rate to both, borrowing and lending rate because expected liquidity costs are negligible. When ν increases, the liquidity markup in the loan rate increases, which in turn reflects in a larger return relative to the IOR rate, that is offered to households. At the same time the borrowing-lending spread rises to compensate for larger expected liquidity costs. For values of $\nu > 2J - J^2/D$ the deposit rate starts to decline. Note here that the inverse of the deposit spread can be seen as a measure for the liquidity premium payed on reserves, which banks are less and less willing to accept if liquidity risk increases. For even larger values of ν, s_d becomes negative, implying that banks demand additional compensation for taking the liquidity risk and the IOR rate exceeds the interest rate on deposits.

Let us next turn to the effects of an increase in excess reserves. Excess reserves can either be supplied via asset purchases ("unconventional open market operations") or refinancing operations such as Long-term Refinancing operations (LTROs). In both cases the central bank buys government or corporate bonds in exchange for reserves. The effect of any excess reserves policy can be decomposed into the direct (positive) effect of mitigating liquidity risk by providing additional reserves, and an indirect negative effect that I will call *the deposit channel.* The deposit channel arises because any decrease in the lending rate R^b will lead to a surge in the demand for funds, which, as loans create deposits, will lead to an expansion of the volume of deposits. This in turn does lead to a relative increase in liquidity risk. To quantify the relative impact of both effects, it is useful to introduce the elasticity of deposits to reserves.

Proposition 5 (Elasticity of deposits to reserves)**.** *If MRR and DLB are slack, for $0 \leq J \leq D$, $0 < D$ and given R^j, the elasticity E_{DJ} of deposits with respect to reserves is*

$$E_{DJ} = \frac{1}{0.5 - (\gamma d'_A J/D\hat{f})^{-1}} \tag{23}$$

and is bounded by $E_{DJ} \in (0, 2)$.

Proof. See Appendix A.5. ■

Intuitively, proposition 5 states that E_{DJ} cannot be negative since trough the demand function d_A, a fall in A would require an increase of R^b. However, R^b can only increase if either J declines or A expands. Similarly, if $E_{DJ} > 2$ the effect of an increase in deposits would outweigh the liquidity effect, thereby increasing R^b in absolute turns, which is inconsistent with an increase in A.

Proposition 6 (Effectiveness of excess reserves)**.** *If MRR and DLB are slack, for $0 \leq J \leq D$, $0 < D$ and a given R^j, any policy that actively increases the supply of excess reserves*

1. *reduces the deposit rate R whenever*

$$\hat{J}\left(1 - 0.5\frac{\hat{J}}{\hat{D}}\right) > \frac{E_{DJ}}{2 - E_{DJ}}, \tag{24}$$

with a pass-trough of $\frac{\partial R}{\partial J/J} = \gamma\hat{J}(0.5\frac{\hat{J}}{\hat{D}} - 1)\hat{f}$ if E_{DJ} is sufficiently small,

2. *always reduces the lending rate $R^b > R^j$ with a pass-trough of*

$$\frac{\partial R^b}{\partial J/J} = \gamma \hat{J}(0.5E_{DJ} - 1)\hat{f}, \tag{25}$$

and decreasing marginal efficiency if E_{DJ} is sufficiently small,

3. *always reduces the borrowing-lending spread s_b with a pass-trough of*

$$\frac{\partial s_b}{\partial J/J} = 0.5\left[0.5\left(\frac{\hat{J}^2}{\hat{D}} - 1\right)E_{DJ} - \frac{\hat{J}^2}{\hat{D}}\right]\hat{f}. \tag{26}$$

Proof. See Appendix A.6. ■

Proposition 6 summarizes the implications of our model for excess reserves policy. For the lending rate (see part 2. of the prop), liquidity effect and the deposit effect run in opposite directions. At the limit, $\lim_{E_{DJ}\to 2} \Delta R^b = 0$ and the effect of reserves policy on the lending rate is exactly zero. Although the effect of reserves policy on the lending rate is always positive, the proposition reveals that the marginal effect of such policy may be close to zero, in particular if the deposit effect is strong, i.e. if E_{DJ} is large. Part 3. suggests states that the effect on the borrowing-lending spread is always negative. Here, the direction of the deposit effect depends on the sign of $(J^2/D - 1)$ but never exceeds the liquidity effect.[8]

Part 1. of proposition 6 documents a key-finding: an increase in the supply of reserves can actually raise the deposit rate, which in general equilibrium will cause households' consumption to fall. As the proposition suggests, this is entirely due to the deposit effect. The LHS term $J\left(1 - 0.5\frac{J}{D}\right)$ in (24) is always positive since reserves cannot exceed deposits and, hence, $\frac{J}{D} < 1$. $\frac{E_{DJ}}{2-E_{DJ}}$ goes from zero to infinity as E_{DJ} increases (remember that E_{DJ} is bounded by $(0,2)$), suggesting that if loan demand responds strongly to reserves policy (i.e. if E_{DJ} is high), a fall in the deposit rate can only be achieved after the central has already supplied large amounts of reserves. The reason is that the deposit channel affects the different spreads differently. Take for example the limit case when $E_{DJ} \to 2$, i.e. loans react strongly to an increase in reserves. In this limit case, R^b will remain unchanged because liquidity and deposit effect exactly cancel out (see prop. 6.2.). However, as the $R^b - R$ spread is always decreasing in J (prop. 6.3.), the deposit rate must be increasing.

Proposition 7 (Effectiveness of IOR policy)**.** *Assume that MRR and DLB are slack, $0 \leq J \leq D$, $0 < D$ and take J as given.*

1. *Any IOR policy*

[8] Mathematically, the ambivalence of the deposit effect on s^b can be seen via

$$\hat{f} = f(\hat{J}|0, \hat{D}) = \frac{1}{\sqrt{\hat{D}}}\varphi\left(\frac{\hat{J}}{\sqrt{\hat{D}}}\right). \tag{27}$$

If $\hat{J}$ is close to zero, the term $\frac{1}{\sqrt{\hat{D}}}$ has a strong discounting effect that dominates the expression as $\hat{D}$ increases.

(a) *has a pass-through to the lending rate R_b of*

$$\frac{\partial R^b}{\partial R^j} = \frac{1}{1 - 0.5\hat{J}E_A\hat{f}} \in (0, 1], \tag{28}$$

(b) *has a pass-through to the deposit rate of*

$$\frac{\partial R}{\partial R^j} = 1 + \gamma 0.25 \left[\hat{J} \left(\frac{\hat{J}}{\hat{D}} - 2 \right) - 1 \right] E_{DR^j}\hat{f} \geq 1, \tag{29}$$

with $E_{DR^j} = \frac{\partial d_A/A}{\partial R^b}\frac{\partial R^b}{\partial R^j}$ being the elasticity of deposits with respect to the IOR rate,

(c) *has ambiguous effect on the borrowing-lending spread s_b with*

$$\frac{\partial s_b}{\partial R^j} = \gamma 0.25 \left(\frac{\hat{J}^2}{\hat{D}} - 1 \right) E_{DR^j}\hat{f}. \tag{30}$$

2. *A stimulative (contractionary) IOR policy moves the economy back to (away from) the MRR.*

Proof. See Appendix A.7. ■

Proposition 7 summarizes the effects of IOR-rate policy for the economy with excess reserves. The pass-through of R^j on R^b is close to perfect and is only moderately demagnified by the deposit channel for reasonable assumptions on the borrowing-lending spread and the demand elasticity of investment. In fact, IOR policy is super-effective in controlling the deposit rate, where as above, pass-through is even amplified by the deposit channel. Finally, the borrowing-lending spread increases in R^j if the amount of excess reserves in the economy is large, but is decreasing otherwise.

A second, important result on the effects of IOR policy from proposition 7 is that any stimulative IOR policy will ultimately lead the economy back to the MRR. This can be decomposed into two effects. The first, direct effect is that a low interest on reserves increases the marginal costs for holding reserves. This is dampened by the second, indirect effect, which comes from the fact that the lending rate decreases with the IOR rate, thereby causing a relative decrease in marginal costs of holding reserves.

3.2 The regime with a binding minimal reserve requirement

Let us now turn to the regime where the MRR is binding. In this case, the banking equilibrium is given by

$$R = (1 - \psi)R^b + \psi R^j + \gamma \left(\psi \left[1 - \hat{F}_\psi \right] - 0.5\hat{f}_\psi \right), \tag{31}$$

$$J = \psi D, \tag{32}$$

and again $d_A(R^b) = \hat{D}$. Note that the deposit rate is a weighted average of R^b and R^j plus a liquidity spread and it always holds that $R < R^b$.[9] Proposition 8 establishes that, when deposits are directly linked to reserves, any exogenous increase in reserves will be reflected by an one-to-one increase in household deposits.

Proposition 8 (Elasticity of deposits to reserves with MRR)**.** *If the MRR is binding and DLB are slack, for* $0 \leq J \leq D$*,* $0 < D$ *and given* R^j*, the elasticity of deposits with respect to reserves* E^{ψ}_{DJ} *is given by*

$$E^{\psi}_{DJ} = 1. \tag{34}$$

Proof. At the MRR it holds that $J = \psi D$. The result follows directly from $d_A(R^b) = \hat{D}$, and $E^{\psi}_{DJ} = \frac{\partial d_A}{\partial J}\frac{J}{A}$. ■

Although seemingly trivial, this result has important implications on the pass-through of reserves policy to the rates in the banking equilibrium, which are summarized in proposition 9. Namely, the lending rate is solely determined by the equilibrium at the funds market, which in turn is directly tied to the supply of reserves via the MRR. The deposit rate is then closely linked to the lending rate, and the spread decreases in J as long as reserves are sufficiently small (or in terms of the proposition, until $\hat{J} = \psi^{-1}$).

Proposition 9 (Effectiveness of open market operations with MRR)**.** *If the MRR is binding and DLB are slack, for* $0 \leq J \leq D$*,* $0 < D$ *and given* R^j*,*

1. *for* $E_A < 0$*, any policy that actively increases the supply of reserves*
   (a) *always reduces the lending rate* R^b *with a pass-trough of*

$$\frac{\partial R^b}{\partial J/J} = E_A^{-1}, \tag{35}$$

   (b) *has a pass-trough to the deposit rate* R_t *of*

$$\frac{\partial R}{\partial J/J} = (1-\psi)E_A^{-1} + \gamma 0.25(1-\psi J)\hat{f} \tag{36}$$

   (c) *has a pass-through on the borrowing-lending spread* s_b *of*

$$\frac{\partial s_b}{\partial J/J} = \psi E_A^{-1} + \gamma 0.25(\psi J - 1)\hat{f}. \tag{37}$$

2. R^b*,* R *and* s^b *are indetermined if* $E_A = 0$*.*

Proof. See Appendix A.8. ■

The equilibrium rates of the banking equilibrium for different levels of reserves are illustrated in figure 4. The solid lines in the red-shaded area to the left represent the

[9] This can be seen by noting that (31) can be expressed as

$$R = R^b - 0.5\hat{f}_{\psi} + \psi MPJ_{\psi}, \tag{33}$$

where $MPJ_{\psi} < 0$ whenever the MRR binds.

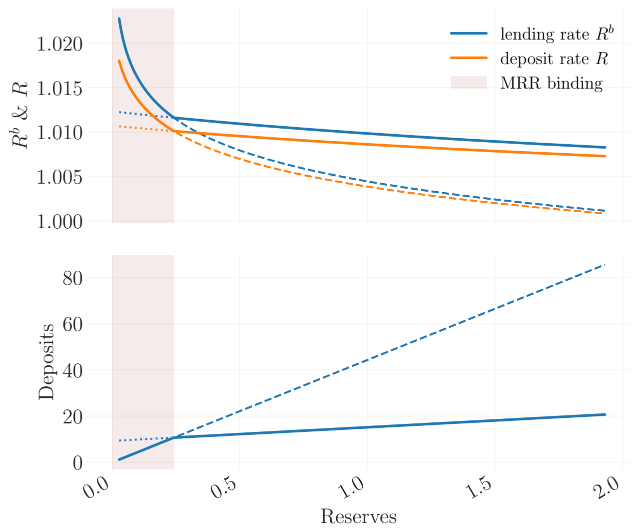

Figure 4: Partial equilibrium responses when varying the quantity of supplied reserves. The inverse loan demand function is $\hat{A} = 0.977{R^b}^{0.005}$, R^j is fixed to 1 and $\phi = 2.26\%$. The dashed (dotted) lines depict the counterfactual equilibrium outcome if the MRR would be binding (slack).

equilibria when the MRR is binding. Borrowing and lending rate decrease sharply when the level of reserves rises, and the lending rate is solely determined by the demand function for funds. Once the MRR is slack, the pass-through of reserves policy to interest rates flattens immediately, and the transmission to deposits is less than one-to-one. Note that, although only slowly, reserves policy with excess reserves successfully reduces the spread between borrowing and lending rate. Figure 5 illustrates the same exercise for a more elastic investment demand function, leading to a more strongly attenuated deposit channel. Importantly, the deposit rate increases with the level of reserves shortly until the MRR becomes slack, and then slowly decreases with the amount of excess reserves supplied. Note that, while the borrowing-lending spread decreases, the deposit rate remains almost constant when reserves increase.

Proposition 10 (Effectiveness of IOR policy with MRR)**.** *If the MRR is binding and DLB are slack, for $0 \leq J \leq D$, $0 < D$ and given J,*

1. *for $E_A < 0$, any IOR policy,*
   (a) *is fully ineffective in altering the lending rate R_b,*
   (b) *has a pass-through on the deposit rate of*

$$\frac{\partial R}{\partial R^j} = \psi, \tag{38}$$

   (c) *has a pass-through on the borrowing-lending spread of*

$$\frac{\partial s^b}{\partial R^j} = -\psi. \tag{39}$$

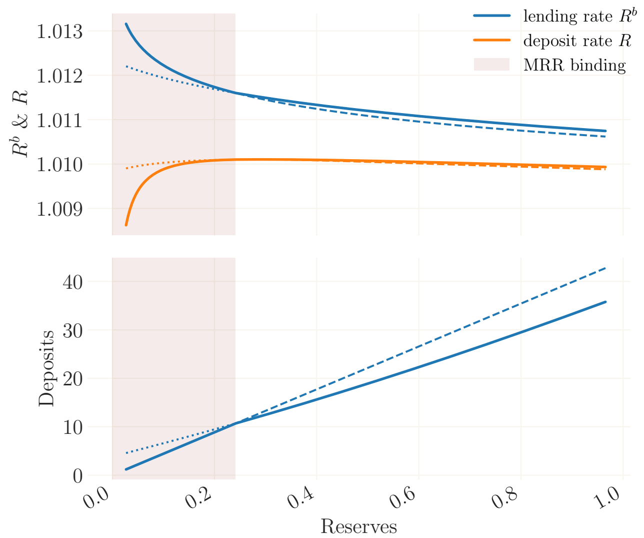

Figure 5: Partial equilibrium responses when varying the quantity of supplied reserves for a larger E_{DJ} than in figure 4. The inverse loan demand function is $A = 0.987{R^b}^{0.0007}$, R^j is fixed to 1 and $\phi = 2.26\%$. The dashed (dotted) lines depict the counterfactual equilibrium outcome if the MRR would be binding (slack).

2. *R^b, R and s^b are indetermined if $E_A = 0$.*

Proof. See Appendix A.9. ■

Finally, proposition 10 documents a very limited transmission of IOR policy onto borrowing and lending rates if the MRR is binding. In fact, because the lending rate is determined by the supply of reserves, it is fully invariant to changes in the IOR rate. Since the deposit rate is a weighted average between lending and IOR rate with weights $1-\psi$ and ψ, it is mainly determined by the lending rate and the pass-through of reserves policy is limited as well. For most countries the MRR is between 1% and 5%. For the Euro Zone, the effective MRR was $\psi_{t<2012} \approx 3.6\%$ before 2012 and $\psi_{t>2012} \approx 1.3\%$ thereafter. This suggests that the pass-through of IOR policy is almost negligible.

Figure 6 represents the equilibria of the banking market for a given range of the IOR rate. The lines in the shaded red area to the left again represent equilibria where the MRR is binding. For the reasons outlined above, the IOR has virtually no impact on equilibrium rates when the MRR is binding. Once the IOR rate exceeds a threshold value (here $R^j = 1$ by construction), the MRR becomes slack and banks wish to hold some of the supplied reserves as excess liquidity. This reflects in a decrease of deposit holdings, which are now endogenous, and the IOR rate becomes an efficient tool to steer equilibrium rates, as implied by proposition 7. In this regime with excess reserves, the IOR rate does only indirectly affect the spreads between the rates through the deposit channel, but, since spreads are mainly determined by the level of reserves, have a large impact on the level of borrowing and lending rate.

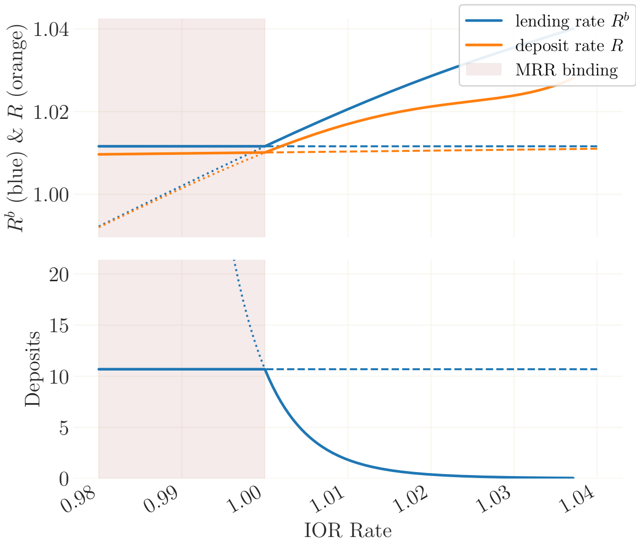

Figure 6: Partial equilibrium responses when varying the IOR rate and keeping $\hat{J}$ fixed. The inverse loan demand function is $\hat{A} = 0.977 {R^b}^{0.005}$, and $\phi = 2.26\%$. The dashed (dotted) lines depict the counterfactual equilibrium outcome if the MRR would be binding (slack).

3.3 The reversal IOR rate and the DLB

My model does not motivate why the IOR rate can or should not be negative. It is easy to see that the optimality conditions of banks remain untouched as long as the deposit rate is positive. A prominent argument against negative IOR rates is that they are costly for banks. This argument is however not entirely convincing because independently of whether the IOR rate is positive or negative, any $R^j < R^b$ implies that holding reserves is associated with opportunity costs.

Correspondingly, the only constraint to this irrelevance result is the household's incentive constraint (6), which is the DLB. The DLB potentially affects the transmission of interest rate policy and reserves policy alike. The following propositions 11 and 12 suggest that the DLB affects the economy differently depending on whether the MRR is binding or not. Proposition 11 states that any IOR policy is ineffective at the MRR if the DLB is binding: the lending rate is fully determined by the supply of reserves while the deposit rate is constrained. This implies that there are no negative side effects of setting the IOR rate below the threshold, simply because there are no effects at all.

Proposition 11 (Effective lower bound)**.** *There exists an effectve lower bound (ELB) to the IOR rate if*

$$r^j < \frac{\psi - 1}{\psi} r^b - \gamma \left(\left[1 - \hat{F}_\psi\right] - \frac{0.5}{\psi} \hat{f}_\psi \right) \tag{40}$$

and

$$r^j - r^b + \gamma \left[1 - \hat{F}_\psi\right] < 0, \tag{41}$$

i.e. when the MRR is binding.

Proof. See Appendix A.10. ■

In contrast, when the MRR is not binding the lending rate directly depends on the IOR rate. This relationship (21) will hold independently of whether the DLB binds or not. This means that the DLB gives rise to a reversal rate at which the households' consumption decision is still indirectly affected by IOR policy changes via the general equilibrium effects of aggregate investment. Proposition 12 summarizes this result.

Proposition 12 (Reversal rate)**.** *Let a "reversal IOR rate" be a IOR rate such that marginal effects of IOR or reserves policy are nonzero but different from propositions 6 to 10. Then there exists a reversal IOR rate if*

$$r^j < \gamma(\hat{F} + 0.5\hat{f} - 1) \tag{42}$$

and

$$\psi d(R^b) < J, \tag{43}$$

i.e. when the MRR is slack.

Proof. See Appendix A.11. ■

Although these effects are different that what is discussed in propositions 6 to 10, this does not mean that the effects of IOR or reserves policy are negative. Also, note that as suggested by proposition 7, a sufficiently large decrease in the IOR rate will push banks back to the MRR. Further, while in practice the reversal IOR rate is likely to be negative, this does not necessarily need to be the case because in theory the deposit rate can also be below the IOR rate (see figure 3).

The results from this subsection contradict some of the more recent findings, that warn against the risks of negative rates. However, some of these findings are based on models where dividend payments and the choice of net worth are not endogenous or rather ad-hoc (e.g. Ulate, 2021; Sims and Wu, 2021). In contrast, my model assigns stimulative (and conventional) effects to any interest rate policy for which $R \geq 1$. Arguably, and as suggested by proposition 12, the reversal IOR rate is only relevant when there are excess reserves. Negative rates in combination with the DLB do indeed decrease profits in the above model, but it is unlikely that banks will immediately exit business or decrease their lending activity. In fact, the model suggests the opposite. Lastly, if for reasons exogenous to this model the central wants to avoid a binding DLB, this implies a careful trade-off between reserves and IOR-policy.

4 A medium Scale DSGE: model and estimation

In this section I first develop a fully-fledged medium-scale DSGE model of the Euro Area (EA) that incorporates the banking sector with liquidity frictions as proposed in section 2. I then specify the setup for nonlinear estimation in terms of data and methodology.

Apart from the banking sector, the backbone of the model by large follows the standard medium-scale setup of Smets and Wouters (2007). The setup of capital producers is adjusted, and I extend the toolbox of the central bank by balance sheet policies and negative interest rate policies. I here focus on the exhibition of these non-standard parts of the model and refer to the original papers for details on the baseline model. In addition, as suggested by Del Negro and Schorfheide (2013), let aggregate productivity be given by

$$Z_t = e^{\gamma t + \frac{1}{1-\alpha}\widetilde{z}_t}, \tag{44}$$

where γ is the steady-state growth rate of the economy and α is the output share of capital. $\widetilde{z}_t$ is the linearly detrended log productivity process that follows the autoregressive law of motion $\widetilde{z}_t = \rho_z \widetilde{z}_{t-1} + \sigma_z \epsilon_z$. For z_t, the growth rate of technology in deviations from γ, it holds that $z_t = \frac{1}{1-\alpha}(\rho_z - 1)\widetilde{z}_t + \frac{1}{1-\alpha}\sigma_z \epsilon_z$.

4.1 Firms

The setup of capital producers is adapted from Boehl et al. (2021) such that the (expected) return on capital R^k_{t+1} can be expressed in terms of the monetary return on physical capital, $Q_t K_{t-1}$. The capital good producer's role in the model is to isolate the investment decision, that becomes dynamic through the introduction of convex investment adjustment costs.[10] At the end of each period, capital good producers buy used capital, restore it and produce new capital goods. Correspondingly, intermediate good producers sell the capital stock that they used for production to the capital producer, which repairs it, and purchase the capital stock that it is going to use in the next period from the capital producer. To finance the purchase of the new capital at the unit price Q_t, it issues a claim for each unit of capital it acquires to banks, which trade at the same price. As above, the interest rate the capital producer has to pay on the loans is R^k_{t+1}. I also assume that the firm incurs costs of capital utilization that are proportional to the amount of capital used, $\Psi(U_t) P_{m,t} K_{t-1}$.[11]

Capital evolves according to the law of motion

$$K_t = (1-\delta) K_{t-1} + e^{v_{i,t}} \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t, \tag{45}$$

where δ is the depreciation rate and the function $S(\cdot)$, indicates a cost of adjusting the level of investment. In steady state it holds that $S = 0, S' = 0$, and $S'' > 0$. and $v_{i,t}$ follows an AR(1) process. The first order condition of capital producers reads

$$\begin{aligned} 1 \quad = \quad & Q_t e^{v_{i,t}} \left(1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right) \\ & + E_t \left\{ \Lambda_{t,t+1} Q_{t+1} e^{v_{i,t+1}} S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \right\}, \end{aligned}$$

and the choices for optimal labor input and optimal capital utilization yield the first order conditions

$$W_t = MC_t (1-\alpha) \frac{Y_t}{L_t}, \tag{46}$$

$$\Psi'(U_t) K_{t-1} = \alpha \frac{Y_t}{U_t} \Leftrightarrow \Psi'(U_t) = \alpha \frac{Y_t}{\overline{K}_t}, \tag{47}$$

[10] Investment adjustment costs are a necessary feature to generate variation in the price of capital.

[11] This assumptions for the utilization costs are set to match the setting in Smets and Wouters (2007).

where MC_t are marginal costs, U_t denotes the level of capital utilization, and $\overline{K}_t$ is the level of effective capital, and ex-post returns are given by

$$R_t^k/\pi_t = \frac{MC_t[\alpha\frac{Y_t}{\overline{K}_t} - \Psi(U_t)] + (1-\delta)Q_t}{Q_{t-1}}. \tag{48}$$

4.2 (Unconventional) Monetary policy

Setting up the monetary policy building block of a model of the post-1999 Euro Area is a nontrivial task. From an aggregate perspective, the MRR was effectively binding (for the majority of banks) until the end of 2008, when the ECB cut rates close to zero in response to the Great Financial Crisis. Until that point the ECB was effectively using the IOR rate as well as open market operations (OMOs) to target a corridor system for the interbank lending rate, and by doing so it was very successful to pin down the interbank lending rate right in the middle of this corridor.[12] While banks started storing small amounts of liquidity in the deposit facility after 2008 – that is, as excess reserves –, they did not accumulate larger amounts of excess reserves before the ECB's 2010 *Security Market Programme*. Before 2009 neither the deposit facility nor the marginal lending facility were used in larger scale because banks could easily refinance in the interbank market. This means that when the MRR was binding, the MRO rate was the IOR rate in effect. However, when the MRR became slack in 2009, the deposit facility rate (DRF) became the relevant IOR rate since it is the rate banks receive on a marginal unit of reserves stored at the central bank.

The EONIA rate – as a measure of the interbank lending rate – was close to the MRO rate before 2009 but quickly converged to the DFR thereafter. Together with the fact that from 2009 to 2010 banks were holding small amounts of excess reserves even in the absence of unconventional monetary policy, this is indication that even before 2009 it was likely that $MPJ_t \approx 0$. This simplifies the setup of monetary policy considerably: under the assumption that $MPJ_t = 0$ at the MRR, J_t is uniquely pinned down given R_t^j (or vice versa), which fully determines the equilibrium of the banking sector.[13]

Hence, assume that the ECB sets the IOR rate to follow a conventional monetary policy rule of the form

$$\frac{R_t^s}{R^s} = \left(\frac{R_{t-1}^s}{R^s}\right)^\rho \left[\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^*}\right)^{\phi_y} \left(\Delta\left(\frac{Y_t}{Y_t^*}\right)\right)^{\phi_{dy}} e^{v_{r,t}}\right]^{1-\rho}, \tag{49}$$

where I refer to the unconstrained nominal policy rate R_t^s as the notional (or shadow) rate. Y_t^* denotes the potential output and $\Delta\left(\frac{Y_t}{Y_t^*}\right)$ denotes the growth in the output

[12]The Euro System knows three policy rates: the deposit facility rate is the rate payed on excess liquidity parked at the central bank, the MRO (marginal refinancing operations) rate is payed on reserves subject to the MRR, and the marginal lending rate is the rate due when borrowing overnight reserves from the ECB.

[13]Note that if a central bank wishes to minimize the spread between borrowing and lending rates they seek to move MPJ_t close to zero. At the MRR it is that $MPJ_t < 0$. Plugging $R^b = R$ into (31) yields $\psi MPJ_t = 0.5\hat{f}_\psi$, which can never be achieved since $\hat{f}_\psi > 0$. However, increasing R^j raises MPJ_t and increasing J reduces $\hat{f}_\psi$.

gap. Parameter ρ expresses an interest rate smoothing motive by the central bank over the notional rate and ϕ_π, ϕ_y, and ϕ_{dy} are feedback coefficients. $v_{r,t}$ is a conventional monetary policy shock that follows an AR(1) process.

As the nominal IOR rate, R_t^j, is under direct control of the central bank there are several options on how to model the policy rate and the effective lower bound. The model from section 2 gives no reason why the IOR rate should not go beyond zero. The model also does not imply that the ECB must respect the zero lower bound on the deposit rate (which must be respected by banks). However, the Federal Reserve Board remained reluctant to set the Federal Funds rate below zero, and the ECB kept the DFR above zero until 2014. This, together with anecdotal evidence, suggests that agents in the EA did not expect that the IOR rate could actually touch negative territory. Such *perceived effective lower bound* (PLB, perceived lower bound), although arguably only an intellectual constraint, can have a large impact on economic dynamics. Hence, assume that the ECB sets the IOR rate to follow the policy rule while maintaining that it does not go below zero:

$$R_t^j = \max\{1, R_t^s\}\, e^{v_{nir,t}}, \tag{50}$$

where we put the stochastic negative interest rate process $v_{nir,t}$ – which follows an AR(1) process – *outside* the max operator to allow for policy innovations that drive the IOR rate into negative territory, as observed in the Euro Area, while having agents to expect a classic zero lower bound ex-ante.

The central bank balance sheet is given by

$$J_t = Q_t^b B_t^{cb} + Q_t K_t^{cb},$$

where I assume that in normal times $K_t^{cb} = 0$. Imposing that in normal times the ECB always supplies enough reserves for banks to satisfy their desired liquidity needs, $J_t = \psi D_t$, the central bank's balance sheet can be written as

$$J_t = \psi D_t + X_t, \tag{51}$$

with X_t as the amount of excess reserves supplied. I assume that X_t follows an AR(2) process

$$x_t = \rho_{x,1} x_{t-1} + \rho_{x,2} x_{t-2} + \epsilon_{x,t}. \tag{52}$$

The advantage of an AR(2) process is that it can capture the hump-shaped response of the asset purchases, thereby also ensuring anticipation and stock effects at the moment the announcement was made. Note that by assumption $X_t = 0$ in steady state.

4.3 The linearized model

The full model is log-linearized around its growth path. By assuming that in steady state $MPJ = 0$ and $J = \psi D$ – that means, the model is linearized exactly at the kink of the banks decision function – a second occasionally binding constraint is avoided. The log-linear counterparts of the novel equations are given by

$$d_t + \frac{Y}{D}\hat{L}_t = \frac{Q^b B}{D}(q_t^b + b_t) + \frac{QK}{D}(q_t + k_t) \tag{53}$$

$$r_t^b = \frac{\kappa Q^b}{1+\kappa Q^b} q_t^b - q_{t-1}^b, \tag{54}$$

$$E_t r_{t+1}^k = E_t r_{t+1}^b + \hat{\epsilon}_t, \tag{55}$$

$$E_t r_{t+1} - \frac{1}{\epsilon_D} r_t = -\frac{\gamma}{2} S_D \left(d_t - \hat{\nu}_t - \frac{\psi J}{\nu}\left(d_t + \hat{\nu}_t - 2j_t\right)\right) + \gamma S_D \hat{\gamma}_t, \tag{56}$$

$$E_t r_{t+1}^b - r_t^j = \gamma S_D \frac{J}{\nu}\left(d_t + \hat{\nu}_t - 2j_t\right) + \gamma S_L \hat{\gamma}_t, \tag{57}$$

$$j_t - d_t = x_t, \tag{58}$$

$$r_t^j = \max\{0, r_t^s\} + v_{nir,t}, \tag{59}$$

with $S_D = 0.5\sqrt{\frac{\nu}{D}}\varphi\left(\frac{J}{\sqrt{\nu D}}\right)$ the steady state value of $s_b(\cdot)$ and $S_L = 1 - \Phi\left(\frac{J}{\sqrt{\nu D}}\right)$ being the steady state lending spread s_l (again, net of γ).[14] I assume that the steady-state deviations of the lending markup, $\hat{\epsilon}_t$, and of the liquidity cost parameter, $\hat{\gamma}_t$, both follow an AR(1) in logs. Note again that the term $v_{nir,t}$ stands outside of the max operator, thereby possibly driving the IOR rate into negative territory. The rest of the linearized model can be found in Appendix B.

4.4 Estimation

The fact that the data includes a long episode in which the PLB binds poses a host of technical challenges. These are related to the solution, likelihood inference and estimation of the model in the presence of an occasionally binding constraint. Boehl and Strobel (2020, henceforth BS) suggest a comprehensive collection of tools to tackle these challenges. To start with, they propose a solution method for occasionally binding constraints that performs roughly four magnitudes faster than alternative methods. For likelihood inference, BS suggest to use the Ensemble Kalman filter (Evensen, 1994), which can be understood as a hybrid of the particle filter and the Kalman filter. The Ensemble Kalman filter allows to efficiently approximate the distribution of states for large-scale nonlinear systems with only a few hundred particles (instead of several million as with the particle filter), which is computationally advantageous.[15] As proposed by BS, I use a nonlinear path-adjustment smoother to obtain the smoothed/historic shock innovations of the high-dimensional nonlinear model. To sample from the posterior distribution I use the differential evolution ensemble Monte Carlo Markov chain method (Ter Braak, 2006; ter Braak and Vrugt, 2008).[16] For further technical details see BS.

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[14] Additionally, linearized aggregated liquidity costs are given by $\gamma D S_D(d_t + \hat{\nu}_t + \hat{\gamma}_t) - \gamma J S_L(j_t + \hat{\gamma}_t)$.

[15] For all estimations and for the numerical analysis, we use an ensemble of 400 particles. This number is chosen to minimize sampling errors during likelihood inference. For the same reason we sample the initial distribution of states from quasi-random low-discrepancy series (e.g. Niederreiter, 1988). For our model, the evaluation of the likelihood for one parameter draw then takes less than 2 seconds on a single CPU. For a more detailed discussion of the properties of the Ensemble Kalman filter, also see Katzfuss et al. (2016).

[16] The fundamental idea is to have a large ensemble of Monte Carlo Markov chains that mutually exchange information. In practice, the posterior "chain" ensemble is initialized with 200 draws sampled

The model is estimated on quarterly data from 1999:II to 2019:IV using a total of eleven observables. As is standard in the estimation of medium-scale models, I include the real per capita growth rates of GDP, consumption, and investment, real wage growth, a measure of labor hours and the GDP deflator. I use the Libor as a proxy of the IOR rate because, as outlined in section 4.2, it closely followed the MRO rate when the MRR was binding, and then moved alongside the DFR rate after 2008. The Libor hence helps to homogenize the MRR and non-MRR parts of the sample. For the interest rate on bank deposits I use the household deposit rate supplied by the statistical data warehouse (SDW) of the ECB and I use the BAA yield as a measure of the lending rate. For unconventional monetary policy, I feed in the time series of reserves held at the ECB divided by required reserves as implied by deposits held by commercial banks and subject to the MRR. In terms of the model's variables this hence reads as $X_t = \frac{J_t}{\psi D_t}$, which uniquely pins down $x_t \approx X_t - 1$. The advantage of this measure is that it is stationary and relatively insensitive to log-linear approximations. These time series are also obtained directly from the SDW.

Instead of using the IOR observable (the Libor) directly, it is further divided into $\text{IOR}^+ = \max\{\text{IOR}, 0\}$ and $\text{NIR} = \min\{\text{IOR}, 0\}$. This helps to clearly identify the negative impact of the PLB and to quantify the effects of the NIR policy. To facilitate the nonlinear filtering, I assume small measurement errors for all variables with a variance that is 0.01 times the variance of the respective time series. Since the IOR^+ and NIR rate and the amount of excess reserves are perfectly observable I divide the measurement error variance here again by 100. Except for the labor supply, the data is not demeaned as I assume the non-stationary model follows a balanced growth path, with a growth rate estimated in line with SW. In total, these eleven observables are matched with eleven economic shock processes.[17] The measurement equations and a detailed description of the data are delegated to Appendix C.

I fix several parameters prior to estimating the others. In line with SW, let the depreciation rate be $\delta = 0.025$, the steady state government share in GDP to $G/Y = 0.484$, and the curvature parameters of the Kimball aggregators for prices and wages to $\epsilon_p = \epsilon_w = 10$. The steady state wage markup is set to $\lambda_w = 1.1$. I set the decay factor for government bonds to 0.975, which implies an average maturity of 40 quarters. Lastly, we calibrate the empirical perceived lower bound of the nominal interest rate to 0.01% quarterly. ψ is fixed to 0.017, which is the relevant value of the MRR until 2012 as identified by the data.[18] I let $\epsilon_D = 0.99$, which is sufficient to guarantee the existence of a local maximum.

Finally, the choice of priors is summarized in table 1. I use standard priors from SW and BS wherever possible. The novel parameters are then ν, γ and the steady state values of the spreads between borrowing and lending rate and the IOR rate. To identify ν I redefine $\nu = \frac{\hat{\nu}}{1-\hat{\nu}}(2\psi - \psi 2)D$, which for $\hat{\nu} = 0.5$ sets ν to maximize the spread between

from the prior distribution. I then let the sampler run 3500 iterations, of which the last 500 ensembles are kept. The posterior parameter distribution is thus represented by $500 \times 200 = 10000$ parameter draws. The full estimation is conducted on a machine with 40 Intel Xeon E5 CPUs and 32 GB RAM and takes about 3 hours.

[17] The economic shocks are: TFP, government spending, marginal efficiency of investment, risk premium, (conventional) monetary policy, price markup, wage markup, loan markup, liquidity costs, liquidity provision, and negative interest rate policy.

[18] The value after 2009 is not relevant since banks were already holding large amounts of excess reserves.

	Prior			Posterior				
	distribution	mean	sd	mean	sd	mode	5% HPD	95% HPD
σ_c	normal	1.500	0.375	1.340	0.102	1.269	1.170	1.497
σ_l	normal	2.000	0.750	1.761	0.523	1.820	0.889	2.535
β_{tpr}	gamma	0.250	0.100	0.096	0.033	0.105	0.044	0.145
h	beta	0.700	0.100	0.672	0.062	0.749	0.579	0.782
S''	normal	4.000	1.500	3.790	1.036	2.801	2.001	5.397
ι_p	beta	0.500	0.150	0.262	0.096	0.229	0.093	0.402
ι_w	beta	0.500	0.150	0.320	0.093	0.329	0.167	0.471
α	normal	0.300	0.050	0.319	0.015	0.311	0.293	0.343
ζ_p	beta	0.500	0.100	0.860	0.026	0.855	0.817	0.901
ζ_w	beta	0.500	0.100	0.807	0.044	0.817	0.735	0.876
Φ_p	normal	1.250	0.125	1.688	0.068	1.667	1.576	1.796
ψ	beta	0.500	0.150	0.303	0.070	0.242	0.186	0.415
ϕ_π	normal	1.500	0.250	1.591	0.188	1.775	1.255	1.878
ϕ_y	normal	0.125	0.050	0.164	0.031	0.140	0.110	0.212
ϕ_{dy}	normal	0.125	0.050	0.204	0.029	0.212	0.154	0.249
ρ	beta	0.750	0.100	0.945	0.016	0.943	0.920	0.970
$\overline{\gamma}$	normal	0.440	0.050	0.329	0.021	0.335	0.294	0.364
$\overline{l}$	normal	0.000	2.000	2.622	0.778	2.834	1.405	3.919
$\overline{\pi}$	gamma	0.625	0.100	0.512	0.057	0.517	0.422	0.605
$\overline{spread}_D$	normal	0.500	0.200	0.200	0.044	0.185	0.123	0.263
$\overline{spread}_K$	normal	0.500	0.200	0.434	0.103	0.344	0.275	0.607
$\hat{\nu}$	beta	0.500	0.250	0.837	0.086	0.905	0.712	0.959

Table 1: Prior distribution and estimation results. The estimates for the parameters governing the exogenous shock processes can be found in Appendix D.

IOR and deposit rate (see figure 3) while mapping $\hat{\nu} \in (0,1) \to \nu \in (0,\infty)$. I estimate $\hat{\nu}$ using a beta distribution with mean 0.5 and a standard deviation of 0.25 as prior, which is a very flat prior. The spread between the deposit rate and IOR rate and the lending rate and the IOR rate are also estimated (both using $\mathcal{N}(0.5, 0.2^2)$ as the prior distribution), which together can be used to pin down γ, S_D and S_L.

5 The quantitative effects of excess reserves

This section presents the results from the estimated model. I first briefly present the posterior distribution of parameters obtained from the estimation and then discuss the dynamics of an unconventional monetary policy (UMP) shock in the estimated model. I then use the estimated model to empirically quantify the effects of the post-2010 UMP measures undertaken by the ECB.

The posterior distribution is summarized in table 1. The estimates of the standard parameters that are in common with the baseline medium-scale model by large reflect the findings of Boehl and Strobel (2020) for the US economy over a similar sample. This includes in particular a low discount factor and estimates of the Calvo (1983) parameters, reflecting rather flat Phillips curves for prices and wages. The mean estimate of the parameter governing the liquidity risk of banks, $\hat{\nu}$, is pinned down at 0.837, which is significantly above its prior value. This value suggests that in terms of figure 3, the

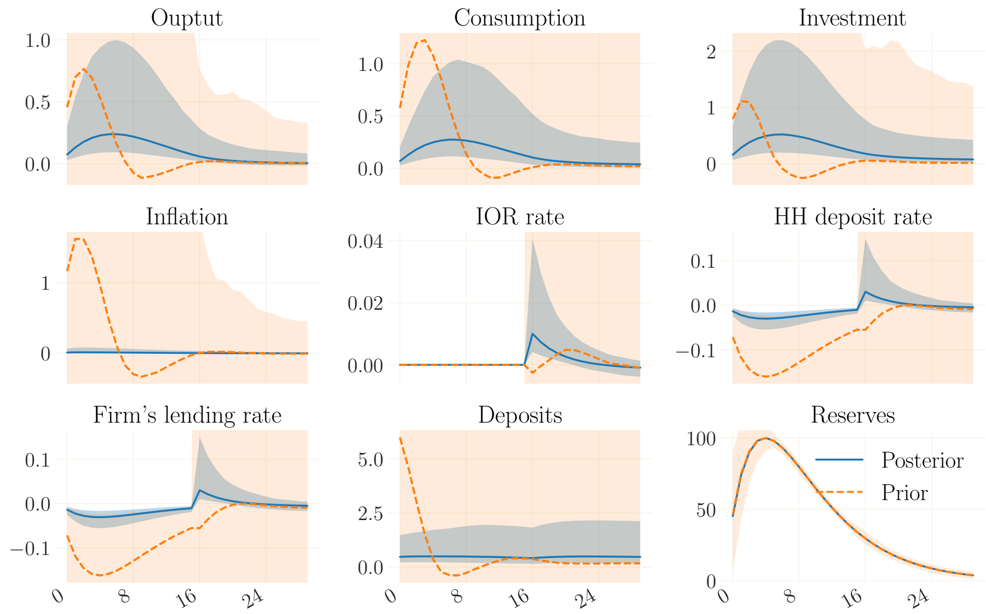

Figure 7: Impulse response functions for an unconventional monetary policy shock that doubles the amount of reserves. Simulations are based on 2500 parameter draws sampled from the posterior (blue) and the prior (orange). Shaded areas illustrate 95% credible sets. The ELB on the IOR rate is enforced for 16 quarters (4 years) to eliminate the effect of the policy rule reacting to the macroeconomic responses of the unconventional monetary policy measures. The AR(2) coefficients for all draws are set to the posterior mean to homogenize the simulations, and the shocksize is set such that the peak response of reserves is 100% of steady state reserves (i.e., reserves are doubled at the peak).

banking equilibrium is at the right side of the dashed line in the region where R_t is decreasing in ν. In this regime, the spread between borrowing and lending rate is mainly determined by the liquidity risk faced by banks. Together with the estimates of the two spreads this results in estimates of $\nu = 0.837$ and $\gamma = 0.005$ at the posterior mean.[19] These two estimates are central for the quantitative results of the model. The AR(2) process of unconventional monetary policy is characterized by $\rho_{x,1} = 1.631$ and $\rho_{x_2} = -0.665$, reflecting the hump-shaped response of asset purchases after announcement. The estimate of capital adjustment costs S'' below its posterior mean is rather uncommon, but entirely due to the assumption on G/Y.[20]

Figure 7 shows impulse response functions for unconventional reserves policy in the model. Specifically, the size of the shock is chosen to double the central banks' steady-state supply of reserves. The blue lines are sampled from the posterior parameter distribution, while the orange dashed lines represent the median over simulations sampled from the prior. Shaded areas represent 95% credible sets. In all simulations the lower

[19] The fact that $\hat{\nu} \approx \nu$ is coincidental.

[20] It is commonly assumed that $G/Y = 0.2$, which is incorrect for the EA when measured average total government expenditures over GDP. The specification used here is preferred by the data in terms of a higher data density, but has no major implications on the effects of UMP.

bound on the IOR rate is enforced for 16 quarters to eliminate the feedback of conventional monetary policy to the UMP shock. For all draws the AR(2) parameters of the UMP shock process, x_t, are fixed to their posterior mean to homogenize the timing of the simulations.

The posterior simulations suggest rather conventional responses of macro variables to UMP: both the household deposit rate and the lending rate decrease moderately, where the impact on both rates is quantitatively similar. This triggers a quarter-percent increase in consumption via the IS curve and a mild increase in investment, both lifting up output. The inflation response is positive but very limited due to the flat Phillips curve. Although consumption increases, income rises such that households are also able to increase savings and hence the volume of deposits also increases. However, this increase in deposit holdings by no means reflects the surge in reserves.

In contrast, the simulations sampled from the prior suggest far stronger median responses – in particular of inflation – and a much larger dispersion. In fact, the responses of output and inflation can in theory also turn negative. This can for example occur when the response of the lending rate is much stronger than that of the borrowing rate. Lower lending rates will cause a decrease in marginal costs, which may trigger a fall in the price level and hence deflation. Deflation in turn, in combination with an only weak response of the household deposit rate to the UMP measures, can cause the households' real interest rate to turn positive, which eventually may depress consumption. A negative consumption response may again reinforce the negative effect on inflation and cause output and investment to fall as well. Boehl et al. (2021) term such deflationary effects *the cost channel of QE* and give account that this channel may have been important for the effects of QE in the US economy. However, such deflationary effects are absent in the Euro Area in the simulations sampled from the posterior. Note that this also documents that none of the effects reported here are actually hardwired into the model. Other than the responses of macroeconomic aggregates, the prior responses of borrowing and lending rates are always negative as suggested by the findings from section 3.

Figure 8 finally shows counterfactual simulations. The methodology is similar to Boehl et al. (2021): I take draws from the posterior distribution. For each draw, I use a nonlinear Bayesian filter to obtain a sequence of shocks that drives the economic dynamics (according to the filter). I then mute the shock that drives the UMP measures and use the rest of the shocks to again simulate a set of time series. The plots then show the net difference between the simulations with all shocks and without the UMP shock. I repeat the same exercise for the shock $v_{nir,t}$ that drives the IOR rate into negative territory. The crucial difference to the impulse response functions in figure 7 is that this exercise takes into account the actual endogenous expected durations of the ELB, which are important when quantifying the impact of the UMP measures.

Overall, the unconventional monetary policy measures lead to a swift and proportional decline in deposit and lending rates. Both rates are affected similarly. The median impact of unconventional reserves policy on output turns out to be quite small with a median response of about 0.25% in 2013:I and again from 2018 to 2019. Notably, the 2018/2019 response was not (much) larger than the earlier spike in 2013, which is due to the fact that in the later period agents are expecting the PLB to be binding for short period (expected PLB durations as implied by the estimation can be found in Appendix E). As already implied by the impulse response functions, the increase in GDP comes from a one-quarter percent median response in consumption, which is quite large in relation

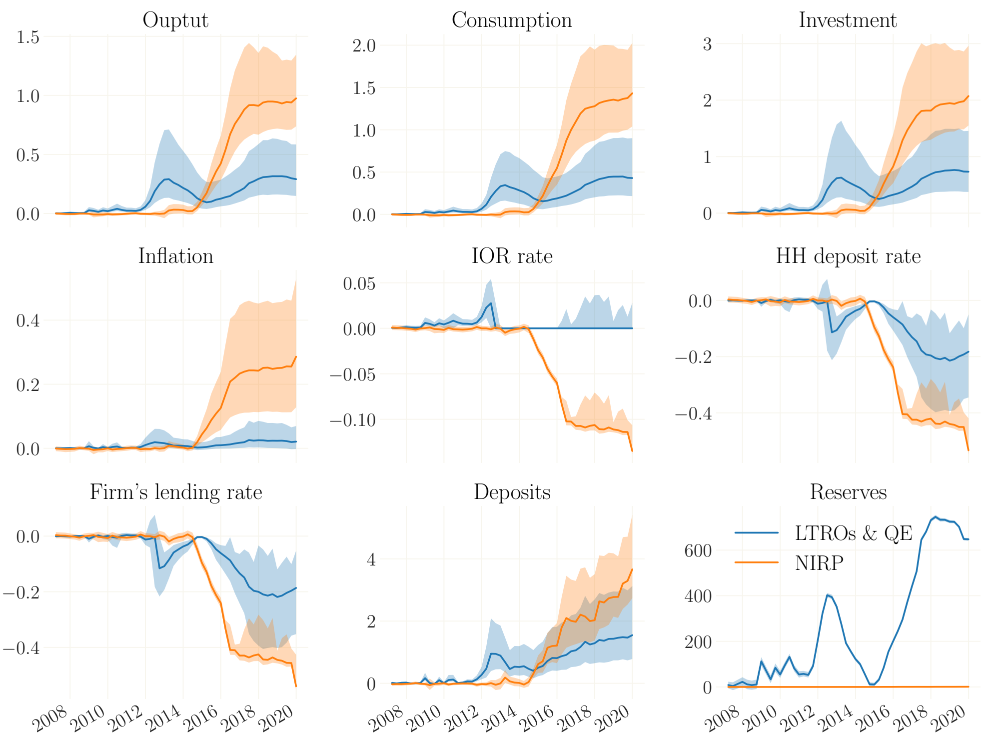

Figure 8: Counterfactual simulations for the effects of reserves policy (blue) and negative interest rate policy (orange). The figure is constructed from 1000 simulated series with and without the shocks driving both policies. All measures in percentage rates. Interest rates and inflation are annualized, the rest is expressed in quartely terms. The nonlinear effects of the ZLB binding in expectations are implicitely included.

to the 0.5% impact on investment. This, in combination with the relatively flat Phillips curve, leads to a muted response of inflation that is quantitatively negligible.

This estimate of the effectiveness of unconventional reserves policy is independent of the specification of the PLM. In fact, an estimated *linear* model will conclude with very similar quantitative findings. However, a linear model would suggest that the dynamics before 2015 would to a large extend be driven by contractionary (conventional) monetary policy shocks, which biases the overall shock decomposition and could have lead to misleading results. Additionally, using in a purely linear model the effects of reserves policy would always scale proportionally, and hence cause the output effects in 2013 and 2018–2019 to differ more strongly.

In contrast, the estimation identifies the gradual decline of the IOR rate into negative territory to be an efficient tool to stimulate both, output and inflation. The peak median response of output lies at about 1% of quarterly GDP, which mainly reflects a sharp increase in consumption and less the rather moderate rise in investment. This strong demand-sided effect of the NIRP sparks a stronger response in inflation of about 0.25% annually. Figure 9 provides estimates of the steady-state reversal IOR rate, whih is given

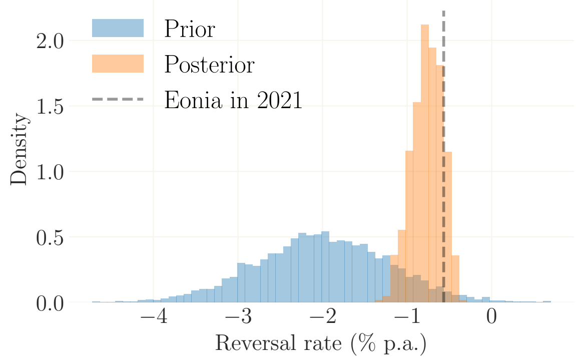

Figure 9: Prior (blue) and posterior (orange) distribution of the annualized steady-state reversal rate.

by (c.f. proposition 12)

$$r^r_{ss} = \gamma(\hat{F}_{ss} + 0.5\hat{f}_{ss} - 1). \tag{60}$$

Note that this neither takes into account the effect of the ECB's measures of liquidity provision, that would *increase* the reversal IOR rate, nor does it take into account any shocks that may increase the liquidity demand by banks, which would lower the actual reversal IOR rata. The prior mean estimate of the reversal rate lies at about -0.76% while the actual Eonia rate was already as low as -0.57% in 2021, which is at the 0.85%-percentile of the posterior distribution. This suggests that the further beneficial effects of the NIRP come with the risk of hitting the DLB rather soon.

6 Conclusion

This paper develops a fully-fledged medium scale DSGE model with a banking sector that supplies inside money. Lending activity creates deposits, and banks use reserves to hedge liquidity frictions associated with deposits. This gives rise to spreads between the borrowing and lending rate, and the interest-on-reserves rate. When the minimal reserve requirement is binding, i.e. when the marginal profit from holding reserves is negative, the central bank can effectively steer borrowing and lending rates through open market operations, but setting the interest on reserves is rather ineffective. Inside money, as measured in terms of the economy wide deposit volume, then increases one-to-one with the volume of reserves provided. Once the minimal reserve requirement is slack and banks are willing to hold excess reserves, a quantitative easing or liquidity provision policy that supplies additional reserves has only limited effect on borrowing and lending rates. Inside money then becomes fully endogenous and depending on banks discretion, and the interest-on-reserves rate is a powerful policy tool.

I show that the general equilibrium effects coming from loan demand – that is, investment demand – can further dampen the effects of LTROs and quantitative easing. If loans create deposits, a decrease in the loan rate triggers more loans and hence more deposits. These deposits however may cause a relative increase in the liquidity risk faced by banks, that may have important quantitative implications. I estimate the model using nonlinear Bayesian methods on data of the 1999-2019 Euro Area while feeding in household deposit rates and various measures of the central bank balance sheet policies such as excess reserves. Counterfactual analysis suggests that the unconventional monetary policy measures undertaken by the ECB had only limited effect on output (about one-quarter percent of quarterly GDP) and almost no measurable impact on inflation.

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Appendix A Proofs of the propositions

Appendix A.1 Proof of proposition 1 (liquidity risk)

To simplify notation, denote bank js expected net inflow of deposits with $Z = \Delta D_{i,t}$, which consists of inflows A from other banks to j and outflows Y from j to other banks. Note that a negative Z implies a net outflow of deposits with $Y > A$. A and Y both follow a binomial distribution with

$$Pr\,(A = x) = f_B(x|D_{-i,t}, \chi\frac{D_{i,t}}{D_t}), \tag{A.1}$$

$$Pr\,(Y = x) = f_B(x|D_{i,t}, \chi\frac{D_{-i,t}}{D_t}), \tag{A.2}$$

that is for A, the number of the units of deposits *not* hold by j that we denote by $D_{-i,t}$, and for each of these units the probability to end up at bank j is the probability χ to get transfered away from its current bank times the probability to be transfered to j, which is given by $\frac{D_{i,t}}{D_t}$. Bank j's deposits after transfers are hence given by the random variable

$$Z = A - Y. \tag{A.3}$$

We cannot directly sum over two Binomial distributions with different probabilities. However, for large sample sizes a Binomial distribution with PDF $f_B(k|n,p)$ can be well approximated by a normal distribution with PDF $f_N\left(k|np, np(1-p)\right)$. Since both $D_{i,t}$

and $D_{-i,t}$ are large (in a stochastic context), we can rewrite

$$A \sim \mathcal{N}\left(\chi \frac{D_{-i,t}D_{i,t}}{D_t}, \chi \frac{D_{-i,t}D_{i,t}}{D_t}\left(1-\chi\frac{D_{i,t}}{D_t}\right)\right), \tag{A.4}$$

$$Y \sim \mathcal{N}\left(\chi \frac{D_{-i,t}D_{i,t}}{D_t}, \chi \frac{D_{-i,t}D_{i,t}}{D_t}\left(1-\chi\frac{D_{-i,t}}{D_t}\right)\right). \tag{A.5}$$

Since now Z is the difference of two normal distributed variables it follows that

$$Z \sim \mathcal{N}\left(0, \frac{D_{i,t}D_{-i,t}}{D_t}(2\chi - \chi^2)\right). \tag{A.6}$$

Note that this results assumes that the *number* of units of deposits, and the real value of deposits is exactly equal. It is however easy to show that the result would hold up to some scaling factor of the variance if I would instead assume that the number of units of deposits is proportial to the value of deposits. For simplicity I hence assume that this scaling factor is included in the parameter for the transfer probability χ.

Appendix A.2 Proof of proposition 2 (liquidity costs)

Using f as the PDF of $Z = \Delta D_{i,t}$ from proposition 1 and the definition of $h(D_{i,t})$ yields

$$g(J_{i,t}, D_{i,t}) = \int_J^\infty (z - J_{i,t}) f(z) dz \tag{A.7}$$

$$= \int_J^\infty z f(z) dz - J_{i,t} \int_J^\infty f(z) dz, \tag{A.8}$$

$$= -h(D_{i,t}) \int_J^\infty f'(z) dz - J_{i,t}(1 - F(J_{i,t})), \tag{A.9}$$

$$= h(D_{i,t}) f(J_{i,t}) - J_{i,t}(1 - F(J_{i,t})). \tag{A.10}$$

Appendix A.3 Proof of proposition 3 (equilibrium of the banking sector)

Bank i's profit maximization problem is

$$\max_{K_{i,t}, B_{i,t}, J_{i,t}, D_{i,t}, L_{i,t}, R^k_{i,t+1}, R_i} E_t \left\{ R^b_{t+1} Q^b_t B_{i,t} + R^k_{i,t+1} Q_t K_{i,t} \right\} + R^j_t J_{i,t} - R_t D_{i,t} - L_{i,t} - \gamma g(J_{i,t}, D_{i,t}) \tag{A.11}$$

s.t.

$$Q_t^b B_{i,t} + Q_t K_{i,t} + J_{i,t} = D_{i,t} + L_{i,t}, \tag{A.12}$$

$$g(J_{i,t}, D_{i,t}) = h(D_{i,t}) f(J_{i,t}, h(D_{i,t})) - J_{i,t}(1 - F(J_{i,t}, h(D_{i,t})) \tag{A.13}$$

$$h(D_{i,t}) = \frac{D_{i,t} D_{-i,t}}{D_{i,t} + D_{-i,t}} (2\chi - \chi^2) \tag{A.14}$$

$$E_t \left\{ \frac{R^k_{i,t+1}}{R^k_{t+1}} \right\} = N^{\frac{1-\epsilon}{\epsilon}} \left(\frac{K_{i,t}}{K_t} \right)^{\frac{1-\epsilon}{\epsilon}}, \tag{A.15}$$

$$\frac{R_{i,t}}{R_t} = N^{\frac{\epsilon_D - 1}{\epsilon_D}} \left(\frac{D_{i,t}}{D_t} \right)^{\frac{\epsilon_D - 1}{\epsilon_D}}, \tag{A.16}$$

$$1 \leq R_{i,t}, \tag{A.17}$$

$$\psi D_{i,t} \leq J_{i,t}. \tag{A.18}$$

For the derivatives with respect to $g(\cdot)$ we can exploit that for the normal distribution it holds that $f'_Z(z) = -\frac{z}{h(D_{i,t})} f_Z(z)$, which simplifies the algebra considerably:

$$\frac{\partial g}{\partial J_{i,t}}(J_{i,t}, D_{i,t}) = -\gamma(1 - F_Z(J_{i,t})), \tag{A.19}$$

$$\frac{\partial^2 g}{\partial J^2_{i,t}}(J_{i,t}, D_{i,t}) = \gamma f_Z(J_{i,t}), \tag{A.20}$$

$$\frac{\partial g}{\partial D_{i,t}}(J_{i,t}, D_{i,t}) = 0.5\gamma h' f_Z(J_{i,t}), \tag{A.21}$$

$$\frac{\partial^2 g}{\partial D^2_{i,t}}(J_{i,t}, D_{i,t}) = 0.5\gamma \left(h'' + 0.5 \left(\frac{J^2}{h} - 1 \right) \frac{h'^2}{h} \right) f_Z(J_{i,t}), \tag{A.22}$$

$$= 0.5\gamma h'' f_Z(J_{i,t}) + 0.25 h'^2 f''_Z(J_{i,t}), \tag{A.23}$$

$$\frac{\partial^2 g}{\partial J_{i,t} D_{i,t}}(J_{i,t}, D_{i,t}) = -\gamma 0.5 J \frac{h'}{h} f_Z(J_{i,t}), \tag{A.24}$$

$$\tag{A.25}$$

where

$$h(D_{i,t}) = \frac{D_{i,t} D_{-i,t}}{D_{i,t} + D_{-i,t}} (2\chi - \chi^2), \tag{A.26}$$

$$h'(D_{i,t}) = \left(\frac{D_{-i,t}}{D_{i,t} + D_{-i,t}} \right)^2 (2\chi - \chi^2), \tag{A.27}$$

$$h''(D_{i,t}) = -2 \frac{D^2_{-i,t}}{(D_{i,t} + D_{-i,t})^3} (2\chi - \chi^2), \tag{A.28}$$

$$= -\frac{2}{D_{i,t} + D_{-i,t}} h'(D_{i,t}). \tag{A.29}$$

Under $\epsilon_D \to 0$ and omitting expectations operators for this proof, the FOCs are

$$J_{i,t} : -R^b_{t+1} + R^j_t + \gamma\left[1 - F(J_{i,t}, h(D_{i,t}))\right] + \mu_{J,t} = 0, \tag{A.30}$$

$$D_{i,t} : -R^b_{t+1} + R_t + \gamma\frac{1}{2}\left(\frac{D_{-i,t}}{D_t}\right)^2 (2\chi - \chi^2) f(J_{i,t}, h(D_{i,t})) + \hat{\mu}_{J,t} + \mu_{D,t} = 0, \tag{A.31}$$

$$R^k_{i,t+1} \& K_{i,t} : R^k_{i,t+1} - \epsilon R^b_{t+1} = 0, \tag{A.32}$$

together with (A.12) to (A.14), (A.17), (A.18) and the (modified) Kuhn-Tucker conditions

$$\hat{\mu}_{J,t} \geq 0 \tag{A.33}$$

$$\mu_{D,t} \geq 0, \tag{A.34}$$

$$\hat{\mu}_{J,t}(\psi D_{i,t} - J_{i,t}) = 0, \tag{A.35}$$

$$\mu_{D,t}(1 - R_{i,t}) = 0. \tag{A.36}$$

Under the assumption of a symmetric equilibrium and no entry and exit ($N = \frac{D_t}{D_{i,t}}$) all N banks make identical choices and with $B^b_t = \sum B_{i,t}$ their best-responses can be aggregated to

$$Q^b_t B^b_t + Q_t K^b_t + J_t = D_t + L_t, \tag{A.37}$$

$$E_t R^b_{t+1} = \frac{1 + \kappa Q^b_{t+1}}{Q^b_t}, \tag{A.38}$$

$$R^b_{t+1} = R^j_t + \gamma\left[1 - \Phi\left(\frac{J_t}{\sqrt{\nu D_t}}\right)\right] + \mu_{J,t}, \tag{A.39}$$

$$R^b_{t+1} = R_t + 0.5\gamma\frac{N-1}{N}\sqrt{\frac{\nu}{D_t}}\varphi\left(\frac{J_t}{\sqrt{\nu D_t}}\right) + \psi\hat{\mu}_{J,t} + \mu_{D,t}, \tag{A.40}$$

$$R^k_{t+1} = \epsilon R^b_{t+1}. \tag{A.41}$$

where $\nu = (N-1)(2\chi - \chi^2)$ is a measure of *liquidity (tail) risk* and with $\Phi(\cdot)$ as the standard normal CDF and $\varphi(\cdot)$ as the standard normal PDF. The result from the proposition follows after assuming $\frac{N-1}{N} \approx 1$.

We must however ensure that this is a (local) maximum of the profit function. This is unproblematic for the constrained case. For the unconstrained case the second partial derivative test requires that the hessian of the profit function is positive,

$$\det H_\Pi(D, J) > 0, \tag{A.42}$$

and that $\Pi_{JJ} < 0$. The latter is easy to see since $g_{JJ} < 0$ always. The condition on the determinant leads to

$$\frac{1-\epsilon_D}{\epsilon_D^2} R > 0.25\gamma h' f(J_{i,t}), \tag{A.43}$$

which implies that some monopsonism with $\epsilon_D < 1$ is necessary for a valid equilibrium (the RHS will always be positive). Plugging in the optimality condition for deposits results in

$$\frac{2-\epsilon_D}{\epsilon_D^2} > \frac{R^b}{R}, \tag{A.44}$$

or

$$\epsilon_D < \frac{\sqrt{1+8R^b/R}-1}{2R^b/R}. \tag{A.45}$$

Appendix A.4 Proof of proposition 4 (liquidity regimes)

Let $\theta = J/D$. For the first part, the deposit spread in terms of ν is given by

$$s_d(\nu) = 1 - \Phi\left(\theta\sqrt{\frac{D}{\nu}}\right) - 0.5\sqrt{\frac{\nu}{D}}\varphi\left(\theta\sqrt{\frac{D}{\nu}}\right), \tag{A.46}$$

with first derivative

$$\frac{\partial s_d(\nu)}{\partial \nu} = \left(0.5\theta\sqrt{D}\nu^{-1.5} - 0.25/\sqrt{\nu D} - 0.25\theta^2\sqrt{D}\nu^{-1.5}\right)\varphi\left(\theta\sqrt{\hat{D}}\right), \tag{A.47}$$

which has a unique root at $\nu = (2\theta - \theta^2)D = 2J - J^2/D$. The second part can be seen by acknowledging that $\partial s_l/\partial\nu > 0$ and then

$$\lim_{\nu\to\infty} s_l(\nu) = \lim_{\nu\to\infty} 1 - \Phi\left(\sqrt{\frac{J}{\nu D}}\right) \iff \lim_{x\to 0} 1 - \Phi(x) = 0.5. \tag{A.48}$$

Appendix A.5 Proof of proposition 5 (elasticity of deposits to reserves)

Express (20) in terms of the standard normal distribution with PDF φ and CDF Φ and insert (17). The lending rate is then given by

$$R^b(\hat{J}) = R^j + \gamma\left(1 - \Phi\left(\hat{J}/\sqrt{d_A(R^b(\hat{J}))}\right)\right). \tag{A.49}$$

Let $d'_A = \frac{\partial d_A}{\partial R^b}$ and define

$$h(J) = d_A(R^b(J))^{-1/2}, \tag{A.50}$$

with first derivative $h' = -0.5A^{-1.5}R^{b'}d'_A$. Inserting into (A.49) and taking the derivative w.r.t. J yields

$$R^{b'} = -\gamma(h + Jh')\phi(\cdot), \tag{A.51}$$

$$= -\gamma h\phi(\cdot) + \gamma J 0.5A^{-1.5}R^{b'}d'_A\phi(\cdot), \tag{A.52}$$

$$= -\gamma\frac{h\phi(\cdot)}{1 - 0.5\gamma JA^{-1.5}d'_A\phi(\cdot)}, \tag{A.53}$$

$$= -\frac{1}{(\gamma\hat{f})^{-1} - 0.5Jd'_A/A}, \tag{A.54}$$

which is negative for all $d'_A < 0$. Since the elasticity of A w.r.t. J is $E_{DJ} = \frac{\partial d_A/A}{\partial J/J} = \frac{\partial d_A}{\partial R^b}\frac{\partial R^b}{\partial J}\frac{J}{A} = d'_A R^{b'} J/A$, the first result from the proposition follows.

(A.52) can also be rewritten as

$$R^{b'} = -\gamma h\phi(\cdot) + \gamma J 0.5 A^{-1.5} R^{b'} d'_A \phi(\cdot), \tag{A.55}$$

$$= \gamma(0.5E_{DJ} - 1)\hat{f}. \tag{A.56}$$

Since $R^{b'} < 0$ and $d'_A < 0$, it follows that

$$\{E_{DJ} \geq 0 \wedge \gamma(0.5E_{DJ} - 1)\phi(\cdot) < 0\} \quad \Longrightarrow \quad E_{DJ} \in [0, 2). \tag{A.57}$$

Appendix A.6 Proof of proposition 6 (effectiveness of asset purchases)

i) The first part follows directly from the proof of proposition 5 in Appendix A.5. For the result on the marginal efficiency, the second derivative is given by

$$R^{b''} = \left[(J^2/A - 1)h' + Jh^3)\phi(\cdot) - 0.5\left(d''_A R^{b'} J/A + d'_A/A - {d'_A}^2 J/A^2 R^{b'}\right)\right] {R^{b'}}^2. \tag{A.58}$$

For $E_A = 0$ it follows that $d'_A = 0$ and $h' = 0$ and the equation collapses to

$$\frac{\partial^2 R^b}{\partial J^2} = \frac{J}{D}\hat{f}, \tag{A.59}$$

which is always positive.

ii) In terms of the standard normal distribution with PDF φ and CDF Φ, the deposit rate is given by

$$R(J) = R^j + \gamma\left(1 - \Phi(Jh) - 0.5h\varphi(Jh)\right), \tag{A.60}$$

with derivative

$$R' = \gamma\left(-(h + Jh') - 0.5h' + 0.5\frac{J}{D}(h + Jh')\right)\varphi(\cdot), \tag{A.61}$$

$$= \gamma\left(-(1 - 0.5E_{DJ}) + 0.5E_{DJ}/J + 0.5\frac{J}{D}(1 - 0.5E_{DJ})\right)\hat{f}, \tag{A.62}$$

$$\frac{\partial R}{\partial J/J} = \gamma\left(J(0.5J/D - 1)(1 - 0.5E_{DJ}) - 0.5E_{DJ}\right)\hat{f}. \tag{A.63}$$

iii) The borrowing-lending spread is given by

$$s_b\left(\hat{D}, \hat{J}\right) = \frac{1}{2\sqrt{\hat{D}}}\varphi\left(\hat{J}/\sqrt{\hat{D}}\right), \tag{A.64}$$

with first derivative

$$s_b{}' = 0.5\left(h' - \frac{J}{D}(h + Jh')\right)\varphi(Jh) \tag{A.65}$$

where from using $Jh' = -0.5E_{DJ}$ it is that

$$s_b{}' = 0.5\left(\frac{J}{D}(0.5E_{DJ} - 1) - 0.5E_{DJ}/J\right)\hat{f}, \tag{A.66}$$

after acknowledging that $E_{DJ} - 2 < 0$ it follows that $s_b' < 0$ iff

$$J\frac{J}{D} > \frac{E_{DJ}}{E_{DJ} - 2}. \tag{A.67}$$

Appendix A.7 Proof of proposition 7 (effectiveness of IOR policy)

1. (a) Again, the lending rate is then given by

$$R^b(\hat{J}) = R^j + \gamma\left(1 - \Phi\left(Jh\right)\right), \tag{A.68}$$

with

$$h(R^j) = d_A(R^b(R^j))^{-1/2}, \tag{A.69}$$

and first derivative $h' = -0.5hR^{b'}d_A'/A$. Again define the elasticity as $E_{DR^j} = \frac{\partial d_A/A}{\partial R^b}\frac{\partial R^b}{\partial R^j} = R^{b'}d_A'/A$. The derivative of the lending rate is then given by

$$R^{b'} = 1 - \gamma Jh'\varphi(Jh) \tag{A.70}$$

$$= \frac{1}{1 - 0.5\gamma JE_A\hat{f}}. \tag{A.71}$$

which is always positive.

(b) The deposit rate is

$$R(J) = R^j + \gamma\left(1 - \Phi(Jh) - 0.5h\varphi(Jh)\right), \tag{A.72}$$

with derivative

$$R' = 1 + 0.5\gamma\left(0.5\frac{J^2}{D} - 0.5 - J\right)E_{DR^j}\hat{f}. \tag{A.73}$$

The conjecture $R' \geq 1$ follows from the fact that $J\frac{J}{D} - J < 0$ since $0 \leq J \leq D$ while $E_{DR^j} < 0$ since $d_A' < 0$.

(c) The borrowing-lending spread is given by

$$s_b = 0.5h\varphi\left(Jh\right), \tag{A.74}$$

with first derivative

$$s_b{}' = 0.5\left(h' - \frac{J}{D}h\right)\varphi(Jh). \tag{A.75}$$

2. The MRR is binding whenever $MPJ_\psi < 0$. Conversely, when the MRR is slack

$$R^j - R^b + \gamma\left[1 - \hat{F}_\psi\right] > 0. \tag{A.76}$$

The economy will hence move towards the MRR for stimulative IOR policy (when $\Delta R^j < 0$) if $\frac{\partial MPJ_\psi}{\partial R^j} > 0$. The term $\gamma \left[1 - \hat{F}_\psi\right]$ is independent of R^j (D is constrained by J at the MRR) and we know from above that

$$R^{b'} = \frac{1}{1 - 0.5\gamma J E_A \hat{f}}. \tag{A.77}$$

Hence,

$$\frac{\partial MPJ_\psi}{\partial R^j} = 1 - \frac{1}{1 - 0.5\gamma J E_A \hat{f}}, \tag{A.78}$$

$$= \frac{-\gamma J E_A \hat{f}}{1 - 0.5\gamma J E_A \hat{f}}, \tag{A.79}$$

$$> 0, \tag{A.80}$$

since $E_A < 0$.

Appendix A.8 Proof of proposition 9 (effectiveness of open market operations with MRR)

i) As above, it is that

$$E^\psi_{DJ} = \frac{\partial d_A/A}{\partial J/J} = \frac{\partial d_A}{\partial R^b}\frac{\partial R^b}{\partial J}\frac{J}{A} = d'_A R^{b'} J/A. \tag{A.81}$$

From $E^\psi_{DJ} = 1$ we can rearrange to

$$\frac{\partial R^b}{\partial J} J = \left(\frac{\partial d_A}{\partial R^b}\right)^{-1} A = E_A^{-1}. \tag{A.82}$$

ii) In terms of the standard normal distribution with PDF φ and CDF Φ, the deposit rate when the MRR is binding is given by

$$R = (1-\psi)R^b + \psi R^j + \gamma\left(\psi\left[1 - \Phi(Jh)\right] - 0.5h\varphi(Jh)\right). \tag{A.83}$$

Taking the derivative w.r.t. J:

$$R' = (1-\psi)R^{b'} + \gamma\left(0.5\frac{J}{D}(h + Jh') - 0.5h' - \psi(h + Jh')\right)\varphi(Jh), \tag{A.84}$$

$$= (1-\psi)R^{b'} - \gamma\left(0.5\psi(h + Jh') + 0.5h'\right)\varphi(Jh). \tag{A.85}$$

Since we know that $Jh' = -.5E_{DJ}h = -.5h$:

$$R' = (1-\psi)R^{b'} - \gamma\left(0.25\psi - 0.25/J\right)\hat{f}. \tag{A.86}$$

iii) The proof follows from the above via

$$s^{b'} = R^{b'} - R'. \tag{A.87}$$

Appendix A.9 Proof of proposition 10 (effectiveness of IOR policy with MRR)

i. The proof follows from the MRR, $\psi D = J$, and the funds market equilibrium, $d_A(R^b) = D$. Since D is fixed because J is given

$$\frac{\partial d_A}{\partial R^j} = \frac{\partial d_A}{\partial R^b}\frac{\partial R^b}{\partial R^j} = 0. \tag{A.88}$$

If $\frac{\partial d_A}{\partial R^b} < 0$ it must be that $\frac{\partial R^b}{\partial R^j} = 0$. If however $\frac{\partial d_A}{\partial R^b} = 0$, $\frac{\partial R^b}{\partial R^j}$ is indetermined.

ii. As above, the deposit rate is

$$R = (1-\psi)R^b + \psi R^j + \gamma\left(\psi\left[1 - \Phi(Jh)\right] - 0.5h\varphi(Jh)\right). \tag{A.89}$$

We know from i. that the derivative of the first term is zero. Since $\frac{\partial d_A}{\partial R^j} = 0$ it follows that $h' = 0$ and the derivative of the last term is also zero.

iii. The proof follows again from

$$s^{b'} = R^{b'} - R'. \tag{A.90}$$

Appendix A.10 Proof of proposition 11 (effective lower bound)

If the MRR is binding, R^b is determined by $d(R^b) = D = \psi J$ and thereby fully independent of R^j. This means that only R is determined by the IOR rate, which at the DLB is fixed at 1.

Appendix A.11 Proof of proposition 12 (reversal rate)

For the slack MRR, the equations follow from (18) and (22). I.e., if the MRR is slack, then for the above $R = 1 + r = 1$ and (21) is inactive. However, R^b is still determined by (20), implying that the borrowing-lendign spread is directly affected by the DLB.

The second condition $\psi d(R^b) < J$ is important because otherwise the MRR binds and we are in the case of proposition 11, where there is no reversal.

Appendix B The linearized model

TBD

Appendix C Data

The observables of GDP, consumption, investment, wages, labor and inflation for the standard Smets and Wouters (2003, 2007) part of my model are also standard and are obtained from the ECB. Due to artificial dynamics in the civilian noninstitutional population series that arise from irregular updating (Edge et al., 2013), I use a 4-quarter trailing moving average to calculate per capita variables. Data on the loan rate and the household deposit rate is obtained directly from the ECB (similar data can be found in the ECB's statistical data warehouse, SDW).

The time series for liquidity provision policy X=JoMRR is using SDW data and is constructed by

- JOMRR: TOTRES/MINRES

- TOTRES: ILM.W.U2.C.L020100.U2.EUR + ILM.W.U2.C.L020200.U2.EUR + ILM.W.U2.C.L020300.U2.EUR
- MINRES: $\psi_{\text{pre-2012}}$*DEPOSITS
- $\psi_{\text{pre-2012}}$ is the mean of (RESERVES/DEPOSITS) until 2011:IV
- RESERVES: BSI.M.U2.N.R.LRE.X.1.A1.3000.Z01.E + BSI.M.U2.N.R.LRR.X.1.A1.3000.Z01.E
- DEPOSITS: BSI.M.U2.N.R.L2A.H.1.A1.3000.Z01.E + BSI.M.U2.N.R.L2B.L.1.A1.3000.Z01.E

Note that as of Jan 2022, the ECB's time series for excess reserves (BSI.M.U2.N.R.LRE.X.1.A1.3000.Z01.E) is missleading because the deposit facility and fixed term deposits are not counted as (excess) reserves, which they are by definition.

Appendix D More parameter estimates

	Prior			Posterior				
	distribution	mean	sd	mean	sd	mode	5% HPD	95% HPD
ρ_r	beta	0.500	0.200	0.340	0.093	0.381	0.187	0.487
ρ_{nir}	beta	0.500	0.200	0.983	0.006	0.984	0.974	0.993
ρ_g	beta	0.500	0.200	0.940	0.016	0.938	0.914	0.967
ρ_z	beta	0.500	0.200	0.992	0.006	0.995	0.986	0.998
ρ_u	beta	0.500	0.200	0.931	0.050	0.894	0.857	0.988
ρ_p	beta	0.500	0.200	0.596	0.149	0.683	0.353	0.818
ρ_w	beta	0.500	0.200	0.828	0.054	0.906	0.748	0.914
ρ_i	beta	0.500	0.200	0.752	0.044	0.709	0.684	0.826
ρ_ϵ	beta	0.500	0.200	0.787	0.046	0.846	0.716	0.860
ρ_γ	beta	0.500	0.200	0.963	0.012	0.970	0.943	0.982
$\text{root}_{x,1}$	beta	0.500	0.200	0.822	0.092	0.767	0.683	0.959
$\text{root}_{x,2}$	beta	0.500	0.200	0.809	0.103	0.862	0.651	0.955
μ_p	beta	0.500	0.200	0.570	0.154	0.628	0.282	0.780
μ_w	beta	0.500	0.200	0.688	0.098	0.807	0.541	0.839
ρ_{gz}	normal	0.500	0.250	1.160	0.132	1.075	0.967	1.382
σ_r	inv.gamma	0.100	0.250	0.063	0.007	0.058	0.054	0.075
σ_{nir}	inv.gamma	0.100	0.250	0.006	0.000	0.007	0.006	0.007
σ_g	inv.gamma	0.100	0.250	0.226	0.019	0.215	0.196	0.256
σ_z	inv.gamma	0.100	0.250	0.199	0.018	0.189	0.172	0.230
σ_i	inv.gamma	0.100	0.250	0.381	0.051	0.399	0.298	0.465
σ_p	inv.gamma	0.100	0.250	0.143	0.015	0.129	0.119	0.167
σ_w	inv.gamma	0.100	0.250	0.204	0.025	0.193	0.165	0.244
σ_u	inv.gamma	0.100	0.250	0.105	0.077	0.177	0.022	0.222
σ_ϵ	inv.gamma	0.100	0.250	0.002	0.000	0.002	0.002	0.002
σ_γ	inv.gamma	0.100	0.250	0.221	0.049	0.246	0.141	0.297
σ_x	inv.gamma	0.100	0.250	0.326	0.024	0.344	0.289	0.365

Table D.2: Estimation results for parameters govering the exogenous shock processes.

Appendix E Estimated expected PLB durations

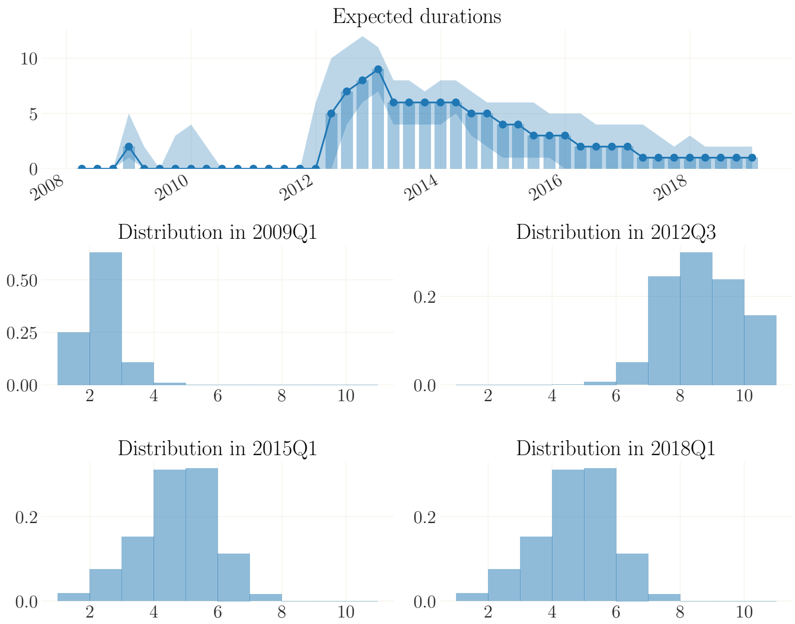

Figure E.10: Estimated expected ELB durations based on the benchmark estimation. Bars in the top panel mark the mean estimate. The shaded area represents 90% credible sets. The lower panels show histograms of the distribution of ELB durations. The last bar to the right marks the probability of a duration of 10 or more quarters.