The Hockey Stick Phillips Curve and the Effective Lower Bound

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January 12, 2023

Abstract

We show that the interplay between a binding effective lower bound (ELB) on nominal interest rates and the costs of external financing weakens the disinflationary effect of financial shocks. In normal times, production factor costs dominate firms' marginal costs and thereby inflation; higher credit spreads are balanced-out by lower nominal rates. At the ELB, higher spreads can offset the effect of lower production factor costs on firms' price setting. The Phillips curve hence features a hockey stick shape: flat at the ELB, but conventional during normal times. This mechanism also weakens forward guidance effects, since such policy reduces spreads and financing costs.

Keywords: Phillips Curve, Financial Frictions, Effective Lower Bound, Disinflation, Forward Guidance
JEL: C62, C63, E31, E32, E44, E52, E58, E63

[⋆] The views expressed in this paper are those of the authors and do not necessarily coincide with the views of the Deutsche Bundesbank or the Eurosystem. We are grateful to Joachim Keller, Keith Kuester, Taisuke Nakata, Johannes Wieland, and participants of several conferences and a seminar at the Bundesbank for helpful discussions and comments. Parts of the research leading to the results in this paper has received financial support from the Alfred P. Sloan Foundation under the grant agreement G-2016-7176 for the MMCI at the IMFS Frankfurt. Gregor Boehl gratefully acknowledges financial support by the DFG under CRC-TR 224 (projects C01 and C05) and under project number 441540692.

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1 Introduction

What is the relationship between inflation and economic activity? Given the fundamental role of these two concepts, it is quite troubling that this question is still puzzling the economic profession. In particular, the Global Financial Crisis of 2007/2008 and the associated financial turmoil led to the *missing disinflation puzzle*: despite substantially negative output gaps, inflation fell only modestly and thus seemed disconnected from economic activity.[1] This observation raised considerable interest in analyzing the seemingly flat Phillips curve (Ball and Mazumder, 2011; Coibion and Gorodnichenko, 2015; Harding et al., 2022). While the explanations put forward are numerous and manifold, we found one key contributing factor yet to be missing: the effective lower bound (ELB) on nominal interest rates, which was reached by several central banks around the globe at the same time as the observed inflation puzzles.

In this paper, we show that the interplay of the ELB and financial frictions may reshape the relationship between inflation and output if financial shocks are prevailing. Recent research documents that financial frictions can be crucial for firms' price setting behavior and, thereby, for inflation dynamics (e.g. Gilchrist et al., 2017). We argue that firms' marginal costs are dominated by the procyclical costs of production factors in normal times. In the presence of financial frictions, however, marginal costs further contain the costs of external financing. These costs consist of the real safe interest rate and a countercyclical credit spread reflecting financial frictions. We show that higher credit spreads can substantially offset lower production factor costs if the nominal rate is constrained by the ELB. In this case, the costs of external financing considerably weaken the supply-side link between output and prices. As a result, financial shocks at the ELB induce only moderate disinflation, and may in extreme cases even be inflationary.

Taking the ELB into account, the resulting *observational Phillips curve*[2] is thus shaped like a hockey stick. For normal times with positive or rather mildly negative output gaps, it exhibits a conventional positive slope in output gap - inflation space. In contrast, the slope is considerably flat for significantly negative output gaps when the ELB is binding. This non-linear, kinked Phillips Curve provides an explanation for the puzzles of missing disinflation consistent with the observed timing of events. As seen in Figure 1, in 2008:Q4, corporate spreads with rating BAA peaked at almost 6%, while the Federal funds rate reached

[1] For example, the US output gap was -5.3% in Q2 2009, accompanied by a core inflation (excluding food and energy) of 1.83%, only slightly below the central bank target of 2%.

[2] We use this term to refer to the reduced-form relationship between realized (equilibrium) values for inflation and output gap, i.e. the *observed* or *empirical* Phillips curve. As discussed below, this is not equivalent to the New Keynesian Phillips curve describing firms' price setting behavior.

its lower bound of almost 0% in 2009:Q1. Both elevated corporate spreads and the binding lower bound persisted until the end of 2015, coincidental with the observed weakening of the relationship between inflation and output.

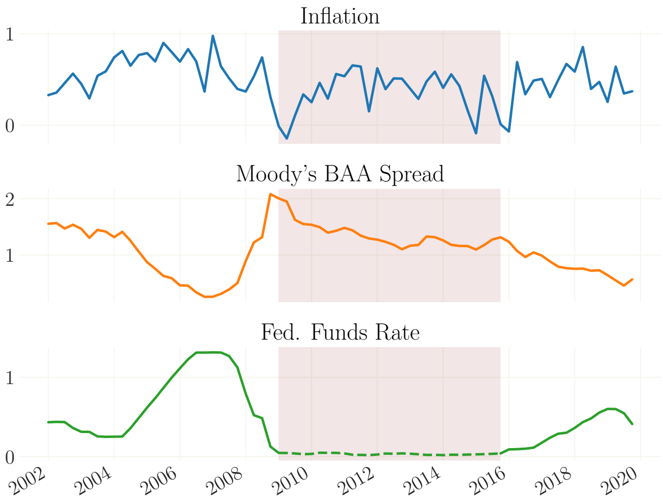


Figure 1: Time series of inflation, BAA spread and US interest rates from 2002 to 2020. Quarterly data in percentage points. The shaded area depicts the episode during which the ELB was binding.

We show these results in a tractable New Keynesian DSGE model featuring financial frictions. In the model, workers need to be paid before production (as in Ravenna and Walsh, 2006), generating external financing needs for the entrepreneurs operating the firms. Due to a costly state verification problem à la Townsend (1979) and Bernanke et al. (1999), the costs of external finance comprise a risk premium in the form of a countercyclical credit spread which depends on entrepreneur leverage. We focus on the effects of risk premium shocks in the spirit of Smets and Wouters (2007). These shocks are known to have a large explanatory power for the joint movement of consumption, investment and inflation following the 2007/2008 recession (Gust et al., 2017; Kulish et al., 2017; Boehl and Strobel, 2020; Boehl et al., forthcoming).

Our first contribution is to provide analytic solutions for macroeconomic dynamics after financial shocks, both for normal times and for a binding ELB. We show that a longer expected ELB duration can be associated with weaker disinflationary effects of financial shocks. This case occurs for large financial shocks if the elasticity of the credit spread with respect to entrepreneur leverage is sufficiently high.[3] The analytic solutions furthermore highlight that even an

[3] Accordingly, this effect depends crucially on the presence of financial frictions and is hence

overall increase of inflation following contractionary financial shocks is possible, and may in particular occur if the ELB is expected to bind for an extended period of time. In this case, financial shocks move output and inflation in opposite directions, thus appearing as supply-type disturbances, in sharp contrast to their usual appearance as demand shocks away from the ELB.

As our second contribution, we discuss the associated implications for monetary policy. Shocks to the monetary policy rule generate macroeconomic dynamics that are very similar to financial shocks. As a consequence, forward guidance shocks with relatively low persistence can even be disinflationary: the effect of keeping expected refinancing costs lower in the future may dominate the long-run effect of increasing the price level by stimulating consumption. Hence, this also provides an explanation for the forward guidance puzzle (Carlstrom et al., 2015; Del Negro et al., 2015a; Kiley, 2016) and suggests that any forward guidance measures must be undertaken with vigor. Furthermore, the central bank might find itself in a knife-edge scenario where the appropriate window for systematic policy responses guaranteeing a determinate equilibrium is rather small.

Our hockey stick Phillips curve is well-supported by recent empirical work showing that financial shocks can be disinflationary if supply-side effects dominate demand effects. Various contributions find empirical evidence in favor of such a (financial) cost channel (Barth III and Ramey, 2001; Chowdhury et al., 2006; Tillmann, 2008; Abbate et al., 2021). Similarly, Gaiotti and Secchi (2006) find this cost channel to be proportional to working capital, using Italian firm-level data. Our paper provides a theoretical foundation for these papers and highlights that both the degree of financial frictions and a binding ELB are particularly relevant.

The issue of missing (dis-)inflation in recent years was first brought up by Ball and Mazumder (2011) and subsequently confirmed for many advanced economies by Friedrich (2016). Manifold explanations were put forward, encompassing anchored expectations (Ball and Mazumder, 2018; Coibion and Gorodnichenko, 2015), various measures of economic slack (Gordon, 2013; Watson, 2014), supply shocks and wage rigidities (Daly and Hobijn, 2014; Harding et al., 2022), optimal monetary policy, potentially in combination with financial frictions (Lieberknecht, 2019; Sims and Wu, 2019; McLeay and Tenreyro, 2020) or global factors (Bobeica and Jarociński, 2019; Forbes, 2019). Compared to this literature, we provide a complementary explanation for inflation dynamics that also matches the particular timing of the observed missing (dis-)inflation: the ELB affects the cyclicality of marginal costs via the costs of external financing, thereby leading to an observational disconnect between inflation and output.

A related strand of the literature investigates these recent inflation dynamics

absent in the standard New Keynesian model.

through the lens of New Keynesian DSGE models, notably Christiano et al. (2015), Del Negro et al. (2015b) and Gilchrist et al. (2017). In line with our paper, these contributions show that adding financial frictions to DSGE models helps to explain the missing disinflation puzzle in the US. Closely related to our work, Gilchrist et al. (2017) explain inflation dynamics via financial distortions, i.e. higher credit spreads in recessions. While our paper shares this argument, we provide additional insights that a binding ELB strongly amplifies the effects of financial frictions such that credit spreads may even dominate inflation dynamics. This is in line with Bianchi and Melosi (2017) and Boehl and Strobel (2020), who find that accounting for the ELB substantially improves the empirical fit of estimated DSGE models. We also shed light on the necessary conditions for inflationary effects of financial shocks – a feature that is present in several financial friction models (Christiano et al., 2010; Meh and Moran, 2010; Gerali et al., 2010).

Lastly, our paper is related to the literature on Neo-Fisherianism, which argues that the causality between the policy rate and inflation is positive even in the short run (Gabaix, 2020; Cochrane, 2011, 2016, 2017). We show that such effects may arise at the ELB, and – theoretically – even in normal times if the elasticity of marginal costs to the credit spread is large enough. This is in contrast to García-Schmidt and Woodford (2019), who argue that Neo-Fisherian effects arise only after credible changes in long-run monetary policy targets.

We continue in Section 2 by outlining the model and discussing the components of marginal costs in this framework. In Section 3, we derive closed-form solutions for macroeconomic dynamics following financial shocks. Section 4 complements by showing numerical solutions and analyzing the resulting observational Phillips curve. In Section 5, we investigate the implications for monetary policy at the ELB. Section 6 concludes.

2 Model

Our analysis is based on a tractable New Keynesian DSGE model featuring financial frictions based on Boehl (2022b) and Lieberknecht (2019). Production is subject to a working capital channel as in Ravenna and Walsh (2006). A distinct role for external finance is motivated via a costly state verification problem in the spirit of Townsend (1979) and Bernanke et al. (1999). Entrepreneurs operating wholesale firms borrow from financial intermediaries to finance production, and their shares are traded on financial markets. Their (homogeneous) good is sold to a monopolistic retail sector where diversification takes place. The resulting final good is bought by a representative household for consumption. The labor market is perfectly competitive. A central bank sets the nominal interest rate subject to an effective lower bound.

2.1 Households

Households maximize the expected present value of lifetime utility by choosing consumption of a composite good C_t and hours supplied to the labor market H_t. They can deposit monetary savings D_t at financial intermediaries (also called banks in the following), for which they receive the gross nominal interest rate R_t in the next period. The household's optimization problem is completely standard and yields the usual Euler equation and an intra-temporal labor supply equation

$$C_t^{-\sigma} = \beta E_t \left[\frac{R_t}{\Pi_{t+1}} U_t C_{t+1}^{-\sigma} \right], \tag{1}$$

$$H_t^{\eta} = W_t C_t^{-\sigma}, \tag{2}$$

where Π_t is gross inflation and W_t is the real wage. U_t is a *financial shock*, i.e. a premium on the risk-free interest rate reflecting the state of the financial system (as in Smets and Wouters, 2007). The parameters σ, η and β are the inverse elasticity of intertemporal substitution, the inverse Frisch elasticity of labor supply and the discount rate, respectively.

2.2 Wholesale and retail firms

The wholesale sector consists of a continuum of firms. Each firm j is operated by a risk-neutral entrepreneur and produces a homogeneous good using a production function that is linear in labor (the only production factor) subject to a a firm-specific idiosyncratic productivity shock. Workers have to be paid before production takes place, while returns are realized at the end of the period. This working capital channel (also labeled the *cost channel*) follows Ravenna and Walsh (2006), is supported by empirical work (see references in the introduction) and motivates a role for external finance. The loan volume demanded by each entrepreneur is the difference between required working capital $W_t H_{j,t}$ and equity $N_{j,t}$.

The realization of the idiosyncratic productivity shock is private information of the entrepreneur; banks can only observe produced output when paying monitoring costs. The contract that solves this costly state verification problem specifies that the interest rate on a loan obtained by an entrepreneur from the intermediary $R_{j,t}^L$ contains an endogenous risk premium on the prevailing real interest rate. The risk premium is a credit spread that depends positively on the individual firm's leverage $LEV_{j,t} = \frac{W_t H_{j,t}}{N_{j,t}}$. It can be shown that all entrepreneurs make identical decisions in equilibrium, such that the aggregate loan rate is given by

$$R_t^L = z \left(\frac{W_t H_t}{N_t} \right) \frac{R_t}{E_t[\Pi_{t+1}]} U_t, \tag{3}$$

where $\nu \equiv z'(\cdot) > 0$ (see Lieberknecht, 2019). Intuitively, when the leverage ratio rises, the premium on external finance increases because less collateral is provided such that the loan becomes more risky.

Since the wholesale sector is assumed to be perfectly competitive, wholesale firms are price takers. In the aggregate, no-arbitrage requires the rate of return on working capital to equal the rate on external funding. It follows that firms' marginal costs are given by

$$MC_t = W_t R_t^L = W_t \, z\left(\frac{W_t H_t}{N_t}\right) \frac{R_t}{E_t[\Pi_{t+1}]} U_t. \tag{4}$$

With respect to equity financing, we assume that entrepreneurs can issue equity in the stock market, which is bought by risk-neutral financial traders associated with the financial intermediaries. Imposing no arbitrage on financial markets and noting that entrepreneurs must be indifferent between external finance and equity finance in equilibrium implies that the expected return on equity equals the loan rate. Invoking rule-of-thumb behavior from financial traders, Boehl (2022b) shows that the evolution of equity can be represented by a function $\Psi(\cdot)$ such that

$$N_t = \Psi\left(Y_t\right), \tag{5}$$

with $\Psi'(\cdot) > 0$, such that equity financing is procyclical with respect to output, as in standard financial accelerator models à la Bernanke et al. (1999). The assumption of equity financing has important implications relative to alternative models with financial frictions as it disbands the necessity for an additional state variable such as entrepreneurs' net worth, and therefor also does not require strong assumptions on the live cycle of entrepreneurs.[4] This setup is also advantageous because it is in practice difficult to find a meaningful empirical counterpart of aggregate entrepreneurs' net worth.

After wholesale goods have been produced, retailers buy the homogeneous good $Y_{j,t}$ on the wholesale market. After differentiation, they sell it in the monopolistically competitive good market. Firms' price setting decisions are subject to nominal rigidities à la Calvo (1983), i.e. they can only adjust their prices each period with a probability of $(1-\zeta)$.

2.3 The central bank

The central bank follows a standard rule for the notional gross nominal interest rate R_t^n,

$$\frac{R_t^n}{R^n} = \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} \exp(v_t), \tag{6}$$

[4] Such assumptions are often necessary to detain entrepreneurs from being independent from external finance in the long run.

where v_t is a monetary policy shock following an AR(1) process. The policy rate R_t is subject to an ELB constraint and cannot fall below $\bar{R}$:

$$R_t = \max\left\{\bar{R}, R_t^n\right\} \tag{7}$$

Note that the max constraint acts on the notional rate R_t^n instead of the monetary policy rule directly. Thus, the shock v_t can hence be understood as a promise by the central bank to keep monetary policy loose (for negative v_t) for an extended period of time. Correspondingly, when the ELB constraint in Equation (7) binds, v_t can be understood as a *forward guidance shock* as, given a sufficiently large autoregression coefficient, it prolongs the expected duration of the ELB.

2.4 Understanding the components of marginal costs

In our framework, financial frictions originate in the firm sector and therefore primarily affect the supply side of the economy. The role of financial frictions for marginal costs and inflation dynamics is thus best understood by studying the New Keynesian Phillips curve. Linearizing around an efficient steady state[5], and denoting log-deviations from steady state via small-case letters, the New Keynesian Phillips curve may be represented in the familiar textbook form

$$\pi_t = \kappa\, mc_t + \beta E_t[\pi_{t+1}], \tag{8}$$

with slope $\kappa \equiv \frac{(1-\zeta\beta)(1-\zeta)}{\zeta}$. Hence, financial frictions do not alter the price setting behavior of firms per se; prices are tied to marginal costs and expectations of future inflation. However, financial frictions determine and affect the components of marginal costs, which are given by

$$mc_t = w_t + (r_t - E_t[\pi_{t+1}]) + s_t, \tag{9}$$

where s_t denotes the linearized credit spread $s_t = z(lev_t) + u_t$. This highlights that marginal costs consist of three components: a) the real wage (called *factor costs* in the following) as in the standard New Keynesian model, b) the risk-free real interest rate (*cost channel*) and c) the credit spread (also known as the *external finance premium*). The latter two components jointly constitute the costs of external finance.

In the following, we take a closer look at the cyclicality of these components, i.e. their co-movement with output. The real wage is unambiguously procyclical: as output increases, firms need to offer a higher real wage in order to attract

[5]Steady state subsidies from the government (financed by lump-sum taxes) can correct for the two inefficiencies arising from monopolistic competition and the presence of financial frictions (Lieberknecht, 2019).

more labor to expand production. The cyclicality of the cost channel depends on the source of aggregate fluctuations. A stable equilibrium requires that the real interest rate rises in response to an increase in inflation to dampen demand. The cost channel is hence procyclical for demand-side shocks (inflation and output move in the same direction), and countercyclical for supply-side shocks (inflation and output move in opposite directions).

Lastly, the credit spread is canonically countercyclical in a financial accelerator economy (Bernanke et al., 1999). The three components of marginal costs are thus characterized by opposing cyclicality over the business cycle: for demand-side shocks, the real wage and the real interest rate are procyclical, whereas the credit spread is countercyclical. Since firms' price setting decisions are tightly connected to marginal costs, the relative dynamics of these components over the business cycle are thus crucial for inflation dynamics.

Before moving to the model analysis, we note that the countercyclicality of the credit spread implies a key parameter restriction in our model setup. With the credit spread depending positively on entrepreneur leverage (see above), leverage is required to be countercyclical as well. Using the household's intra-temporal optimality condition, leverage can be written as

$$lev_t = -(\psi - 1 - \sigma - \eta) y_t, \tag{10}$$

where $\psi = \Psi'(\cdot)$ denotes the elasticity of equity with respect to output. The necessary and sufficient condition for leverage to be countercyclical is thus that the term in brackets is larger than zero, i.e. that the procyclicality of net worth outweighs the procyclicality of the wage bill:

Assumption 1. *The elasticity of net worth with respect to output satisfies*

$$\psi > 1 + \sigma + \eta > 0. \tag{11}$$

The latter inequality follows from the conventional assumptions $\sigma > 0$ and $\eta > 0$.

3 Financial shocks at the effective lower bound

In this section, we analyze how a binding ELB affects the transmission of financial shocks in the model economy. To this end, we derive closed-form general equilibrium solutions for normal times and for a binding ELB. Contrasting these two cases highlights that macroeconomic dynamics at the ELB may be fundamentally different.

3.1 The propagation of financial shocks in normal times

We first analyze the macroeconomic effects of financial shocks in normal times, i.e. when the ELB is not binding. The model can be represented in three equations[6]

$$\pi_t = \kappa\gamma\, y_t + (\beta - \kappa)E_t[\pi_{t+1}] + \kappa r_t + \kappa u_t, \tag{12}$$

$$y_t = -\sigma^{-1}\left(r_t - E_t[\pi_{t+1}] + u_t\right) + E_t[y_{t+1}], \tag{13}$$

$$r_t = \max\{\phi_\pi \pi_t + \phi_y y_t + v_t, \bar{r}\} \tag{14}$$

where $\gamma \equiv \sigma + \eta - \nu(\psi - 1 - \sigma - \eta)$. The exogenous processes for the financial shock u_t and the monetary policy shock v_t are

$$u_t = \rho\, u_{t-1} + \epsilon_t, \tag{15}$$

$$v_t = \rho_r\, v_{t-1} + \epsilon_{r,t}. \tag{16}$$

Equation (12) again represents the New Keynesian Phillips curve, where the third and fourth term reflect the cost channel and the purely exogenous markup effect that arises from financial shocks.[7] Equation (13) is the Euler equation, and Equation (14) is the monetary policy rule setting the (notional) interest rate as a function of inflation and output (gap).[8] In normal times, the ELB constraint does not bind, such that (13) and (14) are identical to the textbook New Keynesian model. Financial frictions thus manifest solely in the New Keynesian Phillips curve, highlighting that the financial accelerator is a supply-side friction that directly affects inflation dynamics via firms' price setting decisions.

We solve the model via the method of undetermined coefficients and guess that the equilibrium responses of endogenous variables are linear functions of the exogenous financial shock.[9]

Proposition 1. *The impact responses of inflation and output to a financial shock in normal times are given by*

$$\pi_t = \lambda_0^\pi\, u_t, \tag{17}$$

$$y_t = \lambda_0^y\, u_t, \tag{18}$$

[6]See the Appendix for more details on this particular representation.

[7]This exogenous effect differentiates financial shocks from pure demand shocks (e.g. natural rate shocks) that appear solely in the Euler equation.

[8]Note that for financial shocks, the responses of output and the output gap are identical: an efficient economy without nominal rigidities and financial frictions does not respond to financial shocks.

[9]We assume that determinacy conditions hold. See Section 5.2 and Footnote 10 for a closer analysis of the corresponding requirements for the coefficients in the monetary policy rule.

where

$$\lambda_0^\pi = -\frac{\kappa\gamma - \kappa\sigma(1-\rho)}{(1-\beta\rho)(\sigma(1-\rho)+\phi_y) + \kappa\gamma(\phi_\pi - \rho) - \kappa\sigma(1-\rho)(\phi_\pi-\rho)}, \tag{19}$$

$$\lambda_0^y = -\frac{1+(\phi_\pi-\rho)\lambda_0^\pi}{\sigma(1-\rho)+\phi_y}. \tag{20}$$

Proof. See Appendix. ■

Lemma 1. *The impact responses of inflation and output to a financial shock in normal times are negative, i.e.*

$$\lambda_0^\pi < 0, \tag{21}$$

$$\lambda_0^y < 0, \tag{22}$$

if the elasticity of the credit spread to entrepreneur leverage satisfies

$$\nu < \frac{\eta + \rho\sigma}{\psi - 1 - \sigma - \eta}. \tag{23}$$

Proof. See Appendix. ■

Proposition 1 and Lemma 1 show that financial shocks are (usually) demand-type shocks. A financial shock increases the wedge between the interest rate controlled by the central bank and the return on deposits for households, thereby reducing current consumption. Thus, an adverse financial shock decreases overall output. Via the New Keynesian Phillips curve, inflation decreases as well, since factor costs dominate over external financing costs.

The analytic solutions from Proposition 1 clearly display the different channels through which the financial shock operates. In λ_0^π, the first term in the numerator is the slope of the Phillips curve with respect to output, whereas the second term captures the exogenous markup effect of the financial shock. Following an adverse financial shock, factor costs decrease, because labor demand falls given the decline in demand (the first part of $\kappa\gamma$). This in turn reduces inflation. At the same time, the financial shock increases the costs of production via the external finance premium, as financial frictions in the firm sector intensify (the second part of $\kappa\gamma$ and the markup effect). This increase in the credit spread partially counteracts the decline in factor costs, weakening the overall disinflationary effect.

The cost channel is represented by the last term in the denominator of λ_0^π. This term features a negative sign and is thus – ceteris paribus – disinflationary. Generally, if the central bank reacts stronger (weaker) to fluctuations in inflation and output (which is captured by ϕ_π and ϕ_y), the denominator is larger (smaller), such that the overall response of inflation is smaller (larger). However, lower nominal interest rates in reaction to an overall decline in inflation also decrease marginal costs directly. This amplifies the disinflationary response. The cost channel thus weakens the overall stabilizing property of the central bank's interest rate policy.

Following financial shocks, the various components of marginal costs thus move in different directions. Whereas factor costs and the cost channel induce a disinflationary response, the credit spread channel weakens it. As seen in Lemma 1, the overall inflation response in normal times is negative, as long as the elasticity of the credit spread to entrepreneur leverage is not excessively large.[10] In this case, the cost channel and credit spreads approximately balance out. As a consequence, the procyclical factor costs dominate the price setting of firms, leading to a positive co-movement of inflation and output.

Nevertheless, as summarized in Lemma 2 below, the analytic solutions also reveal that an overall increase of inflation following adverse financial shocks is in principle possible. This situation – inflation and output move in opposite directions – may occur if the credit spread channel dominates both factor costs and the cost channel because the elasticity of the credit spread to leverage is (excessively) large:

[10]Lemma 1 guarantees that the denominator in λ_0^π is positive, which is required for determinacy (as shown in the Appendix). Intuitively, the model is only determinate if a stronger central bank reaction to deviations from steady state translates into lower deviations in general equilibrium. The combination of a positive numerator from Lemma 1 and determinacy thus yields $\lambda_0^\pi < 0$ (note the minus in front of the fraction).

Lemma 2. *The impact response of inflation to a financial shock in normal times is positive if the elasticity of the credit spread to entrepreneur leverage satisfies*

$$\nu > \frac{\eta + \rho\sigma}{\psi - 1 - \sigma - \eta}. \tag{24}$$

Proof. See Appendix. ■

Note that this result relies crucially on the presence of financial frictions linking marginal costs to credit spreads: in the absence of financial frictions, the policy functions in Proposition 1 are unambiguously negative. In the following, however, we focus on the case in which the financial shock is a classic demand shock in normal times to maintain the analogy to the Global Financial Crisis. We hence generally assume that Equation (24) is not satisfied such that λ_0^π remains negative:

Assumption 2. *The elasticity of the credit spread to entrepreneur leverage satisfies Condition* (23) *from Lemma 1.*

This implies the natural case of an upward sloping Phillips curve for financial shocks in normal times, i.e. a positive relationship between inflation and output.

3.2 The propagation of financial shocks at the ELB

We now turn to the case of a binding ELB. To this end, we assume that a financial shock endogenously brought the economy to the ELB and makes private agents expect the ELB to bind for a certain number of periods (often called the *ELB spell duration*, e.g. Holden, 2019). In this section, we take this ELB spell duration as given and do *not* adjust agents' expectations on the spell duration to any *additional* shocks (we discuss this in Section 4). This scenario hence focuses on *marginal* effects of (further) financial shocks at the ELB that do not alter the expected ELB length. While this perspective abstracts from the mapping between shocks and the expected duration of the ELB, it allows for a straightforward analytical comparison to the case of normal times.

We guess that the equilibrium responses of endogenous variables are linear functions of the exogenous financial shock and the ELB value for the nominal interest rate $\bar{r}$. Again using the method of undetermined coefficients shows that the equilibrium responses of inflation and output can be characterized by recursive policy functions which are conditionally linear given the expected ELB spell duration:

Proposition 2. *Suppose that the ELB on nominal interest rates is expected to bind for $T > 0$ periods. The impact responses of inflation and output to a financial* *shock are then given by*

$$\pi_t = \lambda_T^{\pi}\, u_t + \mu_T^{\pi}\, \bar{r}, \tag{25}$$

$$y_t = \lambda_T^{y}\, u_t + \mu_T^{y}\, \bar{r}, \tag{26}$$

where

$$\lambda_T^{\pi} = \kappa\left(1 - \gamma\sigma^{-1}\right) + \rho\left(\beta - \kappa + \kappa\gamma\sigma^{-1}\right)\, \lambda_{T-1}^{\pi} + \rho\kappa\gamma\, \lambda_{T-1}^{y}, \tag{27}$$

$$\mu_T^{\pi} = \kappa(1 - \gamma\sigma^{-1}) + (\beta - \kappa + \kappa\gamma\sigma^{-1})\, \mu_{T-1}^{\pi} + \kappa\gamma\mu_{T-1}^{y} \tag{28}$$

$$\lambda_T^{y} = -\sigma^{-1} + \rho\sigma^{-1}\, \lambda_{T-1}^{\pi} + \rho\, \lambda_{T-1}^{y}, \tag{29}$$

$$\mu_T^{y} = \sigma^{-1} + \sigma^{-1}\mu_{T-1}^{\pi} + \mu_{T-1}^{y} \tag{30}$$

where $\{\lambda_0^{\pi}, \lambda_0^{y}\}$ *as in Proposition 1 and* $\mu_0^{\pi} = \mu_0^{y} = 0$.

Proof. See Appendix. ■

To interpret Proposition 2, consider the inflation response under an expected ELB duration of one quarter (λ_1^{π}). Under Assumption 2, the impact responses of inflation and output are negative in normal times ($\lambda_0^{\pi} < 0$, $\lambda_0^{y} < 0$). In Equation (27), the second and third term are thus negative. For persistent shocks, these terms induce a monotonically decreasing recursion. This shows the ELB's amplification property: the impact response of inflation increases (ceteris paribus) in the expected ELB spell duration. This reflects the inability of the central bank at the ELB to counteract further contractionary shocks via additional (conventional) monetary stimulus. At the same time, the resulting upward pressure on real interest rates depresses consumption, and accordingly output – the financial shock is contractionary.

However, Proposition 2 also shows that there is an opposing effect on the overall inflation response at the ELB, captured by the first term in Equation (27). This term can be positive, such that there is potential for a policy function for inflation that is *concave* and thus partially increasing in the expected ELB spell duration. In other words, it is possible that the disinflationary effect following adverse financial shocks is *lower* if the ELB is expected to bind for a longer period of time. This requires two necessary conditions, which we postulate in the following Lemma.

Lemma 3. *A concave inflation policy function in the expected ELB spell duration requires that the elasticity of the credit spread with respect to entrepreneur leverage satisfies*

$$\nu > \frac{\eta}{\psi - 1 - \sigma - \eta} \tag{31}$$

and that the size of the financial shock satisfies

$$u_t > -\bar{r} = \beta^{-\sigma} - 1. \tag{32}$$

Proof. See Appendix. ■

The first part of Lemma 3 shows that the overall response of inflation following inflationary shocks depends crucially on the elasticity of the credit spread with respect to entrepreneur leverage. Intuitively, a concave policy function for inflation requires that the credit spread dominates both the factor cost and the cost channel. If financial frictions are sufficiently pronounced such that ν is large, credit spreads may dominate the price setting of firms at the ELB, thereby increasing inflation ceteris paribus. This result also naturally implies that the corresponding effect is absent in the standard NK model (in which $\nu = 0$).[11] For the following analysis, we capture this scenario via the following assumption, a weaker version of Assumption 2:

Assumption 3. *The elasticity of the credit spread with respect to entrepreneur leverage satisfies Condition* (31) *from Lemma 3.*

The second necessary condition refers to the magnitude of the financial shock: a concave policy function in the expected ELB spell duration requires sufficiently large financial shocks. This can be seen by noting that the first term in Equation (27) also shows up in Equation (28). As a consequence, the sum $u_t + \bar{r}$ needs to be positive as well. Intuitively, this highlights once more that the financial shocks needs to be large enough such that the disinflationary effect via the cost channel is outweighed. We capture this necessary condition in the following Assumption:

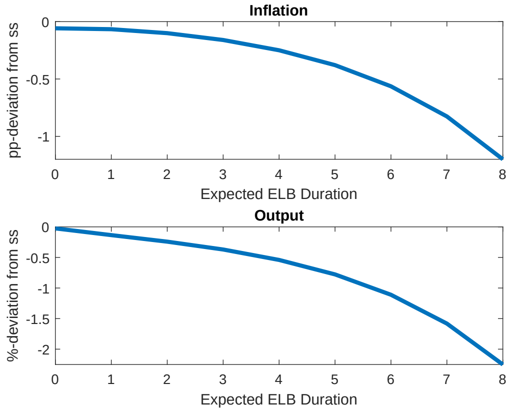
Assumption 4. *The size of the financial shock satisfies Condition* (32) *from Lemma 3.*

Figure 2 displays the policy functions λ_T^{π} and λ_T^{y} under three alternative illustrative calibrations. In the first case, the parameters satisfy Assumptions 1, 2

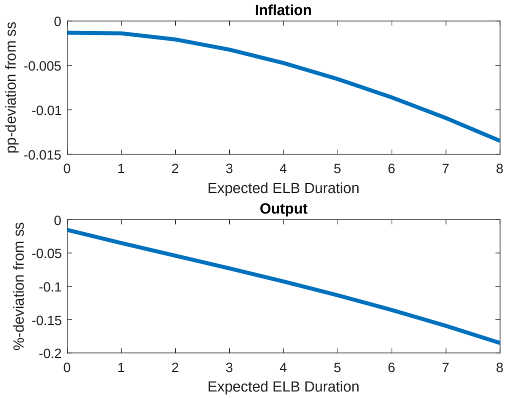
[11] Given financial frictions, a concave policy function is also possible for natural rate shocks. This requires a larger elasticity of the credit spread to compensate for the missing purely exogenous markup effect. It holds that $\lambda_T^{\pi,\mathrm{nrs}} = \lambda_T^{\pi,\mathrm{fs}} - \kappa$, where "nrs" stands for natural rate shock, and "fs" for financial shock.

and 4: the spread is countercyclical, financial shocks have conventional demand-type character in normal times and the shock is relatively large. In the second case, the calibration satisfies Assumption 3 such that financial frictions are relatively severe, but the financial shock is rather small. Case 3 shows the scenario of a large financial shock for severe financial frictions.

(a) Case 1: financial frictions not pronounced, A1, A2 and A4 fulfilled



(b) Case 2: small financial shock, A1, A2 and A3 fulfilled



(c) Case 3: large financial shock and severe financial frictions, A1-A4 fulfilled

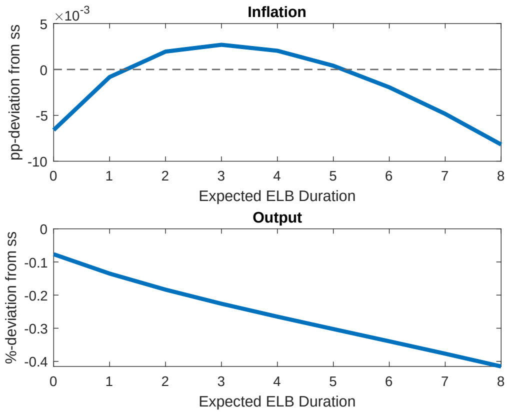


Figure 2: Expected ELB duration and impact response following financial shocks.

In the first two cases, the policy functions for inflation and output are strictly decreasing in the expected ELB spell duration: inflation and output move in the same direction, and a longer expected ELB duration implies a stronger macroeconomic effect of additional financial shocks. In the third case, however, the policy function for inflation is concave in the ELB spell duration, peaking at an expected ELB duration of six quarters in positive territory. In other words, if the ELB is expected to bind for a longer period of time, the overall inflation response may even turn positive. In this case, financial shocks appear as supply-type disturbances, i.e. moving output and inflation in opposite directions. This illustrates that inflation dynamics following financial shocks may be fundamentally different at the ELB compared to normal times. In particular, the slope of the Phillips curve (the ratio of the inflation policy function to the output policy function) may decrease in the expected ELB spell duration and even turn negative in certain cases. We hence explore the Phillips curve in more detail in the next section.

4 Numerical results and the hockey stick Phillips curve

In this section, we complement our closed-form solutions by a numerical analysis of the full general equilibrium rational expectations solution. We employ numerical solution methods to treat the expected ELB spell duration as endogenous. This allows us to present impulse responses to financial shocks and trace out the corresponding observational Phillips curve.

4.1 Calibration and solution method

Throughout this section, we fix the model's structural parameters to standard values taken from Woodford (2003), and adjust them to the most recent estimates (up until 2019) from Boehl and Strobel (2020, BS20 henceforth). We set $\beta = 0.99$, representing the standard view of a quarterly model. We calibrate $\sigma = 1$, which is a common assumption in line with a balanced growth path and also backed by BS20. Following the same line of reasoning, we set $\eta = 0.5$. We calibrate the fraction of non-adjusting price setters ζ to the commonly found textbook value of 0.66. This is conspicuously lower than the larger estimates from Smets and Wouters (2007) and BS20, as we want to avoid assuming a flat New Keynesian Phillips curve ex-ante.

For the parameters pertaining to the financial frictions, we fix $\psi = 8$ such that the output effects of financial shocks are amplified by approx. 20% relative to the standard NK model, which is roughly in line with the amplification degree documented by Bernanke et al. (1999). For our baseline scenario we assume that $\nu = 0.25$, which implies an elasicity of firms' marginal cost w.r.t. the risk spread of one quarter. We conduct comparative exercises with regard to this parameter further below and in section 5.

Regarding monetary policy parameters, we set ϕ_π to 1.5 (a commonly used standard prior), and ϕ_y to 0.2. In line with the estimates of BS20, the latter value is large relative to the standard prior mean of 0.125. As the authors argue, this reflects the strong reaction of the Fed to output during the ELB episode from 2009–2015, during which inflation was close to its target value while the level of output remained persistently depressed. We set $\rho = 0.9$ as suggested by the estimates of Boehl and Strobel (2020); Boehl et al. (forthcoming) for the persistence of such shocks over the post-2009 US and Euro area data. This reflects a lasting, quite persistent financial shock which resembles to the scenario of the Great Recession and its aftermath.

The analytical solutions shown in the previous section hold for the impact period when the shock occurs, under the assumption that the expected duration of the ELB k is given. However, in general and in the absence of special policy measures such as forward guidance, k is an equilibrium outcome to be determined endogenously at each point in time, given the contemporaneous exogenous disturbances that causes the ELB constraint to bind. To solve the model at the ELB, we use the numerical solution method proposed by Boehl (2022a). A brief description of the solution method is outlined in Appendix C.

4.2 Impulse responses to financial shocks

The left panel of Figure 3 displays impulse responses following contractionary financial shocks of differing size. For the impact responses, these correspond to the analytical policy functions in Proposition 1 and Proposition 2. A one-percent shock (yellow line) is not strong enough to cause the ELB to be binding. As a result, the dynamics look conventional (for demand-side shocks), with inflation (and marginal costs) falling in response to the shock. As the shock size increases, the ELB spell duration increases. Respectively, the initial response of inflation shifts upwards, in line with the analytical insight from Assumption 4. For a large value of u_t, the initial response of inflation becomes positive.[12]

In the right panel of Figure 3, we consider a large financial shock, pushing the economy to the ELB, for different values of ν. As highlighted by the graphs, marginal costs decrease less if financial frictions are stronger, which also translates to inflation. For $\nu = 0.25$, inflation actually increases, whereas the same calibration yields regular dynamics in the absence of the ELB (left side of Figure 3). This corresponds to the standard case outlined in the previous section: the elasticity of the credit spread with respect to entrepreneur leverage is large enough to generate a concave inflation policy function, but not so excessively large such that a positive response emerges in normal times.[13]

[12] Note that the lines are simply shifted outwards in case of a larger initial shock, since the responses of endogenous variables are a simple linear map of u_t and u_t decreases each period by $(1 - \rho)$.

[13] As Proposition 2 suggests, the persistence of financial shocks ρ is another central parameter for inflation dynamics, both at the ELB and for the rather extreme Neo-Fisherian case. A lower value of ρ yields a more concave inflation policy function (c.f. Equation 27). A lower ρ also implies a stronger discounting and hence a less dominant effect of the anticipated course of the financial shock. We illustrate this in Figure D.3 in the Appendix. We discuss the role of persistence in more detail in Section 5.

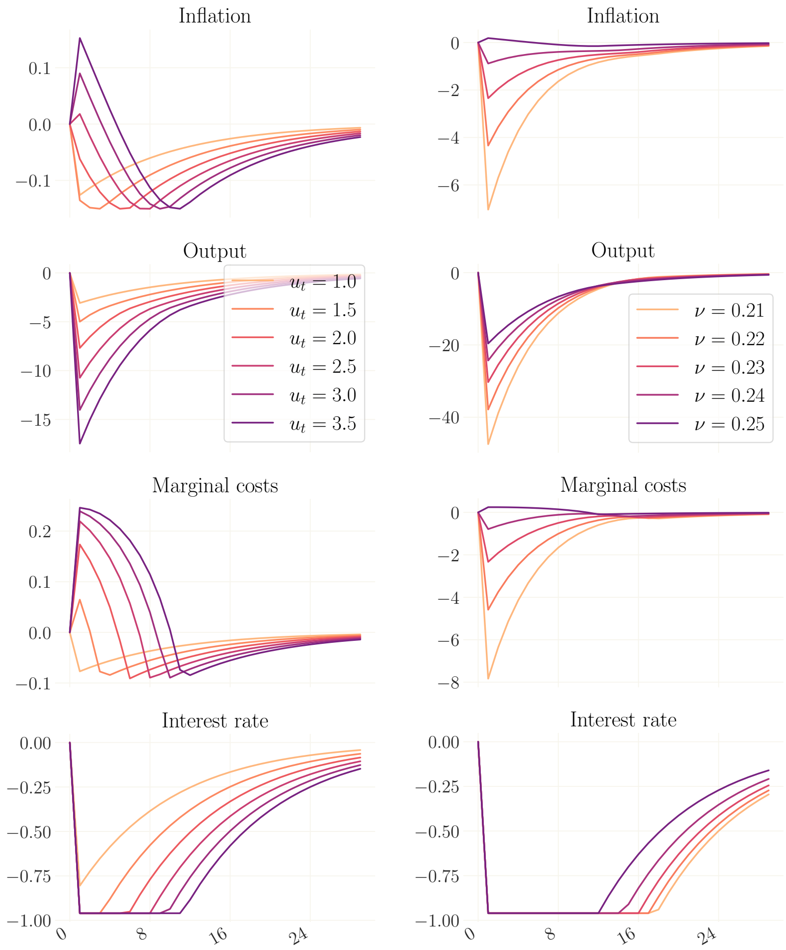


Figure 3: Impulse response functions. *Left:* to different financial shocks for $\nu = 0.25$. *Right:* to a 3% financial shock for different values of ν causing the ELB to be binding.

4.3 The observational hockey stick Phillips curve

Figure 4 plots the impulse responses to financial shocks projected into $\{y_t, \pi_t\}$-space. We interpret this as the *observational Phillips curve*, i.e. the realized values of inflation and output (gap) that would be observed in general equilibrium.[14] This is in contrast to the theoretical New Keynesian Phillips curve – as shown in Equations (8) and (12) – which represents firms' price setting under the assumption of nominal price rigidities. The most remarkable observation in Figure 4 is

[14]The output response following financial shocks is identical to the output gap response, see Footnote 8. As such, the figure can equivalently be interpret as showing the output gap - inflation space.

the striking hockey stick shape of the observational Phillips curve. For positive values of output, the observed slope of the Phillips curve is positive, in line with standard theory. However, for substantially negative values of output (caused by large financial shocks) the observational Phillips curve flattens out at the ELB.

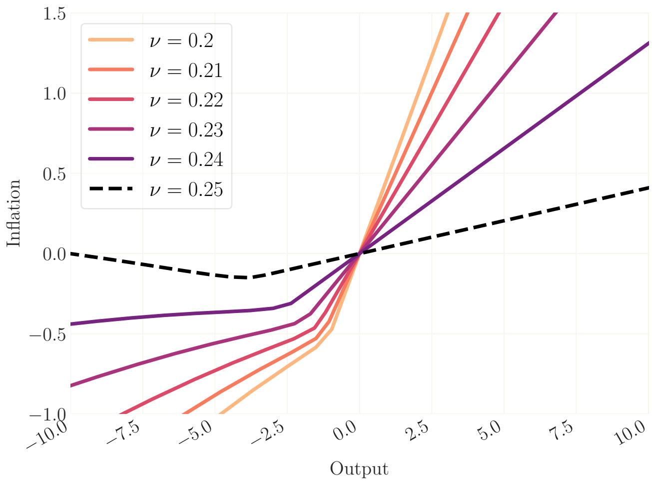


Figure 4: Observed Phillips Curve for an economy facing financial shocks. For each value of ν, we simulate the model for u_t on the interval $[-4, 4]$ and plot the respective combination of π_t and y_t.

As the elasticity of the credit spread to leverage, ν, increases, the Phillips Curve becomes flatter for both regimes: the hockey stick not only rotates in the origin, but also the ratio of the two slopes decreases. For $\nu = 0.24$, the observed slope in the region of -3% output is almost zero, while having a conventional slope in the origin. For a value of $\nu = 0.25$, we observe that the credit spread effect at the ELB is strong enough that inflation actually increases with output, while the Phillips curve is still upwards sloping in normal times.

In other words, an economic observer aiming to infer the slope of the Phillips curve in times of a binding ELB and financial frictions would inherently conclude that the Phillips curve is “dead”. This observation emerges even though the New Keynesian Phillips curve is well and alive: the relationship between firms’ prices and marginal costs, governed by the Calvo parameter, is intact.[15] However, the credit spread channel dominates firms’ price setting at the ELB and thus blurs the supply-side link between output and prices. The flat observational Phillips

[15]Note again that our calibration avoids pre-assuming a flat New Keynesian Phillips curve, with the Calvo parameter $\zeta = 0.66$ being considerably lower than the estimate of $\zeta = 0.85$ in BS20.

curve at the ELB implies that a proper identification of the relationship between inflation and output is challenging if financial shocks are prevailing.

5 Monetary policy at the effective lower bound

From the viewpoint of central banks, the difficulties of interpreting the observational Phillips curve translate into delicate decisions about the appropriate design of monetary policy at the ELB. To make matters worse, the effects of monetary policy itself are also affected by financial frictions and the ELB. We analyze this aspect by considering both monetary policy shocks – in particular forward guidance shocks at the ELB – and the systematic behavior of central banks, governed by the monetary policy rule.

5.1 Reversal effects of forward guidance

The first crucial insight regarding monetary policy shocks v_t is that in normal times, they generate *identical* macroeconomic dynamics as financial shocks. The three-equation representation from Section 3.1 reveals that monetary policy shocks appear in the same places as financial shocks. Therefore, in this framework and away from the ELB, monetary policy shocks and financial shocks are observationally equivalent in terms of inflation and output; they are only distinguishable via the response of the interest rate. As a consequence, all results from the previous sections concerning financial shocks in normal times are valid for monetary policy shocks as well. Notably, this includes the closed-form solutions, which also implies that the possibility of *Neo-Fisherian* effects of monetary policy shocks in normal times (an increase in inflation after rising interest rates) for extreme calibrations. It also follows immediately that the central bank can, in principle, offset financial shocks perfectly in normal times.

The insight that both shocks appear in the same places features major implications for forward guidance monetary policy at the ELB, which is the second important contribution of this paper. At the ELB, through their shock persistence, monetary policy shocks govern the expectations regarding the future interest rate path, acting like explicit forward guidance by the central bank. Forward guidance hence generates the same macroeconomic dynamics at the ELB as financial shocks.[16] However, unfortunately for monetary policy, our previous results thus imply that forward guidance at the ELB might not be particularly effective and may even be associated with unintended effects on inflation. Notably, this includes the possibility that forward guidance at the ELB may be *disflationary*, i.e. inducing Neo-Fisherian effects by *decreasing* inflation, while raising output.

[16] At the ELB, monetary policy shocks v_t and financial shocks u_t are hence not distinguishable, given the same persistence.

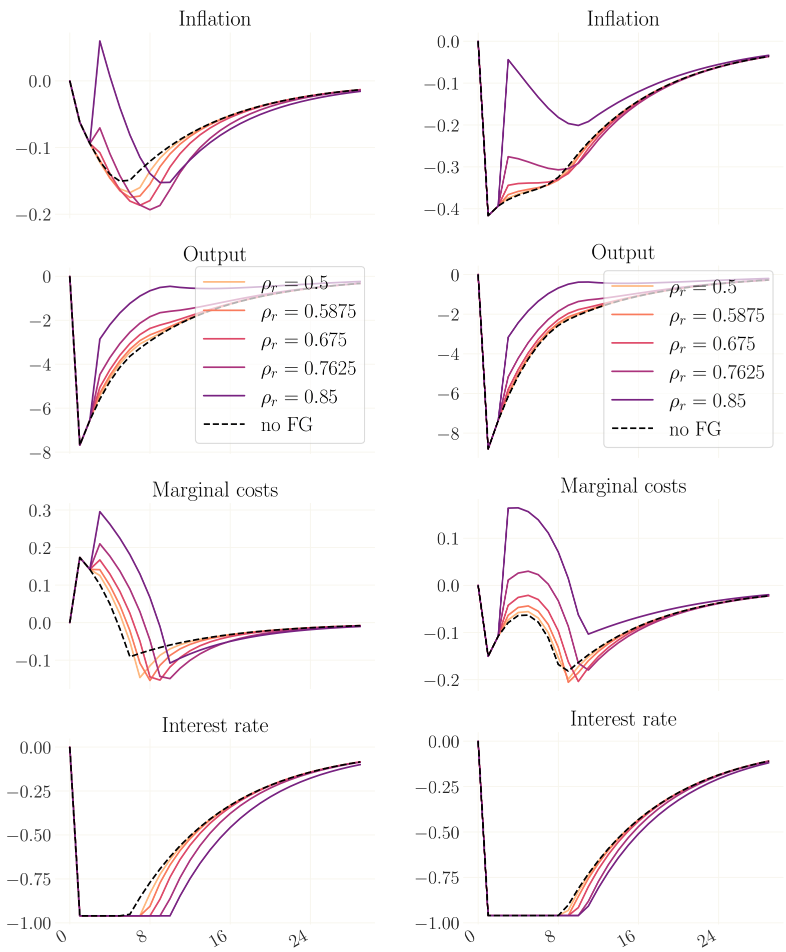


Figure 5: Impulse response functions for a 2% financial shock. Colored lines are the financial shock combined with a forward guidance shock in period 3. Different colors correspond to different persistences of the forward guidance shock. *Left:* for $\nu = 0.25$. For many values of ρ_r, the forward guidance shock is disinflationary. *Right:* for $\nu = 0.24$. For this value of ν the forward guidance shock is not disinflationary.

Intuitively, forward guidance shocks induce three different and partially opposing effects on inflation. First, expected rates are lower, which transmits to the economy via the standard (demand-side) Euler channel. Second, lower expected rates decrease expected marginal costs via the external finance premium. Third, agents expect that the inversion of the policy function will remain active for more periods. The first effect leads to an unambiguous increase in output. The second effect clearly depresses inflation. The third effect prolongs the reversal of the inflation response that is induced by the ELB via the credit channel. As forward guidance raises output, this could also trigger a drop in inflation. Which of these effects dominates depends crucially on the forward guidance persistence and the degree of financial frictions.

As an illustration, Figure 5 shows impulse responses following forward guidance shocks at the ELB given different values for ν. In the left panel, the stronger internal propagation of the forward guidance shock caused by a higher ν leads to a longer ELB period than in the right panel. This means that, given the same financial shock, during the extended ELB period the interest rate is much lower than in the absence of forward guidance, which causes marginal costs to fall. Since expected lower marginal cost are anticipated by firms (via the Phillips curve), the fall in inflation is *larger* than without forward guidance. This effect is absent in the right panel of Figure 5 because the shock does not prologue the ELB period significantly.

Lemma 4. *At the ELB, forward guidance shocks v_t may be associated with Neo-Fisherian effects such that expansionary forward guidance is disinflationary iff*

$$\rho_r < \rho. \tag{33}$$

Note that the condition in Lemma 4 is a necessary, but not a sufficient condition. To see this, assume a combination (ρ, ν) for which a given shock u_t is disinflationary. As the mechanics behind forward guidance and financial shocks are equal, we learn from Equation (27) in Proposition 2 that a smaller ρ (or here: ρ_r) can reduce the weight on the (negative terminal) second and third term. In that sense, a decrease in ρ has a similar effect as an increase in ν. We show this effect in Figure D.3 in the Appendix.

While it is safe to assume a high persistence of the financial shock, the persistence of the forward guidance shock is to some extent a policy parameter that can in principal be chosen by the central bank. However, it also depends on how successful the central bank is in its communication strategies. As illustrated in Figure 5, a monetary policy shock with low persistence can hence trigger negative inflation responses because the short-run effect of decreasing financial costs dominates the longer-term effect that works through the household Euler Equation. As such, non-credible forward guidance may be associated with undesirable macroeconomic dynamics.

5.2 Monetary policy rules at the ELB

We now turn to the systematic behavior of central banks. At first glance, it may seem that these rules are irrelevant at the ELB. However, they are in fact crucial for macroeconomic dynamics because rational private agents take the monetary policy rule into account when forming expectations about future variables and the remaining ELB duration. As such, choosing an appropriate monetary policy rule is of central importance for central banks at the ELB as well. From a policy-making perspective, the minimum requirement that any appropriate rule should satisfy is that it guarantees a determinate equilibrium.

Proposition 3. *The policy parameters in the central bank's monetary policy rule must satisfy the following conditions to guarantee a determinate solution:*

$$\phi_\pi + \frac{1-\beta}{\kappa\gamma}\,\phi_y > 1, \tag{34}$$

$$\kappa(\sigma^{-1}\gamma - 1)\,\phi_\pi + \sigma^{-1}\phi_y > \beta - 1 - \kappa \tag{35}$$

Proof. See Appendix. ■

Equation (34) may be interpreted as a modified Taylor principle for a financial accelerator economy. If the central bank decides to react to inflation only ($\phi_y = 0$), a necessary condition is that the associated coefficient ϕ_π needs to be larger than unity, as in Taylor (1993). If the central bank reacts to output as well ($\phi_y > 0$), determinacy requires the weighted sum of policy coefficients to be larger than unity. Compared to a standard New Keynesian framework, the key difference is that financial frictions affect the degree of substitutability between reacting to inflation and to output. Under Assumption 1, the slope of the New Keynesian Phillips curve with respect to output (the term $\kappa\gamma$) is lower due to the countercyclical credit spread. At first glance, it thus seems that policy responses to output can *substitute* more effectively for policy responses to inflation in the presence of financial frictions.

However, Equation (35) may constitute additional complications for the design of monetary policy rules. To see this, suppose that $(\sigma^{-1}\gamma - 1) < 0$, which is exactly the condition for a concave policy function of inflation at the ELB, i.e. Assumption 3. In this case, Equation (35) implies that the responses to inflation and output are *complements* for some combinations of $\{\phi_\pi, \phi_y\}$, or equivalently constitutes a lower (upper) bound restriction for the response to output (inflation). In other words, a stronger reaction to inflation must be accompanied by a corresponding stronger reaction to output. This clashes with the modified Taylor rule that exhibits the conventional substitutability.

Figure 6 displays this result graphically. As the elasticity of the credit spread with respect to entrepreneur leverage ν increases, a higher value for ϕ_y is necessary to keep the model determined for high values of ϕ_π. For example, in the case of $\nu = 0.2$, $\phi_\pi > 1.76$ requires that $\phi_y > 0$.

Intuitively, abstracting from financial frictions, inflation can be stabilized by raising nominal interest rates appropriately. Higher nominal interest rates amount to higher real interest rates, decreasing consumption and output. As a consequence, real marginal costs fall, and inflation decreases. Whether the hike

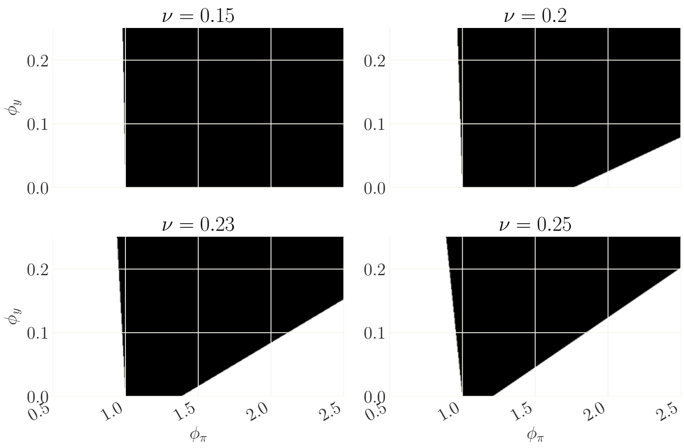


Figure 6: Determinacy regions for different values of ν.

of nominal interest rates constitutes a reaction to (positive) deviations of inflation or output is irrelevant. In the presence of financial frictions, however, an interest rate hike as a reaction to output has the additional effect of increasing marginal costs and thus inflation. Depending on the specific characteristics of the economy, the central bank might find itself in a knife-edge scenario where the appropriate window for systematic policy responses to output deviations is quite small.

Overall, the key message emerging from this section is that the conduct of monetary policy in the presence of financial frictions and a binding ELB may prove difficult. While the hockey stick Phillips curve blurs the relationship between inflation and output at the ELB, conventional monetary policy wisdoms are abolished: short-lived forward guidance shocks may be associated with Neo-Fisherian inflation effects, and determinacy considerations may place rather tight restrictions on appropriate monetary policy rules.

6 Conclusion

This paper argues that a binding effective lower bound (ELB) on nominal interest rates may contribute to an observational disconnect between inflation and economic activity if financial shocks are prevailing. At the ELB, the costs of external financing in the form of credit spreads can dominate firms' price setting and thereby generate inflationary pressure. Via this supply-side mechanism, the Phillips curve features a considerably flatter slope when the ELB binds compared to normal times. As a consequence, the resulting observational Phillips curve is shaped like a hockey stick. These findings constitute a complementary explanation for the recently observed inflation puzzles.

Our results translate into strong implications on the conduct of forward guidance, and provide a potential solution to the forward guidance puzzle: similar to financial shocks, the effects of forward guidance can be decomposed in short-run disinflationary effects via the firms' refinancing cost channel, and a longer-term inflationary effect via real marginal costs. For rather short-lived forward guidance impulses, the first disinflationary effect may dominate and forward guidance can in fact lower inflation. Accordingly, only forward guidance with a high expected persistence succeeds in fostering inflation and growth.

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Appendix A Equilibrium Equations

This section lists the full set of the equations defining the equilibrium. On the household side, we have the inter-temporal Euler equation and the intra-temporal labor-consumption trade-off, Equations (1) and (2) in the main text:

$$C_t^{-\sigma} = \beta E_t \left[\frac{R_t}{\Pi_{t+1}} U_t C_{t+1}^{-\sigma} \right], \tag{A.1}$$

$$H_t^{\eta} = W_t C_t^{-\sigma}. \tag{A.2}$$

On the firm side, we have the aggregate production function, which is obtained by aggregating over the individual linear production functions:

$$Y_t = \frac{H_t}{v_t^p} \tag{A.3}$$

where v_t^p is a measure of price dispersion defined below. Marginal costs are given by Equation (4):

$$MC_t = W_t R_t^L \tag{A.4}$$

The price setting behavior by firms is defined by the following equations, which are standard for Calvo (1983) pricing and make use of two auxiliary variables f_t^1 and f_t^2:

$$f_t^1 = \frac{\varepsilon - 1}{\varepsilon} f_t^2 \tag{A.5}$$

$$f_t^1 = C_t^{-\sigma} MC_t Y_t + \beta\zeta E_t \left[\Pi_{t+1}^{\varepsilon} f_{t+1}^1 \right] \tag{A.6}$$

$$f_t^2 = C_t^{-\sigma} \Pi_t^* Y_t + \beta\zeta E_t \left[\left(\frac{1}{\Pi_{t+1}} \right)^{1-\varepsilon} \left(\frac{\Pi_t^*}{\Pi_{t+1}^*} \right) f_{t+1}^2 \right] \tag{A.7}$$

$$1 = \zeta \left(\frac{1}{\Pi_t} \right)^{1-\varepsilon} + (1-\zeta) \left(\Pi_t^* \right)^{1-\varepsilon} \tag{A.8}$$

$$v_t^p = \zeta \Pi_t^{\varepsilon} v_{t-1}^p + (1-\zeta) \left(\Pi_t^* \right)^{-\varepsilon} \tag{A.9}$$

The interest rate specified in the credit contract is defined by Equation (3):

$$R_t^L = z \left(\frac{W_t H_t}{N_t} \right) \frac{R_t}{E_t[\Pi_{t+1}]} U_t \tag{A.10}$$

Entrepreneur net worth evolves according to Equation (5):

$$N_t = \Psi \left(Y_t \right), \tag{A.11}$$

The central bank operates according to a monetary policy rule shown in Equation (6)

$$\frac{R_t^n}{R^n} = \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} exp(v_t), \tag{A.12}$$

The effective lower bound (ELB) constraint is given by Equation (7):

$$R_t = \max\left\{\bar{R}, R_t^n\right\} \tag{A.13}$$

Finally, the aggregate resource constraint is

$$Y_t = C_t \tag{A.14}$$

These 14 conditions define the equilibrium for the 14 endogenous variables

$$\left(C_t, Y_t, H_t, \Pi_t, \Pi_t^*, W_t, R_t, R_t^L, R_t^n, N_t, MC_t, f_t^1, f_t^2, v_t^p\right), \tag{A.15}$$

together with the evolution of the two exogenous shocks:

$$\begin{aligned} \ln(U_t) &= \rho \ln(U_{t-1}) + \epsilon_t & \text{(A.16)} \\ v_t &= \rho_r\, v_{t-1} + \epsilon_t r, t. & \text{(A.17)} \end{aligned}$$

The linearized equilibrium conditions are as follows:

$$\begin{aligned}
c_t &= -\sigma^{-1}\left(r_t + u_t - E_t\pi_{t+1}\right) + E_t[c_{t+1}], & \text{(A.18)} \\
w_t &= \eta h_t + \sigma c_t, & \text{(A.19)} \\
y_t &= h_t, & \text{(A.20)} \\
mc_t &= w_t + r_t^L, & \text{(A.21)} \\
\pi_t &= \kappa mc_t + \beta E_t[\pi_{t+1}], & \text{(A.22)} \\
r_t^L &= r_t - E_t[\pi_{t+1}] + \nu(w_t + h_t - n_t) + u_t, & \text{(A.23)} \\
n_t &= \psi y_t, & \text{(A.24)} \\
r_t^n &= \phi_\pi \pi_t + \phi_y y_t + v_t, & \text{(A.25)} \\
r_t &= \max\left\{\bar{r}, r_t^n\right\}, & \text{(A.26)} \\
y_t &= c_t, & \text{(A.27)} \\
u_t &= \rho\, u_{t-1} + \epsilon_t, & \text{(A.28)} \\
v_t &= \rho_r\, v_{t-1} + \epsilon_{r,t}, & \text{(A.29)} \\
& & \text{(A.30)}
\end{aligned}$$

where lower-case variables denote log-deviations from steady state.

The three-equation representation shown in Section 3.1 can be obtained by combining Equations (A.19)-(A.24) into one single Phillips curve and using the resource constraint Equation (A.27) to eliminate c_t.

Appendix B Proofs

Proposition 1. *The impact responses of inflation and output to a financial shock in normal times (without a binding ELB on nominal interest rates) are given by:*

$$\pi_t = \lambda_0^\pi \, u_t, \tag{B.1}$$
$$y_t = \lambda_0^y \, u_t, \tag{B.2}$$

where

$$\lambda_0^\pi = -\frac{\kappa\gamma - \kappa\sigma(1-\rho)}{(1-\beta\rho)(\sigma(1-\rho)+\phi_y) + \kappa\gamma(\phi_\pi - \rho) - \kappa\sigma(1-\rho)(\phi_\pi - \rho)}, \tag{B.3}$$
$$\lambda_0^y = -\frac{1 + (\phi_\pi - \rho)\lambda_0^\pi}{\sigma(1-\rho) + \phi_y}. \tag{B.4}$$

Proof. The proof relies on the method of undetermined coefficients. We guess that the solution takes the form $\pi_t = \lambda_0^\pi \, u_t$ and $y_t = \lambda_0^y \, u_t$. Using this guess, the system of equation can be written as

$$(1 - \kappa\phi_\pi - \rho(\beta - \kappa))\lambda_0^\pi u_t = \kappa u_t + \kappa(\gamma + \phi_y)\lambda_0^y u_t, \tag{B.5}$$
$$(1 + \phi_y\sigma^{-1} - \rho)\lambda_0^y u_t = -\sigma^{-1}(\phi_\pi - \rho)\lambda_0^\pi u_t - \sigma^{-1} u_t, \tag{B.6}$$

where the nominal interest rate is replaced using the (unconstrained) Taylor rule. Note that expectations of future variables can be replaced by using the law of motion for the financial shocks under rational expectations. The solution is obtained by dividing both equations by u_t, substituting for λ_0^y in the first equation using the second equation and rearranging. ■

Lemma 1. *The impact responses of inflation and output to a financial shock in normal times (without a binding ELB on nominal interest rates) are negative, i.e.*

$$\lambda_0^\pi < 0, \tag{B.7}$$
$$\lambda_0^y < 0, \tag{B.8}$$

if the elasticity of the credit spread to entrepreneur leverage satisfies

$$\nu < \frac{\eta + \rho\sigma}{\psi - 1 - \sigma - \eta}. \tag{B.9}$$

Proof. The proof consists of three parts. First, we show that the model's determinacy conditions imply that the denominator of λ_0^π is positive. Second, the sign of λ_0^π then depends on its numerator, which is equivalent to the parameter restriction in the Lemma. Third, the sign of λ_0^y follows from λ_0^π.

First, let us consider the determinacy conditions. The forward looking components of our model can be expressed as

$$M\mathbf{x}_t = E_t[\mathbf{x}_{t+1}], \tag{B.10}$$

with $\mathbf{x}_t = (y_t, \pi_t)'$. To arrive at this formulation, we can rewrite Equations (13) and (14) (ignoring exogenous innovations and the ELB) as

$$(1+\sigma^{-1}\phi_y)y_t = -\sigma^{-1}\left(\phi_\pi \pi_t - E_t[\pi_{t+1}]\right) + E_t[y_{t+1}], \tag{B.11}$$

$$(1-\kappa\phi_\pi)\pi_t = \kappa(\gamma+\phi_y)y_t + \beta_\kappa E_t[\pi_{t+1}], \tag{B.12}$$

where we define $\beta_\kappa = \beta - \kappa$ for convenience. Then, we can rewrite

$$A\mathbf{x}_t = B\mathbf{x}_{t+1}, \tag{B.13}$$

$$\begin{bmatrix} 1+\sigma^{-1}\phi_y & \sigma^{-1}\phi_\pi \\ -\kappa(\gamma+\phi_y) & 1-\kappa\phi_\pi \end{bmatrix} \mathbf{x}_t = \begin{bmatrix} 1 & \sigma^{-1} \\ 0 & \beta_\kappa \end{bmatrix} \mathbf{x}_{t+1}. \tag{B.14}$$

It is straightforward that

$$B^{-1} = \frac{1}{\beta_\kappa}\begin{bmatrix} \beta_\kappa & -\sigma^{-1} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\beta_\kappa^{-1}\sigma^{-1} \\ 0 & \beta_\kappa^{-1} \end{bmatrix}, \tag{B.15}$$

and hence

$$M = AB^{-1} = \begin{bmatrix} 1+\sigma^{-1}\phi_y & \sigma^{-1}\phi_\pi \\ -\kappa(\gamma+\phi_y) & 1-\kappa\phi_\pi \end{bmatrix} \begin{bmatrix} 1 & -\beta_\kappa^{-1}\sigma^{-1} \\ 0 & \beta_\kappa^{-1} \end{bmatrix}, \tag{B.16}$$

$$= \begin{bmatrix} 1+\sigma^{-1}\phi_y & -\beta_\kappa^{-1}\sigma^{-1}(1+\sigma^{-1}\phi_y - \phi_\pi) \\ -\kappa(\gamma+\phi_y) & \beta_\kappa^{-1}\sigma^{-1}\kappa(\gamma+\phi_y) + \beta_\kappa^{-1}(1-\kappa\phi_\pi) \end{bmatrix}, \tag{B.17}$$

$$= \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix}. \tag{B.18}$$

The eigenvalues of the system are given by $|M - \lambda I| = \lambda^2 + p\lambda + q$, where

$$p = -(m_1+m_4) = -\left(1+\sigma^{-1}\phi_y + \beta_\kappa^{-1}\sigma^{-1}\kappa(\gamma+\phi_y) + \beta_\kappa^{-1}(1-\kappa\phi_\pi)\right) \tag{B.19}$$

is the negative of the trace and

$$q = m_1 m_4 - m_2 m_3 = \beta_\kappa^{-1}(1+\sigma^{-1}\phi_y - \kappa\phi_\pi + \sigma^{-1}\phi_\pi\kappa\gamma) \tag{B.20}$$

is the determinant. As there are no endogenous states, determinacy under the conditions by Blanchard and Kahn (1980) requires the modulus of both eigenvalues of M to be larger than zero. We can find a representation of the absolute value of these eigenvalues in terms of the elements of M as

$$|\lambda_{1,2}^r| = \begin{cases} -p/2 + \sqrt{p^2/4 - q} > 1 \\ -p/2 - \sqrt{p^2/4 - q} > 1 \end{cases} \qquad \text{if } p^2/4 \geq q, \tag{B.21}$$

$$|\lambda_{1,2}^i| = \sqrt{p^2/2 - q} > 1 \qquad \text{if } p^2/4 < q. \tag{B.22}$$

$|\lambda_{1,2}^r|$ are the real eigenvalues if the respective condition for the square root is satisfied, $|\lambda_{1,2}^i|$ are corresponding imaginary eigenvalues otherwise. Using the condition in Equation (B.21) in the second case implies that $-p/2 > 1$, or equivalently

$$p < -2. \tag{B.23}$$

Rearranging the second case in Equation (B.21) also implies

$$1 + p + q > 0. \tag{B.24}$$

Together with Equation (B.23), this implies

$$q > 1. \tag{B.25}$$

Equation (B.25) is also a necessary condition for the case of imaginary eigenvalues. Similarly, one can show that Equation (B.23) and Equation (B.24) imply that Equation (B.22) holds. Therefore, Equations (B.23)-(B.25) are jointly sufficient for both eigenvalues to be larger than one in modulus.

In our model, the three necessary condition $1 + p + q > 0$, $p < -2$ and $q > 1$ thus read

$$\phi_\pi + \frac{1-\beta}{\kappa\gamma}\phi_y > 1, \tag{B.26}$$

$$\sigma^{-1}\phi_y + \beta_\kappa^{-1}\sigma^{-1}(\kappa\gamma + \kappa\phi_y) + \beta_\kappa^{-1}(1 - \kappa\phi_\pi) > 1, \tag{B.27}$$

$$1 + \sigma^{-1}(\kappa\gamma\phi_\pi + \phi_y) - \kappa\phi_\pi > \beta_\kappa. \tag{B.28}$$

As a second step, we can use these determinacy conditions to derive a sign for the denominator of λ_0^π. Let us suppose that the denominator is positive, i.e.

$$(1 - \beta\rho)(\sigma(1 - \rho) + \phi_y) + \kappa\gamma(\phi_\pi - \rho) - \kappa\sigma(1 - \rho)(\phi_\pi - \rho) > 0. \tag{B.29}$$

This can be rearranged to

$$\left(\phi_\pi + \frac{1-\beta}{\kappa\gamma}\phi_y - 1\right) + \frac{1-\rho}{\kappa\gamma}\left(\kappa\gamma + \beta\phi_y + \sigma(1 - \beta\rho - \kappa(\phi_\pi - \rho))\right) > 0. \tag{B.30}$$

The first term in large brackets is positive, which can be seen directly from the necessary condition in Equation (B.26). After some algebraic manipulations, one can show that Equation (B.27) implies that the second term in brackets is also positive. This shows that the denominator of λ_0^π is indeed positive.

With the denominator being positive, the sign of λ_0^π depends on the numerator, including the minus in front of the fraction. The condition for $\lambda_0^\pi < 0$ is thus

$$\kappa\gamma - \kappa\sigma(1-\rho) > 0. \tag{B.31}$$

Using the definition of γ, this is equivalent to

$$\sigma + \eta - \nu(\psi - 1 - \sigma - \eta) > \sigma(1-\rho). \tag{B.32}$$

Rearranging yields the parameter restriction in terms of the elasticity of the credit spread to entrepreneur leverage.

As a last step, the sign of λ_0^y can be determined given the solution for λ_0^π. The denominator of λ_0^y is positive for conventional parameters, such that the sign is determined by the numerator, including the minus. Inserting λ_0^π, this is given by

$$-1 + (\phi_\pi - \rho)\frac{\kappa\gamma - \kappa\sigma(1-\rho)}{Z}, \tag{B.33}$$

where Z denotes the denominator of λ_0^π. After some algebraic manipulations, this is equivalent to

$$-Z^{-1}\Big((1-\beta\rho)(\sigma(1-\rho)+\phi_y) + (1-\rho)^2\kappa\sigma\Big), \tag{B.34}$$

which is unambiguously negative for $0 \leq \rho \leq 1$. ∎

Lemma 2. *The impact response of inflation to a financial shock in normal times (without a binding ELB on nominal interest rates) is positive if the elasticity of the credit spread to entrepreneur leverage satisfies*

$$\nu > \frac{\eta + \rho\sigma}{\psi - 1 - \sigma - \eta}. \tag{B.35}$$

Proof. This is the converse case of Lemma 1. As argued in the corresponding proof, determinacy of the model requires the denominator of λ_0^π to be positive. The condition for $\lambda_0^\pi > 0$ is hence that the numerator (including the minus in front of the fraction) is positive. This is equivalent to

$$\sigma(1-\rho) > \gamma. \tag{B.36}$$

Using the definition of γ to obtain

$$\sigma(1-\rho) > \sigma + \eta - \nu(\psi - 1 - \sigma - \eta) \tag{B.37}$$

and rearranging yields the desired result. ■

Proposition 2. *Suppose that the ELB on nominal interest rates is expected to bind for $T > 0$ periods. The impact responses of inflation and output to a financial shock are then given by*

$$\pi_t = \lambda_T^{\pi}\, u_t + \mu_T^{\pi}\, \bar{r}, \tag{B.38}$$
$$y_t = \lambda_T^{y}\, u_t + \mu_T^{y}\, \bar{r}, \tag{B.39}$$

where

$$\lambda_T^{\pi} = \kappa\left(1 - \gamma\sigma^{-1}\right) + \rho\left(\beta - \kappa + \kappa\gamma\sigma^{-1}\right)\, \lambda_{T-1}^{\pi} + \rho\kappa\gamma\, \lambda_{T-1}^{y}, \tag{B.40}$$
$$\mu_T^{\pi} = \kappa(1 - \gamma\sigma^{-1}) + (\beta - \kappa + \kappa\gamma\sigma^{-1})\, \mu_{T-1}^{\pi} + \kappa\gamma\mu_{T-1}^{y} \tag{B.41}$$
$$\lambda_T^{y} = -\sigma^{-1} + \rho\sigma^{-1}\, \lambda_{T-1}^{\pi} + \rho\, \lambda_{T-1}^{y}, \tag{B.42}$$
$$\mu_T^{y} = \sigma^{-1} + \sigma^{-1}\mu_{T-1}^{\pi} + \mu_{T-1}^{y} \tag{B.43}$$

where $\{\lambda_0^{\pi}, \lambda_0^{y}\}$ as in Proposition 1 and $\mu_0^{\pi} = \mu_0^{y} = 0$.

Proof. Similar to Proposition 1, the proof relies on the method of undetermined coefficients. Suppose that the ELB is expected to bind for $k \geq 1$ periods. We guess that the responses of inflation and output are linear functions of the financial shock and the ELB value, as shown in Equations (B.38) and (B.39). Combining Equations (12)-(13) yields

$$\pi_t = \kappa(1 - \gamma\sigma^{-1})\bar{r} + (\beta - \kappa + \kappa\gamma\sigma^{-1})E_t[\pi_{t+1}] + \kappa\gamma E_t[y_{t+1}] + \kappa(1 - \gamma\sigma^{-1})u_t \tag{B.44}$$

where the interest rate is replaced by the ELB value. Expectations of future variables can be replaced by the corresponding policy functions for the case of an expected ELB duration of $k - 1$ under rational expectations, using the law of motion for the financial shocks. This yields

$$\pi_t = \kappa(1 - \gamma\sigma^{-1})\bar{r} + (\beta - \kappa + \kappa\gamma\sigma^{-1})(\rho\lambda_{T-1}^{\pi}u_t + \mu_{T-1}^{\pi}\bar{r}) + \kappa\gamma(\rho\lambda_{T-1}^{y}u_t + \mu_{T-1}^{y}\bar{r}) + \kappa(1 - \gamma\sigma^{-1})u_t \tag{B.45}$$

Collecting terms and matching coefficients yields the solution for λ_T^{π} and μ_T^{π}. The values for λ_T^{y} and μ_T^{y} can be found similarly: start from Equation (13), replace the interest rate by the ELB value and expectations by policy functions for an ELB duration of $T - 1$, match coefficients. ■

Lemma 3. *A concave inflation policy function in the expected ELB spell duration requires that the elasticity of the credit spread with respect to entrepreneur leverage* *satisfies*

$$\nu > \frac{\eta}{\psi - 1 - \sigma - \eta} \tag{B.46}$$

and that the size of the financial shock satisfies

$$u_t > -\bar{r} = \beta^{-\sigma} - 1. \tag{B.47}$$

Proof. If the first term $\kappa\left(1 - \gamma\sigma^{-1}\right)$ in Equations (27) and (28) is negative, λ_T^π and μ_T^π are negative for all T under Assumption 2 for conventional calibrations. Conversely, the recursion can only be increasing if $\kappa\left(1 - \gamma\sigma^{-1}\right)$ is larger than zero, which yields the first parameter restriction shown above. The second restriction follows from noting that the same term appears also in μ_T^π. Hence, the sum $u_t + \bar{r}$ must be positive. Note that $\bar{r}$ is a negative value, i.e. the negative percentage point deviation of interest rates at the ELB from their steady state. This ELB value is determined by the household preference parameters β and σ as shown in the Lemma. ■

Proposition 3. *The policy parameters in the central bank's monetary policy rule must satisfy the following conditions to guarantee a determinate solution:*

$$\phi_\pi + \frac{1-\beta}{\kappa\gamma}\phi_y > 1, \tag{B.48}$$

$$\kappa(\sigma^{-1}\gamma - 1)\,\phi_\pi + \sigma^{-1}\phi_y > \beta - 1 - \kappa \tag{B.49}$$

Proof. The first equation follows directly from the condition $1 + p + q > 0$, which is required to satisfy the Blanchard and Kahn (1980) conditions. This is Equation (B.26) in the proof for Proposition 1. The second equation can be obtained by rearranging the condition $q > 1$, which is Equation (B.28) above. ■

Appendix C Numerical Solution Method

For the sake of clarity, we use a different representation of the policy functions to outline the solution procedure. The analytic solutions in Section 3 are expressed in terms recursive policy functions of u_t. A different, non-recursive way of presenting these policy functions is suggested in Boehl (2022a). The simplicity of our model allows to ease the notation therein and express our model with $\mathbf{x}_t = (\pi_t, y_t)'$ in matrix form as

$$\mathbf{x}_t + \mathbf{c}\max\{\mathbf{d}\mathbf{x}_t, \bar{r}\} = \mathbf{N}E_t\mathbf{x}_{t+1} + \mathbf{c}u_t, \tag{C.1}$$

where $\mathbf{N}$ is the system matrix of the constrained system, $\mathbf{c}$ contains the coefficients that determine how $\mathbf{x}_t$ is affected by r_t (and thereby also by u_t) and $\mathbf{d}$ contains the parameters of the monetary policy rule. $\bar{r} < 0$ is the actual model-implied lower bound of r_t.

Assume again that the economy is at the ELB for k periods. Then

$$\mathbf{x}_t + \mathbf{c}\bar{r} = \mathbf{N}E_t\mathbf{x}_{t+1} + \mathbf{c}u_t, \tag{C.2}$$

$$E_t\mathbf{x}_{t+1} + \mathbf{c}\bar{r} = \mathbf{N}E_t\mathbf{x}_{t+2} + \mathbf{c}u_{t+1}, \tag{C.3}$$

$$\dots$$

$$E_t\mathbf{x}_{t+k-1} + \mathbf{c}\bar{r} = \mathbf{N}E_t\mathbf{x}_{t+k} + \mathbf{c}u_{t+k-1}, \tag{C.4}$$

$$E_t\mathbf{x}_{t+k} = \mathbf{A}(0)u_{t+k}. \tag{C.5}$$

Recursively inserting (C.5) into (C.4) yields, acknowledging that $E_t u_{t+s} = \rho^s u_t$,

$$\mathbf{x}_t = \mathbf{N}^k\mathbf{A}(0)\rho^k u_t + \sum_{i=0}^{k-1}\mathbf{N}^i\mathbf{c}\rho^i u_t - \sum_{i=0}^{k-1}\mathbf{N}^i\mathbf{b}\bar{r}, \tag{C.6}$$

$$= \mathbf{A}(k)u_t + \mathbf{a}(k)\bar{r}. \tag{C.7}$$

Rewriting (C.6) yields

$$\pi_t = A_\pi(k)u_t + a_\pi(k)\bar{r}, \tag{C.8}$$

$$y_t = A_y(k)u_t + a_y(k)\bar{r}. \tag{C.9}$$

In verbal terms, this implies that depending on the expected number of periods at the ELB k, we can express the vector of controls $\mathbf{x}_t$ as a linear map $A_j(k)$ of u_t and the (constant) vector $a_j(k)$. Both terms are nonlinear functions of k defined on $\mathbb{N}_0$. In other words: given k, the policy function is simply a two dimensional linear projection of the scalar u_t.

Definition 1 recapitulates the conditions for k to be an equilibrium value under the assumption that each shock causes the ELB to hold instantly without any transition period.

Definition 1 (equilibrium k)**.** *For each period t, an equilibrium value of $k \in \mathbb{N}_0$ must satisfy that the ELB binds in expectations exactly until period $t+k$. Hence,*

$$\mathbf{d}\mathbf{x}_t > \bar{r} \implies k = 0, \tag{C.10}$$

while for $k > 0$ it must hold that

$$\mathbf{d}E_t\mathbf{x}_{t+k} > \bar{r}, \tag{C.11}$$

and

$$\mathbf{d}E_t\mathbf{x}_{t+k-1} \leq \bar{r}. \tag{C.12}$$

The parsimonious nature of our model allows that, for each u_t, a k can simply be found by iterating over $k \in \mathbb{N}_0$ (where, naturally, k is likely to be small). More sophisticated iteration schemes for a general formulation of the dynamic system can be found in Boehl (2022a).

To provide some quantitative impression given our model, for $\nu = 0.2$, a 1% risk premium shock will cause the ELB to initially bind for $k = 2$ periods, a 2% shock will cause $k = 9$ and a 3% shock an endogenous duration of $k = 12$ periods.

In Figure C.1 we show the reduced-form slope of the Phillips Curve, based only on the dynamic effect in response to the risk premium shock. The figure confirms that the slope is considerably high if away from the ELB, but drops once the ELB is reached and remains consistently low as the number of expected durations at the ELB increases.

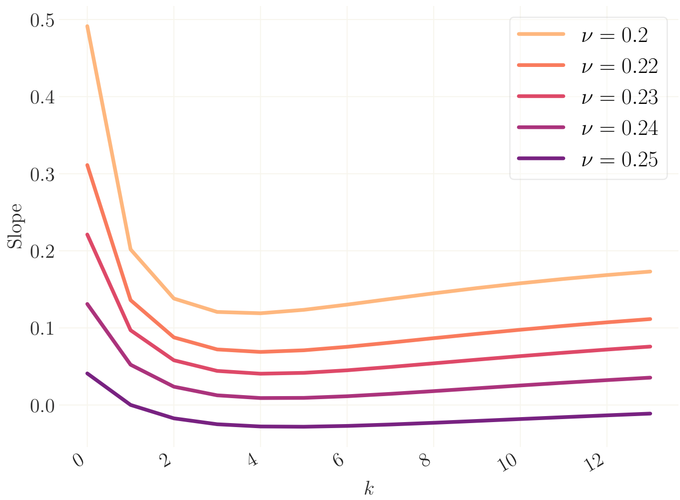

Figure C.1: Theoretical Phillips Curve slope $A_\pi(k)/A_y(k)$. This exercise ignores the static effect of the ELB, that is captured by $a_\pi(k)$ and $a_y(k)$.

Figure C.2 plots the non-recursive policy functions for π_t. For a more moderate value of ν of 0.2, the mapping $A_\pi(k)$ from $u_t \to \pi_t$ decreases with k while the linear part $a_\pi(k)$ increases in about the same fashion. As larger shocks are necessary to cause a higher k, the dynamic effect of the shock dominates the static effect and inflation falls. For $\nu = 0.22$, $A_\pi(k)$ becomes more convex, meaning that the coefficient that translates financial shocks to inflation increases for low expected durations. This effect is not offset by the static effect of a longer anticipated ELB period, which leads to a more muted inflation response. For a value of $\nu = 0.24$, the dynamic response approaches zero while for $\nu = 0.25$, $A_\pi(k)$ turns positve for values of k larger than two. As the static effect is again too weak to counteract, this leads to an increase of inflation on impact, as it is captured in Figure 3.

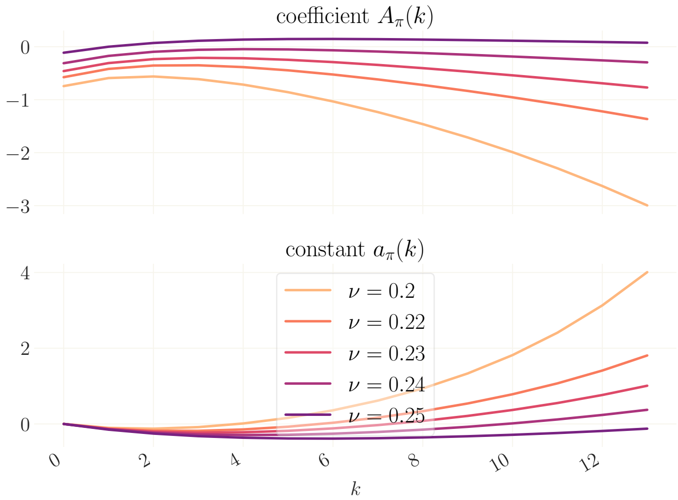

Figure C.2: Expected ELB Duration and Impact Response

Appendix D Additional Figures

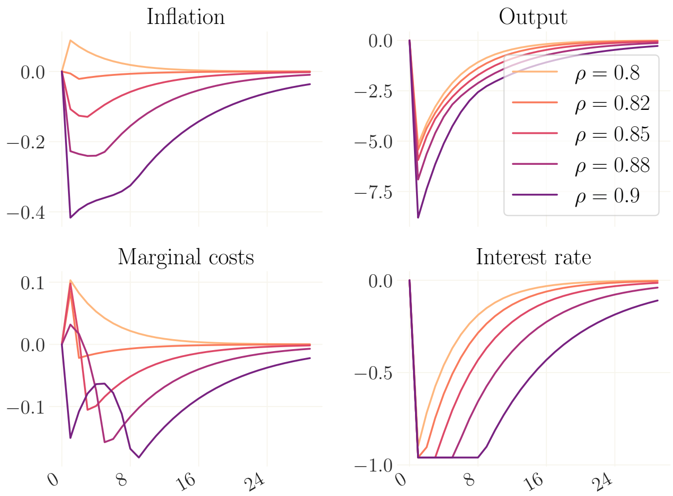

Figure D.3: Impulse responses to 2% risk premium shocks for different values of ρ, given $\nu = 0.24$.