# The Hockey Stick Phillips Curve and the Effective Lower Bound

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**Abstract**

We show that the interplay between a binding effective lower bound (ELB) on nominal interest rates and the costs of external financing weakens the disinflationary effect of financial shocks. In normal times, factor costs dominate firms' marginal costs and thereby inflation; larger credit spreads are balanced-out by lower nominal rates. At the ELB, larger spreads can offset the effect of lower factor costs on firms' price setting. The Phillips curve hence features a hockey stick shape: flat at the ELB, but conventional during normal times. This mechanism also weakens forward guidance effects, since such policy reduces spreads and financing costs.

*Keywords:* Phillips Curve, Financial Frictions, Effective Lower Bound, Disinflation, Forward Guidance

*JEL:* C62, C63, E31, E32, E44, E52, E58, E63

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## 1 Introduction

What is the relationship between inflation and economic activity? Given the fundamental role of these two concepts, it is troubling that this question is still puzzling the economic profession. After the Global Financial Crisis of 2007/2008 and the associated financial turmoil, inflation seemed disconnected from economic activity: despite substantially negative output gaps, inflation fell only modestly.[1] This "missing disinflation puzzle" raised considerable interest in analyzing the seemingly flat Phillips curve (Ball and Mazumder, 2011; Coibion and Gorodnichenko, 2015; Harding et al., 2022). While the explanations put forward are numerous and manifold, we found one key contributing factor yet to be missing: the effective lower bound (ELB) on nominal interest rates, which was reached by several central banks around the globe at the same time as the observed inflation puzzles.

In this paper, we show how the interplay of the ELB and financial frictions can reshape the relation between inflation and output if financial shocks are prevailing. Recent research documents that financial distortions can be crucial for firms' price setting behavior and, thereby, for inflation dynamics (e.g. Gilchrist et al., 2017). We argue that during normal times, firms' marginal costs are dominated by the procyclical costs of production factors, which hence determine their price setting. In the presence of financial frictions, however, marginal costs further contain the costs of external financing. These consist of the real safe interest rate and a countercyclical credit spread reflecting financial frictions. While these two components roughly balance out in normal times, larger credit spreads can substantially offset lower production factor costs if the nominal rate is constrained by ELB. In this case, the costs of external financing considerably weaken the supply-side link between output and prices. As a result, financial shocks at the ELB induce only moderate disinflationary responses, and may in extreme cases even be inflationary.

Taking the ELB into account, the resulting *observational Phillips curve*[2] is thus shaped like a hockey stick: For normal times with positive or rather mildly negative output gaps, it exhibits a conventional positive slope in output gap - inflation space. In contrast, the slope is considerably flat for significantly negative output gaps when the ELB is binding. This hockey

[1] For example, the US output gap was -5.3% in Q2 2009, accompanied by a core inflation (excluding food and energy) of 1.83%, only slightly below the central bank target of 2%.

[2] We use this term to refer to the reduced-form relationship between realized (equilibrium) values for inflation and output gap, i.e. the *observed* or *empirical* Phillips curve. As discussed below, this is not equivalent to the New Keynesian Phillips curve describing firms' price setting behavior.

stick Phillips Curve not only provides an explanation for the puzzles of missing disinflation, it also is consistent with the observed timing of events. As seen in Figure 1, in 2008:Q4, corporate spreads with rating BAA peaked at almost 6%, while the Federal funds rate reached its lower bound of almost 0% in 2009:Q1. Both elevated corporate spreads and the binding lower bound persisted until the end of 2015, coincidental with the observed weakening of the relationship between inflation and output.

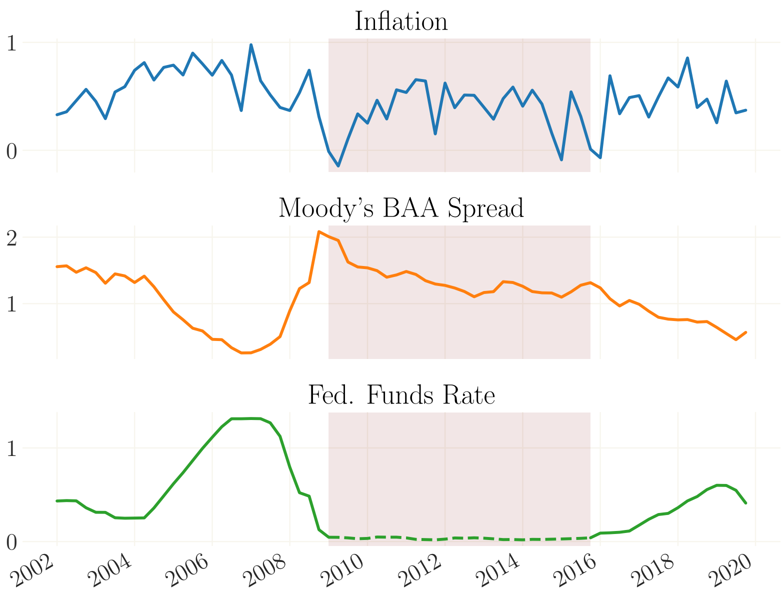


Figure 1: Time series of inflation, BAA spread and US interest rates from 2002 to 2020. Quarterly data in percentage points. The shaded area depicts the episode during which the ELB was binding.

We show these results in a tractable New Keynesian DSGE model featuring financial frictions. In the model, workers need to be paid before production (as in Ravenna and Walsh, 2006), generating external financing needs for the entrepreneurs operating the firms. Due to a costly state verification problem à la Townsend (1979) and Bernanke et al. (1999), the costs of external finance comprise a risk premium in the form of a countercyclical credit spread which depends on entrepreneur leverage. We focus on the effects of financial shocks in the form of risk premium shocks in the spirit of Smets and Wouters (2007). These shocks are known to have a large explanatory power for the joint movement of consumption and investment following the 2007/2008 recession (Gust et al., 2017; Kulish et al., 2017; Boehl and Strobel, 2020; Boehl et al., forthcoming).

Our first contribution is to show that the expectation of a longer ELB duration can be associated with weaker disinflationary effects of financial shocks. We provide analytic solutions both for normal times and for a binding ELB, which reveal that this case occurs for large financial shocks if the elasticity of the credit spread with respect to entrepreneur leverage is sufficiently large.[3] The analytic solutions furthermore highlight that even an overall increase of inflation following contractionary financial shocks is possible, and may in particular occur if the ELB is expected to bind for an extended period of time. In this case, financial shocks move output and inflation in opposite directions, thus appearing as supply-type disturbances, in sharp contrast to their usual appearance as demand shocks.

As our second contribution, we discuss the associated implications for monetary policy. We show that monetary policy shocks generate macroeconomic dynamics that are highly similar to financial shocks. As a consequence, forward guidance shocks with relatively low persistence can even be disinflationary: the effect of keeping expected refinancing costs lower in the future may dominate the long-run effect of increasing the price level by stimulating consumption. Hence, this also provides an explanation for the forward guidance puzzle (Carlstrom et al., 2015; Del Negro et al., 2015a; Kiley, 2016) and suggests that any forward guidance measures must be undertaken with vigor. Furthermore, the central bank might find itself in a knife-edge scenario where the appropriate window for systematic policy responses guaranteeing a determinate equilibrium are rather smaller.

The issue of missing (dis-)inflation in the recent years was first brought up by Ball and Mazumder (2011) and subsequently confirmed for many advanced economies by Friedrich (2016). Manifold of explanations were put forward, encompassing anchored expectations (Ball and Mazumder, 2018; Coibion and Gorodnichenko, 2015), various measures of economic slack (Gordon, 2013; Watson, 2014), supply shocks and wage rigidities (Daly and Hobijn, 2014; Harding et al., 2022), optimal monetary policy, potentially in combination with financial frictions (Lieberknecht, 2019; Sims and Wu, 2019; McLeay and Tenreyro, 2020) or global factors (Bobeica and Jarociński, 2019; Forbes, 2019). Compared to this literature, we provide a complementary explanation for inflation dynamics that also matches the particular timing of the observed missing (dis-)inflation: the ELB affects the cyclicality of marginal costs via the costs of external financing, thereby leading to an observational disconnect between inflation and output.

A related strand of the literature investigates these recent inflation dynamics through the lens of New Keynesian DSGE models, notably Christiano et al. (2015), Del Negro et al. (2015b) and Gilchrist et al. (2017). In line with our paper, these contributions show that adding financial

[3] Accordingly, this effect depends crucially on the presence of financial frictions and is hence absent in the standard New Keynesian model.

frictions to DSGE models helps to explain the missing disinflation puzzle in the US. Closely related to our work, Gilchrist et al. (2017) explain inflation dynamics via financial distortions, i.e. larger credit spreads in recessions. While our paper shares this argument, we provide additional insights that a binding ELB strongly amplifies the effects of financial frictions, such that credit spreads may even dominate inflation dynamics. This is in line with Bianchi and Melosi (2017) and Boehl and Strobel (2020), who find that accounting for the ELB substantially improves the empirical fit of estimated DSGE models. We also shed light on the necessary conditions for inflationary effects of financial shocks – a feature that is present in several financial friction models (Christiano et al., 2010; Meh and Moran, 2010; Gerali et al., 2010).

Our hockey-stick Phillips curve is also well-supported by recent empirical work showing that financial shocks can be disinflationary if supply-side effects dominate demand effects. Various contributions find empirical evidence in favor of such a (financial) cost channel (Barth III and Ramey, 2001; Chowdhury et al., 2006; Tillmann, 2008; Abbate et al., 2021). Similarly, Gaiotti and Secchi (2006) find this cost channel to be proportional to working capital, using Italian firm-level data. Our paper provides a theoretical foundation for these papers and highlights that both the degree of financial frictions and a binding ELB are particularly relevant.

Lastly, our paper is related to the literature on Neo-Fisherianism, which argues that the causality between the policy rate and inflation is positive even in the short run (Gabaix, 2020; Cochrane, 2011, 2016, 2017). We show that such effects may arise at the ELB, and – theoretically – even in normal times if the elasticity of marginal costs to the risk spread is large enough. This is in contrast to García-Schmidt and Woodford (2019), who argue that Neo-Fisherian effects arise only after credible changes in long-run monetary policy targets.

We continue in Section 2 by outlining the model and discussing the components of marginal costs in this framework. In Section 3, we derive closed-form solutions for macroeconomic dynamics following financial shocks. Section 4 complements by showing numerical solutions and analyzing the resulting observational Phillips curve. In Section 5, we investigate the implications for monetary policy at the ELB. Section 6 concludes.

## 2 Model

Our analysis is based on a tractable New Keynesian DSGE model featuring financial frictions. The setup is based on Boehl (2017) and Lieberknecht (2019), to which we refer for further details. Production is subject to a working capital channel as in Ravenna and Walsh (2006). A distinct role for external finance is motivated via a costly state verification problem in the spirit of Townsend (1979) and Bernanke et al. (1999). Entrepreneurs operating wholesale firms borrow from financial intermediaries to finance production, and their shares are traded on financial markets. Their (homogeneous) good is sold to a monopolistic retail sector where diversification takes place. The resulting final goods are sold to a representative household, who consumes and supplies labor in a perfectly competitive labor market. A central bank sets the nominal interest rate subject to an effective lower bound.

### *2.1 Households*

Households maximize the expected present value of lifetime utility by choosing consumption of a composite good $C_t$ and hours devoted to the labor market $H_t$. They can deposit monetary savings $D_t$ at financial intermediaries (banks in the following), for which they receive the gross nominal interest rate $R_t$ in the next period. The household's optimization problem is completely standard and yields the usual Euler equation and an intra-temporal labor supply equation

$$C_t^{-\sigma} = \beta E_t \left[ \frac{R_t}{\Pi_{t+1}} U_t C_{t+1}^{-\sigma} \right], \tag{1}$$

$$H_t^{\eta} = W_t C_t^{-\sigma}, \tag{2}$$

where $\Pi_t$ is gross inflation and $W_t$ is the real wage. $U_t$ is a *financial shock*, i.e. a premium on the risk-free interest rate reflecting the state of the financial system (as in Smets and Wouters, 2007). This type of shock features the highest explanatory power regarding the post-2000 macroeconomic dynamics across all standard shocks, and can explain a large share of the joint dynamics of consumption, investment and inflation following the 2007/2008 financial crisis (Gust et al., 2017; Kulish et al., 2017; Boehl et al., forthcoming). The parameters $\sigma, \eta$ and $\beta$ are the inverse elasticity of intertemporal substitution, the inverse Frisch elasticity of labor supply and the discount rate, respectively.

### *2.2 Wholesale and retail firms*

The wholesale sector consists of a continuum of firms indexed by $j$. Each firm is operated by a risk-neutral entrepreneur and produces a homogeneous good using a production function that is linear in labor (the only production factor) subject to a a firm-specific idiosyncratic productivity shock. Workers have to be paid before production takes place, while returns are realized at the end of the period. This working capital channel (also labeled the *cost channel*) follows Ravenna and Walsh (2006), is supported by empirical work (see references in the introduction) and motivates a role for external finance. The loan volume demanded by each entrepreneur is the difference between her desired working capital $W_t H_{j,t}$ and her equity $N_{j,t}$.

The realization of the idiosyncratic productivity shock is private information of the entrepreneur; banks can only observe produced output when paying monitoring costs. The contract that solves this costly state verification problem specifies that the interest rate on a loan obtained by an entrepreneur from the intermediary $R^L_{j,t}$ contains an endogenous risk premium on the prevailing real interest rate. The risk premium is a credit spread that depends positively on the individual firm's leverage $LEV_{j,t} = \frac{W_t H_{j,t}}{N_{j,t}}$. It can be shown that all entrepreneurs take identical choices in equilibrium, such that the aggregate loan rate is given by

$$R^L_t = z\left(\frac{W_t H_t}{N_t}\right)\frac{R_t}{E_t[\Pi_{t+1}]}U_t, \tag{3}$$

where $z'(\cdot) > 0$. Intuitively, when the leverage ratio rises, the premium on external finance increases because less collateral is provided such that the loan becomes more risky. Since the wholesale sector is assumed to be perfectly competitive, wholesale firms are price takers. In the aggregate, no-arbitrage requires the rate of return on working capital to equal the rate on external funding. It follows that firms' marginal costs are given by

$$MC_t = W_t R^L_t = W_t\, z\left(\frac{W_t H_t}{N_t}\right)\frac{R_t}{E_t[\Pi_{t+1}]}U_t. \tag{4}$$

With respect to equity financing, we assume that entrepreneurs can issue equity in the stock market, which is bought by risk-neutral financial traders associated with the financial intermediaries. Imposing no arbitrage on financial markets and noting that entrepreneurs must be indifferent between external finance and equity finance in equilibrium implies that the expected return on equity equals the loan rate. Invoking rule-of-thumb behavior from financial traders, it can be shown that the evolution of equity is given by

$$N_t = \Psi\left(Y_t\right), \tag{5}$$

with $\Psi'(\cdot) > 0$, such that equity financing is procyclical with respect to output, as in standard financial accelerator models à la Bernanke et al. (1999).

After wholesale goods have been produced, retailers buy the homogeneous good $Y_{j,t}$ on the wholesale market. After differentiation, they sell it in the monopolistically competitive good market. Firms' price setting decisions are subject to nominal rigidities à la Calvo (1983), which gives rise to a classic New Keynesian Phillips curve.

### *2.3 The central bank*

The central bank follows a standard rule for the notional gross nominal interest rate $R_t^n$,

$$\frac{R_t^n}{R^n} = \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} \exp(v_t), \tag{6}$$

where $v_t$ is a monetary policy shock following an AR(1) process. The interest rate on deposits $R_t$ is subject to a ELB constraint and cannot fall below $\bar{R}$,

$$R_t = \max\left\{\bar{R}, R_t^n\right\}. \tag{7}$$

Note that when the ELB constraint in Equation (7) binds, $v_t$ can be understood as a *forward guidance shock* as it prolongs the expected duration of the ELB.

### *2.4 Understanding the components of marginal costs*

In our framework, financial frictions originate in the firm sector and therefore primarily affect the supply side of the economy. The role of financial frictions for marginal costs and inflation dynamics is thus best understood by studying the New Keynesian Phillips curve. Linearizing around an efficient steady state[4], and denoting log-deviations from steady state via small-case letters, the New Keynesian Phillips curve may be represented in the familiar textbook form

$$\pi_t = \kappa\, mc_t + \beta E_t[\pi_{t+1}], \tag{8}$$

with slope $\kappa = \frac{(1-\zeta\beta)(1-\zeta)}{\zeta}$. Hence, financial frictions do not alter the price setting behavior of firms per se; prices are tied to marginal costs and expectations of future inflation. However, financial frictions determine and affect the components of marginal costs, which are given by

$$mc_t = w_t + (r_t - E_t[\pi_{t+1}]) + s_t, \tag{9}$$

[4]Steady state subsidies from the government (financed by lump-sum taxes) can correct for the two inefficiencies arising from monopolistic competition and the presence of financial frictions (Lieberknecht, 2019).

where $s_t$ denotes the linearized credit spread $s_t = z(lev_t) + u_t$. This highlights that marginal costs consist of three components: a) the real wage (also called *factor costs* in the following), as in the standard NK model, b) the risk-free real interest rate and c) the credit spread. The latter two components jointly constitute the costs of external finance.

In the following, we take a closer look at the cyclicality of these components. In a financial accelerator economy, the credit spread (also known as the external finance premium) is countercyclical (Bernanke et al., 1999). The elasticity of the credit spread with respect to entrepreneur leverage $\nu = z'(\cdot)$ can be shown to be positive (Lieberknecht, 2019). In turn, this implies that a countercyclical spread requires entrepeneur leverage to be countercyclical as well. Using the household's intra-temporal optimality condition, leverage is given by

$$lev_t = -(\psi - 1 - \sigma - \eta)y_t, \tag{10}$$

where $\psi = \Psi'(\cdot)$ denotes the elasticity of equity with respect to output. The necessary and sufficient condition for leverage to be countercyclical is thus that the term in brackets is larger than zero, i.e. that the procyclicality of net worth outweighs the procyclicality of factor costs. This implies the following parameter restriction:

**Assumption 1.** *The elasticity of net worth with respect to output satisfies*

$$\psi > 1 + \sigma + \eta. \tag{11}$$

Using these insights about entrepreneur leverage, marginal costs can be written as

$$mc_t = \gamma\, y_t + (r_t - E_t[\pi_{t+1}]) + u_t, \tag{12}$$

with

$$\gamma \equiv \sigma + \eta - \nu(\psi - 1 - \sigma - \eta) \tag{13}$$

capturing the elasticity of marginal costs with respect to output. The factor cost component, i.e. term $\sigma + \eta$ in $\gamma$, is procyclical. As output increases, expanding production requires firms to offer a larger real wage in order to increase labor supply. The credit spread term is countercyclical given Assumption 1 such that the term $-\nu(\psi - 1 - \sigma - \eta)$ is negative. The financial shock $u_t$ is also countercyclical, as output falls for positive realizations of $u_t$. The cyclicality of the cost channel is ambiguous and depends on the source of aggregate fluctuations, as this determines the endogenous nominal interest rate reaction by the central bank. For demand-side shocks – like the financial shock we consider – that reduce inflation, the real interest rate is procyclical. The three components of marginal costs are thus characterized by opposing cyclicality over the business cycle: factor costs and the interest rate are procyclical, whereas the external finance premium is countercyclical. Since firms' price setting is tightly connected to marginal costs, the relative dynamics of these components over the business cycle are thus crucial for inflation dynamics.

## 3 Financial shocks at the effective lower bound

In this section, we analyze how a binding ELB affects the transmission of financial shocks in the economy. To this end, we derive closed-form general equilibrium solutions for normal times and for when the economy is at the ELB. Contrasting these two cases highlights that macroeconomic dynamics at the ELB may be fundamentally different.

### *3.1 The propagation of financial shocks in normal times*

We first analyze the macroeconomic effects of financial shocks in normal times, i.e. when the ELB is non-binding. The model can be represented in three equations:[5]

$$\pi_t = \kappa\gamma\, y_t + (\beta - \kappa)E_t[\pi_{t+1}] + \kappa(r_t + u_t), \tag{14}$$

$$y_t = -\sigma^{-1}\left(r_t - E_t[\pi_{t+1}] + u_t\right) + E_t[y_{t+1}], \tag{15}$$

$$r_t = \max\left\{\phi_\pi \pi_t + \phi_y y_t + v_t, \bar{r}\right\}. \tag{16}$$

The exogenous processes for the financial shock $u_t$ and the monetary policy shock $v_t$ are

$$u_t = \rho\, u_{t-1} + \epsilon_t, \tag{17}$$

$$v_t = \rho_r\, v_{t-1} + \epsilon_{r,t}. \tag{18}$$

Equation (14) again represents the New Keynesian Phillips curve, where the third and fourth term reflect the cost channel and the purely exogenous markup that arises from financial shocks increasing the credit spread.[6] Equation (15) is the Euler equation, and Equation (16) is the

[5]See the Appendix for more details on this particular representation.

[6]This exogenous effect differentiates financial shocks from pure demand shocks (e.g. natural rate shocks) that appear solely in the Euler equation.

monetary policy rule setting the (notional) interest rate.[7] In normal times, the ELB constraint does not bind, such that (15) and (16) are identical to the textbook New Keynesian model. Financial frictions thus manifest solely in the New Keynesian Phillips curve, highlighting that the financial accelerator is a supply-side friction that directly affects inflation dynamics.

We solve the model via the method of undetermined coefficients and guess that the equilibrium responses of endogenous variables are linear functions of the exogenous financial shock.[8]

**Proposition 1.** *The impact responses of inflation and output to a financial shock in normal times are given by*

$$\pi_t = a_0\, u_t, \tag{19}$$

$$y_t = b_0\, u_t, \tag{20}$$

*where*

$$a_0 = -\frac{\kappa\gamma - \kappa\sigma(1-\rho)}{(1-\beta\rho)(\sigma(1-\rho)+\phi_y) + \kappa\gamma(\phi_\pi - \rho) - \kappa\sigma(1-\rho)(\phi_\pi-\rho)}, \tag{21}$$

$$b_0 = -\frac{1+(\phi_\pi-\rho)a_0}{\sigma(1-\rho)+\phi_y}. \tag{22}$$

*Proof.* See Appendix. ■

In combination with Proposition 1, the following Lemma 1 shows that financial shocks are (usually) a particular form of demand shocks. A positive financial shock increases the wedge between the interest rate controlled by the central bank and the return on bonds held by households, thereby reducing current consumption. Thus, a positive financial shock decreases overall output. Via the New Keynesian Phillips curve, inflation decreases as well as factor costs dominate over external financing costs.

**Lemma 1.** *The impact responses of inflation and output to a financial shock in normal times are negative, i.e.*

$$a_0 < 0, \tag{23}$$

$$b_0 < 0, \tag{24}$$

[7] Note that for financial shocks, the responses of output and the output gap are identical: an efficient economy without nominal rigidities and financial frictions does not respond to financial shocks.

[8] We assume that determinacy conditions hold. See Section 5.2 and Footnote 9 for a closer analysis.

*iff the elasticity of the credit spread to entrepreneur leverage satisfies*

$$\nu < \frac{\eta + \rho\sigma}{\psi - 1 - \sigma - \eta}. \tag{25}$$

*Proof.* See Appendix. ■

The analytic solutions from Proposition 1 display precisely the different channels through which the financial shock operates. In $a_0$, the first term in the numerator is the slope of the Phillips curve with respect to output, whereas the second term captures the exogenous markup effect of the financial shock. Following a positive financial shock, factor costs decrease, because labor demand falls given the decline in demand (the first part of $\kappa\gamma$). This in turn reduces inflation. At the same time, the financial shock increases the costs of production via the external finance premium, as financial frictions in the firm sector intensify (the second part of $\kappa\gamma$ and the markup effect). This increase in the credit spread partially counteracts the decline in factor costs, weakening the overall disinflationary effect.

The cost channel is represented by the last term in the denominator in $a_0$. This term features a negative sign and is thus – ceteris paribus – disinflationary. Generally, if the central bank reacts stronger (weaker) to fluctuations in inflation and output, the denominator is larger (smaller), such that the overall response of inflation is smaller (larger). However, lower nominal interest rates in reaction to the overall decline in inflation also decrease marginal costs directly. This amplifies the disinflationary response and the cost channel thus weakens the overall stabilizing property of the central bank's interest rate policy.

Following financial shocks, the various components of marginal costs thus move in different directions. Whereas factor costs and the cost channel amplify the disinflationary response, the credit spread channel weakens it. As seen in Lemma 1, the overall inflation response in normal times is negative, as long as the elasticity of the credit spread to entrepreneur leverage is not excessively large.[9] In this case, factor costs dominate the price setting of firms, whereas the interest rates and credit spreads approximately balance out.

Nevertheless, as summarized in Lemma 2 below, the analytic solutions reveal that an overall increase of inflation following positive financial shocks is in principle possible. This situation

[9] Lemma 1 guarantees that the denominator in $a_0$ is positive, which is required for determinacy (as shown in the Appendix). Intuitively, the model is only determinate if a stronger central bank reaction to deviations from steady state translates into lower deviations in general equilibrium. The combination of a positive numerator from Lemma 1 and determinacy thus yields $a_0 < 0$ (note the minus in front of the fraction).

may occur if the credit spread channel dominates both factor costs and the cost channel because the credit spread sensitivity to leverage is (excessively) large:

**Lemma 2.** *The impact response of inflation to a financial shock in normal times is positive if the elasticity of the credit spread to entrepreneur leverage satisfies*

$$\nu > \frac{\eta + \rho\sigma}{\psi - 1 - \sigma - \eta}. \tag{26}$$

*Proof.* See Appendix. ■

Note that this results directly from the presence of financial frictions linking credit spreads to marginal costs: in the absence of financial frictions, the policy functions in Proposition 1 are unambiguously negative. In the following, we want to focus on the case in which our financial shocks is a classic demand shock to maintain the analogy to the Global Financial Crisis. We hence generally assume that Equation (26) is not satisfied such that $a_0$ remains negative:

**Assumption 2.** *The elasticity of the credit spread to entrepreneur leverage satisfies Condition* (25) *from Lemma 1.*

This implies the natural case of an upward sloping Phillips curve for financial shocks in normal times, i.e. a positive relationship between inflation and output.

### *3.2 The propagation of financial shocks at the ELB*

We now turn to the case of a binding ELB. To this end, we assume that a financial shock endogenously brought the economy to the ELB and makes private agents expect the ELB to bind for a certain number of periods (often called the *ELB spell duration*, e.g. Holden, 2019). In this section, we take this ELB spell duration as given and do *not* adjust agents' expectations on the spell duration to any *additional* shocks, which we discuss in Section 4. This scenario hence focuses on *marginal* effects of (further) financial shocks at the ELB. While this perspective abstracts from the mapping between shocks and the expected duration of the ELB, it allows for a straightforward analytical comparison to the case of normal times.

The equilibrium responses of inflation and output can be characterized by recursive policy functions which are conditionally linear given the expected ELB spell:

**Proposition 2.** *Suppose that the ELB on nominal interest rate is expected to bind for $k > 0$ periods. Then, the impact responses of inflation and output to a financial shock are given by*

$$\pi_t = a_k\, u_t, \tag{27}$$

$$y_t = b_k\, u_t, \tag{28}$$

*where*

$$a_k = \kappa\left(1 - \gamma\sigma^{-1}\right)\left(1 + \frac{\bar{r}}{u_t}\right) + \rho\left(\beta - \kappa + \kappa\gamma\sigma^{-1}\right) a_{k-1} + \rho\kappa\gamma\, b_{k-1}, \tag{29}$$

$$b_k = -\sigma^{-1}\left(1 + \frac{\bar{r}}{u_t}\right) + \rho\sigma^{-1}\, a_{k-1} + \rho\, b_{k-1}, \tag{30}$$

*and $\{a_0, b_0\}$ as in Proposition 1.*

*Proof.* See Appendix. ■

To interpret Proposition 2, consider the inflation response for an expected ELB duration of one quarter ($k = 1$), i.e. $a_1$, and recall that Assumption 2 guarantees negative policy functions $a_0$ and $b_0$. This implies that both the second and third term in Equation (29) are negative. The term in front of $a_0$ is close to unity for persistent shocks and shows the ELB's amplification property: the impact response of inflation increases (ceteris paribus) in the expected length of the ELB. This reflects the inability of the central bank at the ELB to counteract further contractionary shocks via additional (conventional) monetary stimulus. At the same time, the resulting upward pressure on real interest rates depresses consumption, and accordingly output.

However, there is an opposing effect on the overall inflation response, captured by the first term in Equation (29). This term can be positive, such that there is potential for a policy function for inflation that is concave in the expected ELB spell duration. In other words, it is possible that the disinflationary effect following positive financial shocks is *lower* if the ELB is expected to bind for a longer period of time. A necessary condition for a concave inflation policy function is that $1 > \gamma\sigma^{-1}$, which is equivalent to the following Lemma 3.

**Lemma 3.** *The inflation policy function is concave if the elasticity of the credit spread with respect to entrepreneur leverage satisfies*

$$\nu > \frac{\eta}{\psi - 1 - \sigma - \eta}. \tag{31}$$

It thus follows from Lemma 3 that the overall response of inflation following inflationary shocks depends crucially on the elasticity of the credit spread with respect to entrepreneur leverage. An alternative way to see this is to note that

$$\frac{\partial a_k}{\partial \nu} = \frac{\partial a_k}{\partial \gamma}\frac{\partial \gamma}{\partial \nu} > 0. \tag{32}$$

The first term in Equation (29) depends negatively on $\gamma$. The second depends positively on $\gamma$, but following the recursion brings up $a_0$, which is negative following Assumption 2. The last term is positive in $\gamma$ as well, while $b_{k-1} < 0$ for all reasonable calibrations. The effect of an increase in $\nu$ is hence unambiguous: a larger elasticity of the credit spread with respect to entrepreneur leverage ceteris paribus increases the inflationary effect of financial shocks.

Intuitively, a concave policy function for inflation requires that the credit spread (the left-hand side in Equation (31)) dominates both the factor cost and the safe interest rate (the right-hand side in Equation (31)). If financial frictions are sufficiently pronounced such that $\nu$ is large, credit spreads may dominate the price setting of firms, thereby increasing inflation ceteris paribus. This result also naturally implies that the corresponding effect is absent in the standard NK model (in which $\nu = 0$).[10] For the following analysis, we capture this scenario via the following assumption:

**Assumption 3.** *The elasticity of the credit spread with respect to entrepreneur leverage satisfies Condition* (31) *from Lemma 3.*

Note that Assumption 3 is weaker than the counterpart in Assumption 2. A further requirement for a concave policy function is that financial shocks are sufficiently large. This can be seen by inspecting the term $(1 + \frac{\bar{r}}{u_t})$, which is only positive if the following Assumption holds:

**Assumption 4.** *The financial shock size satisfies*

$$u_t > -\bar{r} = \beta^{-\sigma} - 1. \tag{33}$$

Figure 2 displays the policy functions $a_k$ and $b_k$ under two illustrative calibrations. In the first case, the parameters satisfy Assumptions 1, 2 and 4: the spread is countercyclical, financial

[10]Given financial frictions, a concave policy function is also possible for natural rate shocks. This requires a larger elasticity of the credit spread to compensate for the missing purely exogenous markup effect. It holds that $a_k^{\text{nrs}} = a_k^{\text{fs}} - \kappa$, where "nrs" stands for natural rate shock, and "fs" for financial shock.

shocks have conventional effects in normal times and the shock is relatively large. In the second case, the calibration additionally satisfies Assumption 3. In the first case, the policy functions for inflation and output are strictly decreasing in the expected ELB spell duration; a longer expected ELB duration implies a stronger macroeconomic effect of additional financial shocks. In the second case, however, the policy function for inflation is concave, peaking at an expected ELB duration of six quarters in positive territory. In other words, if the ELB is expected to bind for a longer period of time, the overall inflation response may even turn positive. In this case, financial shocks appear as supply-type disturbances, i.e. moving output and inflation in opposite directions, whereas they usually belong to the class of demand shocks. This illustrates that inflation dynamics following financial shocks may be fundamentally different at the ELB compared to normal times.

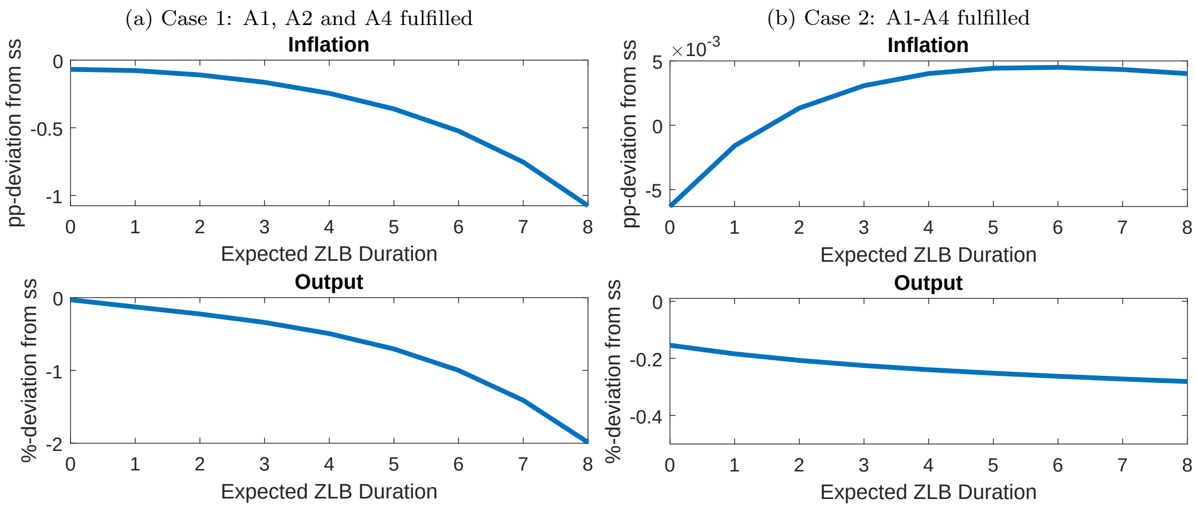


Figure 2: Expected ELB duration and impact response following financial shocks.

## 4 Numerical results and the hockey stick Phillips curve

In this section, we supplement our closed-form solutions by a numerical analysis of the full general equilibrium rational expectations solution. We employ numerical solution methods to treat the expected ELB spell as endogenous to present impulse responses to financial shocks and trace out the corresponding observational Phillips curve.

### *4.1 Calibration and solution method*

Throughout this section, we fix the model's structural parameters to standard values taken from Woodford (2003), and adjust them to the most recent estimates (up until 2019) from Boehl and Strobel (2020, BS20 henceforth). We set $\beta = 0.99$, representing the standard view of a quarterly model. We calibrate $\sigma = 1$, which is a common assumption in line with a balanced growth path and also backed by BS20. Following the same line of reasoning, we set $\eta = 0.5$. We calibrate the fraction of non-adjusting price setters $\zeta$ to the commonly found textbook value of 0.66. This is conspicuously lower than the larger estimates from Smets and Wouters (2007) and BS20, as we want to avoid assuming a flat New Keynesian Phillips curve ex-ante.

For the parameters pertaining to the financial frictions, we fix $\psi = 8$ such that the output effects of financial shocks are amplified by approx. 20% relative to the standard NK model, which is roughly in line with the amplification degree documented by Bernanke et al. (1999). For our baseline szenaro we assume that $\nu = 0.25$, which implies an elasicity of firms' marginal cost w.r.t. the risk spread of one quarter. We conduct comparative exercises with regard to this parameter further below and in section 5.

Regarding monetary policy parameters, we set $\phi_\pi$ to 1.5 (a commonly used standard prior), and $\phi_y$ to 0.2. In line with the estimates of BS20, the latter value is large relative to the standard prior mean of 0.125. As the authors argue, this reflects the strong reaction of the Fed to output during the ELB episode from 2009–2015, during which inflation was close to its target value while the level of output remained persistently depressed. We set $\rho = 0.9$ as suggested by the estimates of Boehl and Strobel (2020); Boehl et al. (forthcoming) for the persistence of such shocks over the post-2009 US and Euro area data. This reflects a lasting, quite persistent financial shock which resembles to the scenario of the Great Recession and its aftermath.

The analytical solutions shown in the previous section hold for the impact period when the shock occurs, under the assumption that the expected duration of the ELB $k$ is given. However, in general and in the absence of special policy measures such as forward guidance, $k$ is an equilibrium outcome to be determined endogenously at each point in time, given the contemporaneous exogenous disturbances that causes the ELB constraint to bind. To solve the model at the ELB, we use the numerical solution method proposed by Boehl (2021). A brief description of the solution method is outlined in Appendix C.

### *4.2 Impulse responses to financial shocks*

The left panel of Figure 3 displays impulse responses following contractionary financial shocks of differing size. For the impact responses, these correspond to the analytical policy functions in Proposition 1 and Proposition 2. A one-percent shock (yellow line) is not strong enough to cause the ELB to be binding. As a result, the dynamics look conventional (for demand-side shocks), with inflation (and marginal costs) falling in response to the shock. As the shock size increases, the ELB spell duration increases. Respectively, the initial response of inflation shifts upwards, in line with the analytical insight from Assumption 4. For a large value of $u_t$, the initial response of inflation becomes positive.[11]

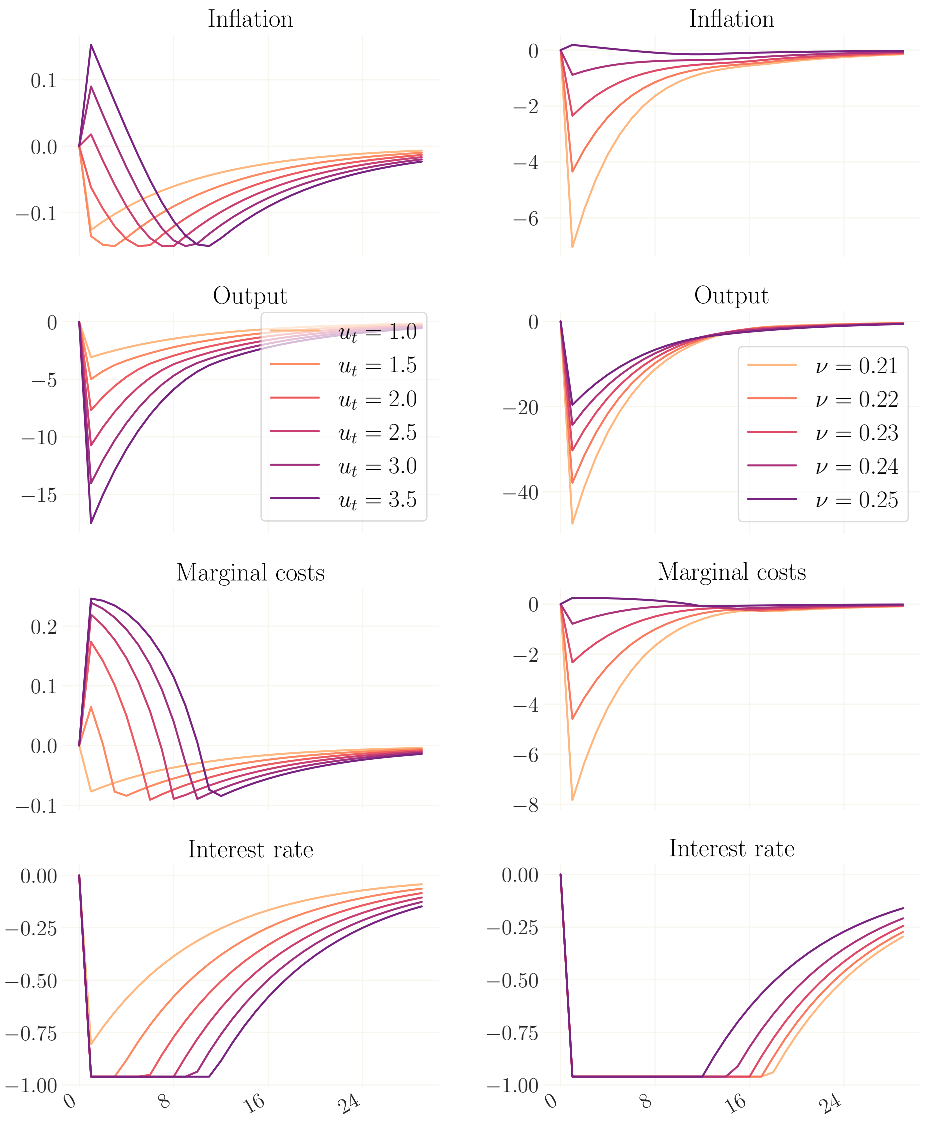


Figure 3: Impulse response functions. *Left:* to different financial shocks for $\nu = 0.25$. *Right:* to a 3% financial shock for different values of $\nu$ causing the ELB to be binding.

In the right panel of Figure 3, we consider a large financial shock, pushing the economy to the ELB, for different values of $\nu$. As highlighted by the graphs, marginal costs decrease less if financial frictions are stronger, which also translates to inflation. For $\nu = 0.25$, inflation

[11]Note that the lines are simply shifted outwards in case of a larger initial shock, since the responses of endogenous variables are a simple linear map of $u_t$ and $u_t$ decreases each period by $(1 - \rho)$.

actually increases, whereas the same calibration yields regular dynamics in the absence of the ELB (left side of Figure 3). This corresponds to the standard case outlined in the previous section: the elasticity of the credit spread with respect to entrepreneur leverage is large enough to generate a concave inflation policy function, but not so excessively large such that a positive response emerges in normal times.[12]

### *4.3 The observational hockey stick Phillips curve*

Figure 4 plots the impulse responses to financial shocks projected into $\{y_t, \pi_t\}$-space. We interpret this as the *observational Phillips curve*, i.e. the realized values of inflation and output (gap) that would be observed in general equilibrium.[13] This is in contrast to the theoretical New Keynesian Phillips curve – as shown in Equations (8) and (14) – which represents firms' price setting under the assumption of nominal price rigidities. The most remarkable observation in Figure 4 is the striking hockey stick shape of the observational Phillips curve. For positive values of output, the observed slope of the Phillips curve is positive, in line with standard theory. However, for substantially negative values of output (caused by large financial shocks) the observational Phillips curve flattens out at the ELB.

As the elasticity of the credit spread to leverage, $\nu$, increases, the Phillips Curve becomes flatter for both regimes: the hockey stick not only rotates in the origin, but also the ratio of the two slopes decreases. For $\nu = 0.24$, the observed slope in the region of -3% output is almost zero, while having a conventional slope in the origin. For a value of $\nu = 0.25$, we observe that the credit spread effect at the ELB is strong enough that inflation actually increases with output, while the Phillips curve is still upwards sloping in normal times.

In other words, an economic observer aiming to infer the slope of the Phillips curve in times of a binding ELB and financial frictions would inherently conclude that the Phillips curve is "dead". This observation emerges even though the New Keynesian Phillips curve is well and alive: the relationship between firms' prices and marginal costs, governed by the Calvo parameter, is intact.[14] However, the credit spread channel dominates firms' price setting at the ELB and

[12]As Proposition 2 suggests, the persistence of financial shocks $\rho$ is another central parameter for inflation dynamics, both at the ELB and for the rather extreme Neo-Fisherian case. A lower value of $\rho$ yields a more concave inflation policy function (c.f. Equation 29). A lower $\rho$ also implies a stronger discounting and hence a less dominant effect of the anticipated course of the financial shock. We illustrate this in Figure D.3 in the Appendix. We discuss the role of persistence in more detail in Section 5.

[13]Note again that the output response following financial shocks is identical to the output gap response, see Footnote 7. As such, the figure can equivalently be interpret as showing the output gap - inflation space.

[14]Note again that our calibration avoids pre-assuming a flat New Keynesian Phillips curve, with the Calvo parameter $\zeta = 0.66$ being considerably lower than the estimate of $\zeta = 0.85$ in BS20.

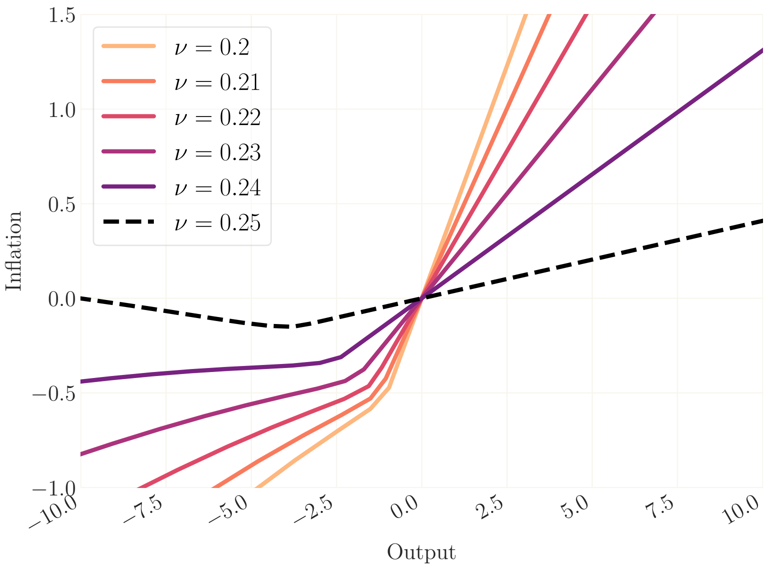


Figure 4: Observed Phillips Curve for an economy facing financial shocks. For each value of $\nu$, we simulate the model for $u_t$ on the interval $[-4, 4]$ and plot the respective combination of $\pi_t$ and $y_t$.

thus blurs the supply-side link between output and prices. The flat observational Phillips curve at the ELB implies that a proper identification of the relationship between inflation and output is challenging if financial shocks are prevailing.

## 5 Monetary policy at the effective lower bound

From the viewpoint of central banks, the difficulties of interpreting the observational Phillips curve translate into delicate decisions about the appropriate design of monetary policy at the ELB. To make matters worse, the effects of monetary policy itself are also affected by financial frictions and the ELB. We analyze this aspect by considering both monetary policy shocks – in particular forward guidance shocks at the ELB – and the systematic behavior of central banks, governed by the monetary policy rule.

### *5.1 Reversal effects of forward guidance*

The first crucial insight regarding monetary policy shocks $v_t$ is that in normal times, they generate *identical* macroeconomic dynamics as financial shocks. The three-equation representation from Section 3.1 reveals that monetary policy shocks appear in the same places as financial shocks. Therefore, in this framework and away from the ELB, monetary policy shocks and financial shocks are observationally equivalent in terms of inflation and output; they are only distinguishable via the response of the interest rate. As a consequence, all results from the previous sections concerning financial shocks in normal times are valid for monetary policy shocks as well. Notably, this includes the closed-form solutions, which also implies that the possibility of *Neo-Fisherian* effects of monetary policy shocks in normal times (an increase in inflation after rising interest rates) for extreme calibrations. It also follows immediately that the central bank can, in principle, offset financial shocks perfectly in normal times.

The insight that both shocks appear in the same places features major implications for forward guidance monetary policy at the ELB, which is the second important contribution of this paper. At the ELB, through their shock persistence, monetary policy shocks govern the expectations regarding the future interest rate path, acting like explicit forward guidance by the central bank. Forward guidance hence generates the same macroeconomic dynamics at the ELB as financial shocks.[15] However, unfortunately for monetary policy, our previous results thus imply that forward guidance at the ELB might not be particularly effective and may even be associated with unintended effects on inflation. Notably, this includes the possibility that forward guidance at the ELB may be *disflationary*, i.e. inducing Neo-Fisherian effects by *decreasing* inflation, while raising output.

Intuitively, forward guidance shocks induce three different and partially opposing effects on inflation. First, expected rates are lower, which transmits to the economy via the standard (demand-side) Euler channel. Second, lower expected rates decrease expected marginal costs via the external finance premium. Third, agents expect that the inversion of the policy function will remain active for more periods. The first effect leads to an unambiguous increase in output. The second effect clearly depresses inflation. The third effect prolongs the reversal of the inflation response that is induced by the ELB via the credit channel. As forward guidance raises output, this could also trigger a drop in inflation. Which of these effects dominates depends crucially on the forward guidance persistence and the degree of financial frictions.

As an illustration, Figure 5 shows impulse responses following forward guidance shocks at the ELB given different values for $\nu$. In the left panel, the stronger internal propagation of the forward guidance shock caused by a higher $\nu$ leads to a longer ELB period than in the right panel. This means that, given the same financial shock, during the extended ELB period the interest rate is much lower than in the absence of forward guidance, which causes marginal costs to fall. Since expected lower marginal cost are anticipated by firms (via the Phillips curve), the fall in inflation is *larger* than without forward guidance. This effect is absent in the right panel

[15]At the ELB, monetary policy shocks $v_t$ and financial shocks $u_t$ are hence not distinguishable, given the same persistence.

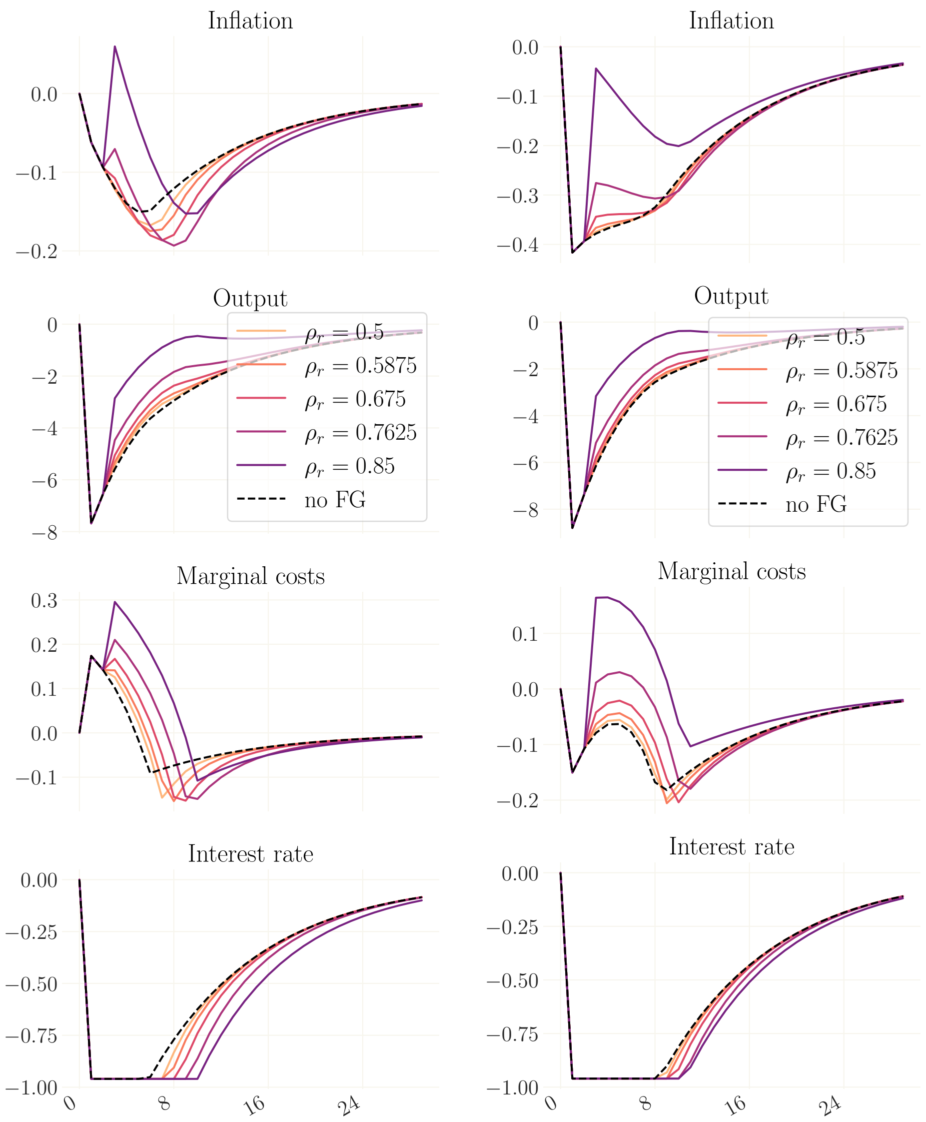


Figure 5: Impulse response functions for a 2% financial shock. Colored lines are the financial shock combined with a forward guidance shock in period 3. Different colors correspond to different persistences of the forward guidance shock. *Left:* for $\nu = 0.25$. For many values of $\rho_r$, the forward guidance shock is disinflationary. *Right:* for $\nu = 0.24$. For this value of $\nu$ the forward guidance shock is not disinflationary.

of Figure 5 because the shock does not prologue the ELB period significantly.

**Lemma 4.** *At the ELB, forward guidance shocks $v_t$ may be associated with Neo-Fisherian effects such that expansionary forward guidance is disinflationary iff*

$$\rho_r < \rho. \tag{34}$$

Note that the condition in Lemma 4 is a necessary, but not a sufficient condition. To see this, assume a combination $(\rho, \nu)$ for which a given shock $u_t$ is disinflationary. As the mechanics behind forward guidance and financial shocks are equal, we learn from Equation (29) in Proposition 2 that a smaller $\rho$ (or here: $\rho_r$) can reduce the weight on the (negative terminal) second and third term. In that sense, a decrease in $\rho$ has a similar effect as an increase in $\nu$. We show this effect in Figure D.3 in the Appendix.

While it is safe to assume a high persistence of the financial shock, the persistence of the forward guidance shock is to some extent a policy parameter that can in principal be chosen by the central bank. However, it also depends on how successful the central bank is in its communication strategies. As illustrated in Figure 5, a monetary policy shock with low persistence (i.e. low credibility) can hence trigger negative inflation responses because the short-run effect of decreasing financial costs dominates the longer-term effect that works through the household Euler Equation. As such, non-credible forward guidance may be associated with undesirable macroeconomic dynamics.

### *5.2 Monetary policy rules at the ELB*

We now turn to the systematic behavior of central banks. At first glance, it may seem that these rules are irrelevant at the ELB. However, they are in fact crucial for macroeconomic dynamics because rational private agents take the monetary policy rule into account when forming expectations about future variables and the remaining ELB duration. As such, choosing an appropriate monetary policy rule is of central importance for central banks at the ELB as well. From a policy-making perspective, the minimum requirement that any appropriate rule should satisfy is that it guarantees a determinate equilibrium.

**Proposition 3.** *The policy parameters in the central bank's monetary policy rule must satisfy the following conditions to guarantee a determinate solution:*

$$\phi_\pi + \frac{1-\beta}{\kappa\gamma}\,\phi_y > 1, \tag{35}$$

$$\kappa(\sigma^{-1}\gamma - 1)\,\phi_\pi + \sigma^{-1}\phi_y > \beta - 1 - \kappa \tag{36}$$

*Proof.* See Appendix. ■

Equation (B.44) may be interpreted as a modified Taylor principle for a financial accelerator economy. If the central bank decides to react to inflation only ($\phi_y = 0$), a necessary condition is that the associated coefficient $\phi_\pi$ needs to be larger than unity, as in Taylor (1993). If the central bank reacts to output as well ($\phi_y > 0$), determinacy requires the weighted sum of policy coefficients to be larger than unity. Compared to a standard New Keynesian framework, the key difference is that financial frictions affect the degree of substitutability between reacting to inflation and to output. Under Assumption 1, the slope of the New Keynesian Phillips curve with respect to output (the term $\kappa\gamma$) is lower due to the countercyclical credit spread. At first glance, it thus seems that policy responses to output can *substitute* more effectively for policy responses to inflation in the presence of financial frictions.

However, Equation (B.45) may constitute additional complications for the design of monetary policy rules. To see this, suppose that $(\sigma^{-1}\gamma - 1) < 0$, which is exactly the condition for a concave policy function of inflation at the ELB, i.e. Assumption 3. In this case, Equation (B.45) implies that the responses to inflation and output are *complements* for some combinations of $\{\phi_\pi, \phi_y\}$, or equivalently constitutes a lower (upper) bound restriction for the response to output (inflation). In other words, a stronger reaction to inflation must be accompanied by a corresponding stronger reaction to output. This clashes with the modified Taylor rule that exhibits the conventional substitutability.

Figure 6 displays this result graphically. As the elasticity of the credit spread with respect to entrepreneur leverage $\nu$ increases, a higher value for $\phi_y$ is necessary to keep the model determined for high values of $\phi_\pi$. For example, in the case of $\nu = 0.2$, $\phi_\pi > 1.76$ requires that $\phi_y > 0$.

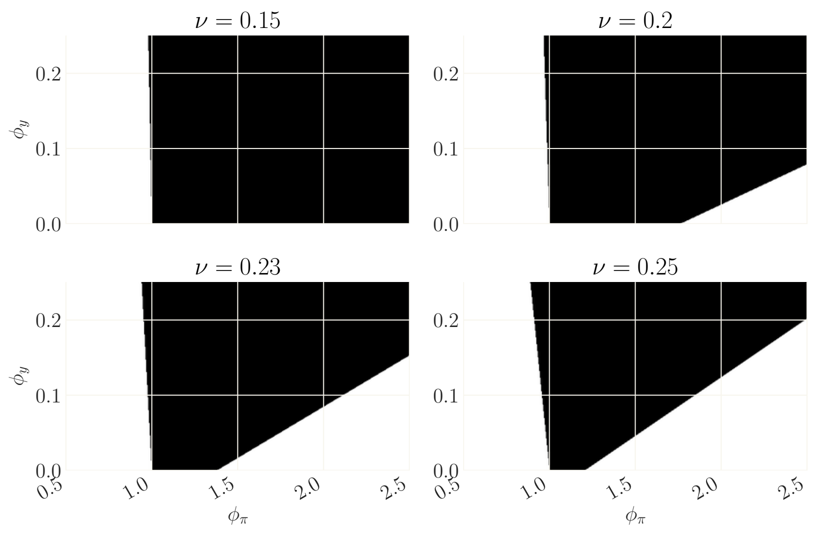


Figure 6: Determinacy regions for different values of $\nu$.

Intuitively, abstracting from financial frictions, inflation can be stabilized by raising nominal interest rates appropriately. Higher nominal interest rates amount to higher real interest rates, decreasing consumption and output. As a consequence, real marginal costs fall, and inflation decreases. Whether the hike of nominal interest rates constitutes a reaction to (positive) deviations of inflation or output is irrelevant. In the presence of financial frictions, however, an interest rate hike as a reaction to output has the additional effect of increasing marginal costs and thus inflation. Depending on the specific characteristics of the economy, the central bank might find itself in a knife-edge scenario where the appropriate window for systematic policy responses to output deviations is quite small.

Overall, the key message emerging from this section is that the conduct of monetary policy in the presence of financial frictions and a binding ELB may prove difficult. While the hockey stick Phillips curve blurs the relationship between inflation and output at the ELB, conventional monetary policy wisdoms are abolished: short-lived forward guidance shocks may be associated with Neo-Fisherian inflation effects, and determinacy considerations may place rather tight restrictions on appropriate monetary policy rules.

## 6 Conclusion

This paper argues that a binding effective lower bound (ELB) on nominal interest rates may contribute to an observational disconnect between inflation and economic activity if financial shocks are prevailing. At the ELB, the costs of external financing in the form of credit spreads can dominate firms' price setting and thereby generate inflationary pressure. Via this supply-side mechanism, the Phillips curve features a considerably flatter slope when the ELB binds compared to normal times. As a consequence, the resulting observational Phillips curve is shaped like a hockey stick. These findings constitute a complementary explanation for the recently observed inflation puzzles.

Our results translate into strong implications on the conduct of forward guidance, and provide a potential solution to the forward guidance puzzle: similar to financial shocks, the effects of forward guidance can be decomposed in short-run disinflationary effects via the firms' refinancing cost channel, and a longer-term inflationary effect via real marginal costs. For rather short-lived forward guidance impulses, the first disinflationary effect may dominate and forward guidance can in fact lower inflation. Accordingly, only forward guidance with a high expected persistence succeeds in fostering inflation and growth.

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## Appendix A Equilibrium Equations

This section lists the full set of the equations defining the equilibrium. On the household side, we have the inter-temporal Euler equation and the intra-temporal labor-consumption trade-off, Equations (1) and (2) in the main text:

$$C_t^{-\sigma} = \beta E_t \left[ \frac{R_t}{\Pi_{t+1}} U_t C_{t+1}^{-\sigma} \right], \tag{A.1}$$

$$H_t^{\eta} = W_t C_t^{-\sigma}. \tag{A.2}$$

On the firm side, we have the aggregate production function, which is obtained by aggregating over the individual linear production functions:

$$Y_t = \frac{H_t}{v_t^p} \tag{A.3}$$

where $v_t^p$ is a measure of price dispersion defined below. Marginal costs are given by Equation (4):

$$MC_t = W_t R_t^L \tag{A.4}$$

The price setting behavior by firms is defined by the following equations, which are standard for Calvo (1983) pricing and make use of two auxiliary variables $f_t^1$ and $f_t^2$:

$$f_t^1 = \frac{\varepsilon - 1}{\varepsilon} f_t^2 \tag{A.5}$$

$$f_t^1 = C_t^{-\sigma} MC_t Y_t + \beta\zeta E_t \left[ \Pi_{t+1}^{\varepsilon} f_{t+1}^1 \right] \tag{A.6}$$

$$f_t^2 = C_t^{-\sigma} \Pi_t^* Y_t + \beta\zeta E_t \left[ \left( \frac{1}{\Pi_{t+1}} \right)^{1-\varepsilon} \left( \frac{\Pi_t^*}{\Pi_{t+1}^*} \right) f_{t+1}^2 \right] \tag{A.7}$$

$$1 = \zeta \left( \frac{1}{\Pi_t} \right)^{1-\varepsilon} + (1-\zeta) \left( \Pi_t^* \right)^{1-\varepsilon} \tag{A.8}$$

$$v_t^p = \zeta \Pi_t^{\varepsilon} v_{t-1}^p + (1-\zeta) \left( \Pi_t^* \right)^{-\varepsilon} \tag{A.9}$$

The interest rate specified in the credit contract is defined by Equation (3):

$$R_t^L = z \left( \frac{W_t H_t}{N_t} \right) \frac{R_t}{E_t[\Pi_{t+1}]} U_t \tag{A.10}$$

Entrepreneur net worth evolves according to Equation (5):

$$N_t = \Psi\left(Y_t\right), \tag{A.11}$$

The central bank operates according to a monetary policy rule shown in Equation (6)

$$\frac{R_t^n}{R^n} = \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} exp(v_t), \tag{A.12}$$

The effective lower bound (ELB) constraint is given by Equation (7):

$$R_t = \max\left\{\bar{R}, R_t^n\right\} \tag{A.13}$$

Finally, the aggregate resource constraint is

$$Y_t = C_t \tag{A.14}$$

These 14 conditions define the equilibrium for the 14 endogenous variables

$$\left(C_t, Y_t, H_t, \Pi_t, \Pi_t^*, W_t, R_t, R_t^L, R_t^n, N_t, MC_t, f_t^1, f_t^2, v_t^p\right), \tag{A.15}$$

together with the evolution of the two exogenous shocks:

$$\ln(U_t) = \rho\, \ln(U_{t-1}) + \epsilon_t \tag{A.16}$$

$$v_t = \rho_r\, v_{t-1} + \epsilon_t r, t. \tag{A.17}$$

The linearized equilibrium conditions are as follows:

$$c_t = -\sigma^{-1}\left(r_t + u_t - E_t\pi_{t+1}\right) + E_t[c_{t+1}], \tag{A.18}$$

$$w_t = \eta h_t + \sigma c_t, \tag{A.19}$$

$$y_t = h_t, \tag{A.20}$$

$$mc_t = w_t + r_t^L, \tag{A.21}$$

$$\pi_t = \kappa mc_t + \beta E_t[\pi_{t+1}], \tag{A.22}$$

$$r_t^L = r_t - E_t[\pi_{t+1}] + \nu(w_t + h_t - n_t) + u_t, \tag{A.23}$$

$$n_t = \psi y_t, \tag{A.24}$$

$$r_t^n = \phi_\pi \pi_t + \phi_y y_t + v_t, \tag{A.25}$$

$$r_t = \max\left\{\bar{r}, r_t^n\right\}, \tag{A.26}$$

$$y_t = c_t, \tag{A.27}$$

$$u_t = \rho\, u_{t-1} + \epsilon_t, \tag{A.28}$$

$$v_t = \rho_r\, v_{t-1} + \epsilon_{r,t}, \tag{A.29}$$

$$\tag{A.30}$$

where lower-case variables denote log-deviations from steady state.

The three-equation representation shown in Section 3.1 can be obtained by combining Equations (A.19)-(A.24) into one single Phillips curve and using the resource constraint Equation (A.27) to eliminate $c_t$.

## Appendix B Proofs

**Proposition 1.** *The impact responses of inflation and output to a financial shock in normal times (without a binding ELB on nominal interest rates) are given by:*

$$\pi_t = a_0\, u_t, \tag{B.1}$$

$$y_t = b_0\, u_t, \tag{B.2}$$

*where*

$$a_0 = -\frac{\kappa\gamma - \kappa\sigma(1-\rho)}{(1-\beta\rho)(\sigma(1-\rho)+\phi_y) + \kappa\gamma(\phi_\pi - \rho) - \kappa\sigma(1-\rho)(\phi_\pi - \rho)}, \tag{B.3}$$

$$b_0 = -\frac{1+(\phi_\pi - \rho)a_0}{\sigma(1-\rho)+\phi_y}. \tag{B.4}$$

*Proof.* The proof relies on the method of undetermined coefficients. We guess that the solution is given by $\pi_t = a_0\, u_t$ and $y_t = b_0\, u_t$. Using this guess, the system of equation can be written as

$$(1-\kappa\phi_\pi - \rho(\beta-\kappa))a_0 u_t = \kappa u_t + \kappa(\gamma+\phi_y)b_0 u_t, \tag{B.5}$$

$$(1+\phi_y\sigma^{-1} - \rho)b_0 u_t = -\sigma^{-1}(\phi_\pi - \rho)a_0 u_t - \sigma^{-1} u_t, \tag{B.6}$$

where we replaced the nominal interest rate using the (unconstrained) Taylor rule. Note that expectations of future variables can be replaced by using the law of motion for the financial shocks under rational expectations. The solution is obtained by dividing both equations by $u_t$, substituting for $b_0$ in the first equation using the second equation and rearranging. ■

**Lemma 1.** *The impact responses of inflation and output to a financial shock in normal times (without a binding ELB on nominal interest rates) are negative, i.e.*

$$a_0 < 0, \tag{B.7}$$

$$b_0 < 0, \tag{B.8}$$

*iff the elasticity of the credit spread to entrepreneur leverage satisfies*

$$\nu < \frac{\eta + \rho\sigma}{\psi - 1 - \sigma - \eta}. \tag{B.9}$$

*Proof.* The proof consists of three parts. First, we show that the model's determinacy conditions imply that the denominator of $a_0$ is positive. Second, the sign of $a_0$ then depends on its numerator, which is equivalent to the parameter restriction in the Lemma. Third, the sign of $b_0$ follows from $a_0$.

First, let us consider the determinacy conditions. The forward looking components of our model can be expressed as

$$M\mathbf{x}_t = E_t[\mathbf{x}_{t+1}], \tag{B.10}$$

with $\mathbf{x}_t = (y_t, \pi_t)'$. To arrive at this formulation, we can rewrite Equations (15) and (16) (ignoring exogenous innovations and the ELB) as

$$(1 + \sigma^{-1}\phi_y)y_t = -\sigma^{-1}\left(\phi_\pi \pi_t - E_t[\pi_{t+1}]\right) + E_t[y_{t+1}], \tag{B.11}$$

$$(1 - \kappa\phi_\pi)\pi_t = \kappa(\gamma + \phi_y)y_t + \beta_\kappa E_t[\pi_{t+1}], \tag{B.12}$$

where we define $\beta_\kappa = \beta - \kappa$ for convenience. Then, we can rewrite

$$A\mathbf{x}_t = B\mathbf{x}_{t+1}, \tag{B.13}$$

$$\begin{bmatrix} 1 + \sigma^{-1}\phi_y & \sigma^{-1}\phi_\pi \\ -\kappa(\gamma + \phi_y) & 1 - \kappa\phi_\pi \end{bmatrix} \mathbf{x}_t = \begin{bmatrix} 1 & \sigma^{-1} \\ 0 & \beta_\kappa \end{bmatrix} \mathbf{x}_{t+1}. \tag{B.14}$$

It is straightforward that

$$B^{-1} = \frac{1}{\beta_\kappa} \begin{bmatrix} \beta_\kappa & -\sigma^{-1} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\beta_\kappa^{-1}\sigma^{-1} \\ 0 & \beta_\kappa^{-1} \end{bmatrix}, \tag{B.15}$$

and hence

$$M = AB^{-1} = \begin{bmatrix} 1 + \sigma^{-1}\phi_y & \sigma^{-1}\phi_\pi \\ -\kappa(\gamma + \phi_y) & 1 - \kappa\phi_\pi \end{bmatrix} \begin{bmatrix} 1 & -\beta_\kappa^{-1}\sigma^{-1} \\ 0 & \beta_\kappa^{-1} \end{bmatrix}, \tag{B.16}$$

$$= \begin{bmatrix} 1 + \sigma^{-1}\phi_y & -\beta_\kappa^{-1}\sigma^{-1}(1 + \sigma^{-1}\phi_y - \phi_\pi) \\ -\kappa(\gamma + \phi_y) & \beta_\kappa^{-1}\sigma^{-1}\kappa(\gamma + \phi_y) + \beta_\kappa^{-1}(1 - \kappa\phi_\pi) \end{bmatrix}, \tag{B.17}$$

$$= \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix}. \tag{B.18}$$

The eigenvalues of the system are given by $|M - \lambda I| = \lambda^2 + p\lambda + q$, where

$$p = -(m_1 + m_4) = -\left(1 + \sigma^{-1}\phi_y + \beta_\kappa^{-1}\sigma^{-1}\kappa(\gamma + \phi_y) + \beta_\kappa^{-1}(1 - \kappa\phi_\pi)\right) \tag{B.19}$$

is the negative of the trace and

$$q = m_1 m_4 - m_2 m_3 = \beta_\kappa^{-1}(1 + \sigma^{-1}\phi_y - \kappa\phi_\pi + \sigma^{-1}\phi_\pi\kappa\gamma) \tag{B.20}$$

is the determinant. As there are no endogenous states, determinacy under the conditions by Blanchard and Kahn (1980) requires the modulus of both eigenvalues of $M$ to be larger than zero. We can find a representation of the absolute value of these eigenvalues in terms of the elements of $M$ as

$$|\lambda_{1,2}^r| = \begin{cases} -p/2 + \sqrt{p^2/4 - q} > 1 \\ -p/2 - \sqrt{p^2/4 - q} > 1 \end{cases} \qquad \text{if } p^2/4 \geq q, \tag{B.21}$$

$$|\lambda_{1,2}^i| = \sqrt{p^2/2 - q} > 1 \qquad \text{if } p^2/4 < q. \tag{B.22}$$

$|\lambda_{1,2}^r|$ are the real eigenvalues if the respective condition for the square root is satisfied, $|\lambda_{1,2}^i|$ are corresponding imaginary eigenvalues otherwise. Using the condition in Equation (B.21) in the second case implies that $-p/2 > 1$, or equivalently

$$p < -2. \tag{B.23}$$

Rearranging the second case in Equation (B.21) also implies

$$1 + p + q > 0. \tag{B.24}$$

Together with Equation (B.23), this implies

$$q > 1. \tag{B.25}$$

Equation (B.25) is also a necessary condition for the case of imaginary eigenvalues. Similarly, one can show that Equation (B.23) and Equation (B.24) imply that Equation (B.22) holds. Therefore, Equations (B.23)-(B.25) are jointly sufficient for both eigenvalues to be larger than one in modulus.

In our model, the three necessary condition $1 + p + q > 0$, $p < -2$ and $q > 1$ thus read

$$\phi_\pi + \frac{1-\beta}{\kappa\gamma}\phi_y > 1, \tag{B.26}$$

$$\sigma^{-1}\phi_y + \beta_\kappa^{-1}\sigma^{-1}(\kappa\gamma + \kappa\phi_y) + \beta_\kappa^{-1}(1 - \kappa\phi_\pi) > 1, \tag{B.27}$$

$$1 + \sigma^{-1}(\kappa\gamma\phi_\pi + \phi_y) - \kappa\phi_\pi > \beta_\kappa. \tag{B.28}$$

As a second step, we can use these determinacy conditions to derive a sign for the denominator of $a_0$. Let us suppose that the denominator is positive, i.e.

$$(1 - \beta\rho)(\sigma(1-\rho) + \phi_y) + \kappa\gamma(\phi_\pi - \rho) - \kappa\sigma(1-\rho)(\phi_\pi - \rho) > 0. \tag{B.29}$$

This can be rearranged to

$$\left(\phi_\pi + \frac{1-\beta}{\kappa\gamma}\phi_y - 1\right) + \frac{1-\rho}{\kappa\gamma}\left(\kappa\gamma + \beta\phi_y + \sigma(1 - \beta\rho - \kappa(\phi_\pi - \rho))\right) > 0. \tag{B.30}$$

The first term in large brackets is positive, which can be seen directly from the necessary condition in Equation (B.26). After some algebraic manipulations, one can show that Equation (B.27) implies that the second term in brackets is also positive. This shows that the denominator of $a_0$ is indeed positive.

With the denominator being positive, the sign of $a_0$ depends on the numerator, including the minus in front of the fraction. The condition for $a_0 < 0$ is thus

$$\kappa\gamma - \kappa\sigma(1-\rho) > 0. \tag{B.31}$$

Using the definition of $\gamma$, this is equivalent to

$$\sigma + \eta - \nu(\psi - 1 - \sigma - \eta) > \sigma(1-\rho). \tag{B.32}$$

Rearranging yields the parameter restriction in terms of the elasticity of the credit spread to entrepreneur leverage.

As a last step, the sign of $b_0$ can be determined given the solution for $a_0$. The denominator of $b_0$ is positive for conventional parameters, such that the sign is determined by the numerator, including the minus. Inserting $a_0$, this is given by

$$-1 + (\phi_\pi - \rho)\frac{\kappa\gamma - \kappa\sigma(1-\rho)}{Z}, \tag{B.33}$$

where $Z$ denotes the denominator of $a_0$. After some algebraic manipulations, this is equivalent to

$$-Z^{-1}\Big((1-\beta\rho)(\sigma(1-\rho)+\phi_y)+(1-\rho)^2\kappa\sigma\Big), \tag{B.34}$$

which is unambiguously negative for $0 \leq \rho \leq 1$. ■

**Lemma 2.** *The impact response of inflation to a financial shock in normal times (without a binding ELB on nominal interest rates) is positive whenever $a_0$ is positive, i.e. if the elasticity of the credit spread to entrepreneur leverage satisfies*

$$\nu > \frac{\eta + \rho\sigma}{\psi - 1 - \sigma - \eta}. \tag{B.35}$$

*Proof.* This is the converse case of Lemma 1. As argued in the corresponding proof, determinacy of the model requires the denominator of $a_0$ to be positive. The condition for $a_0 > 0$ is hence that the numerator (including the minus in front of the fraction) is positive. This is equivalent to

$$\sigma(1-\rho) > \gamma. \tag{B.36}$$

Using the definition of $\gamma$ to obtain

$$\sigma(1-\rho) > \sigma + \eta - \nu(\psi - 1 - \sigma - \eta) \tag{B.37}$$

and rearranging yields the desired result. ■

**Proposition 2.** *Suppose that the ELB on nominal interest rate is expected to bind for $k \geq 1$ periods. Then, the impact responses of inflation and output to a financial shock are given by:*

$$\pi_t = a_k\, u_t, \tag{B.38}$$

$$y_t = b_k\, u_t, \tag{B.39}$$

*where*

$$a_k = \kappa\left(1-\gamma\sigma^{-1}\right)\left(1+\frac{\bar{r}}{u_t}\right)+\rho\left(\beta-\kappa+\kappa\gamma\sigma^{-1}\right)a_{k-1}+\rho\kappa\gamma\, b_{k-1}, \tag{B.40}$$

$$b_k = -\sigma^{-1}\left(1+\frac{\bar{r}}{u_t}\right)+\rho\sigma^{-1}\,a_{k-1}+\rho\, b_{k-1}. \tag{B.41}$$

*Proof.* Similar to Proposition 1, the proof relies on the method of undetermined coefficients. Suppose that the ELB on nominal interest is expected to bind for $k \geq 1$ periods. Denoting the corresponding policy functions for by $a_k$ and $b_k$, respectively, we can rewrite the system of equations as

$$a_k\, u_t = \kappa\gamma\, b_k u_t + \kappa(\bar{r}+u_t)+(\beta-\kappa)\rho\, a_{k-1}u_t, \tag{B.42}$$

$$b_k\, u_t = -\sigma^{-1}(\bar{r}+u_t)+\rho\sigma^{-1}a_{k-1}u_t+\rho\, b_{k-1}u_t, \tag{B.43}$$

where the central bank interest rate is replaced by the ELB value. Note that expectations of future variables can be replaced by the corresponding policy functions for the case of an expected ELB duration of $k-1$ under rational expectations, using the law of motion for the financial shocks. The solution is obtained by dividing both equations by $u_t$, substituting for $b_k$ in the first equation using the second equation and rearranging. ■

**Proposition 3.** *The policy parameters in the central bank's monetary policy rule must satisfy the following conditions to guarantee a determinate solution:*

$$\phi_\pi + \frac{1-\beta}{\kappa\gamma}\,\phi_y > 1, \tag{B.44}$$

$$\kappa(\sigma^{-1}\gamma-1)\,\phi_\pi+\sigma^{-1}\phi_y > \beta-1-\kappa \tag{B.45}$$

*Proof.* The first equation follows directly from the condition $1+p+q>0$, which is required to satisfy the Blanchard and Kahn (1980) conditions. This is Equation (B.26) in the proof for Proposition 1. The second equation can be obtained by rearranging the condition $q>1$, which is Equation (B.28) above. ■

## Appendix C Numerical Solution Method

For the sake of clarity, we use a different representation of the policy functions to outline the solution procedure. The analytic solutions in Section 3 are expressed in terms recursive policy functions of $u_t$. A different, non-recursive way of presenting these policy functions is suggested in Boehl (2021). The simplicity of our model allows to ease the notation therein and express our model with $\mathbf{x}_t = (\pi_t, y_t)'$ in matrix form as

$$\mathbf{x}_t + \mathbf{c} \max \{\mathbf{d}\mathbf{x}_t, \bar{r}\} = \mathbf{N} E_t \mathbf{x}_{t+1} + \mathbf{c} u_t, \tag{C.1}$$

where $\mathbf{N}$ is the system matrix of the constrained system, $\mathbf{c}$ contains the coefficients that determine how $\mathbf{x}_t$ is affected by $r_t$ (and thereby also by $u_t$) and $\mathbf{d}$ contains the parameters of the monetary policy rule. $\bar{r} < 0$ is the actual model-implied lower bound of $r_t$.

Assume again that the economy is at the ELB for $k$ periods. Then

$$\mathbf{x}_t + \mathbf{c}\bar{r} = \mathbf{N} E_t \mathbf{x}_{t+1} + \mathbf{c} u_t, \tag{C.2}$$

$$E_t \mathbf{x}_{t+1} + \mathbf{c}\bar{r} = \mathbf{N} E_t \mathbf{x}_{t+2} + \mathbf{c} u_{t+1}, \tag{C.3}$$

$$\dots$$

$$E_t \mathbf{x}_{t+k-1} + \mathbf{c}\bar{r} = \mathbf{N} E_t \mathbf{x}_{t+k} + \mathbf{c} u_{t+k-1}, \tag{C.4}$$

$$E_t \mathbf{x}_{t+k} = \mathbf{A}(0) u_{t+k}. \tag{C.5}$$

Recursively inserting (C.5) into (C.4) yields, acknowledging that $E_t u_{t+s} = \rho^s u_t$,

$$\mathbf{x}_t = \mathbf{N}^k \mathbf{A}(0) \rho^k u_t + \sum_{i=0}^{k-1} \mathbf{N}^i \mathbf{c} \rho^i u_t - \sum_{i=0}^{k-1} \mathbf{N}^i \mathbf{b} \bar{r}, \tag{C.6}$$

$$= \mathbf{A}(k) u_t + \mathbf{a}(k) \bar{r}. \tag{C.7}$$

Rewriting (C.6) yields

$$\pi_t = A_\pi(k) u_t + a_\pi(k) \bar{r}, \tag{C.8}$$

$$y_t = A_y(k) u_t + a_y(k) \bar{r}. \tag{C.9}$$

In verbal terms, this implies that depending on the expected number of periods at the ELB $k$, we can express the vector of controls $\mathbf{x}_t$ as a linear map $A_j(k)$ of $u_t$ and the (constant) vector $a_j(k)$. Both terms are nonlinear functions of $k$ defined on $\mathbb{N}_0$. In other words: given $k$, the policy function is simply a two dimensional linear projection of the scalar $u_t$.

Definition 1 recapitulates the conditions for $k$ to be an equilibrium value under the assumption that each shock causes the ELB to hold instantly without any transition period.

**Definition 1** (equilibrium $k$)**.** *For each period $t$, an equilibrium value of $k \in \mathbb{N}_0$ must satisfy that the ELB binds in expectations exactly until period $t+k$. Hence,*

$$\mathbf{d}\mathbf{x}_t > \bar{r} \implies k = 0, \tag{C.10}$$

*while for $k > 0$ it must hold that*

$$\mathbf{d}E_t\mathbf{x}_{t+k} > \bar{r}, \tag{C.11}$$

*and*

$$\mathbf{d}E_t\mathbf{x}_{t+k-1} \leq \bar{r}. \tag{C.12}$$

The parsimonious nature of our model allows that, for each $u_t$, a $k$ can simply be found by iterating over $k \in \mathbb{N}_0$ (where, naturally, $k$ is likely to be small). More sophisticated iteration schemes for a general formulation of the dynamic system can be found in Boehl (2021).

To provide some quantitative impression given our model, for $\nu = 0.2$, a 1% risk premium shock will cause the ELB to initially bind for $k = 2$ periods, a 2% shock will cause $k = 9$ and a 3% shock an endogenous duration of $k = 12$ periods.

In Figure C.1 we show the reduced-form slope of the Phillips Curve, based only on the dynamic effect in response to the risk premium shock. The figure confirms that the slope is considerably high if away from the ELB, but drops once the ELB is reached and remains consistently low as the number of expected durations at the ELB increases.

Figure C.2 plots the non-recursive policy functions for $\pi_t$. For a more moderate value of $\nu$ of 0.2, the mapping $A_\pi(k)$ from $u_t \to \pi_t$ decreases with $k$ while the linear part $a_\pi(k)$ increases in about the same fashion. As larger shocks are necessary to cause a higher $k$, the dynamic effect of the shock dominates the static effect and inflation falls. For $\nu = 0.22$, $A_\pi(k)$ becomes more convex, meaning that the coefficient that translates financial shocks to inflation increases for low expected durations. This effect is not offset by the static effect of a longer anticipated ELB period, which leads to a more muted inflation response. For a value of $\nu = 0.24$, the dynamic response approaches zero while for $\nu = 0.25$, $A_\pi(k)$ turns positve for values of $k$ larger than two. As the static effect is again too weak to counteract, this leads to an increase of inflation on impact, as it is captured in Figure 3.

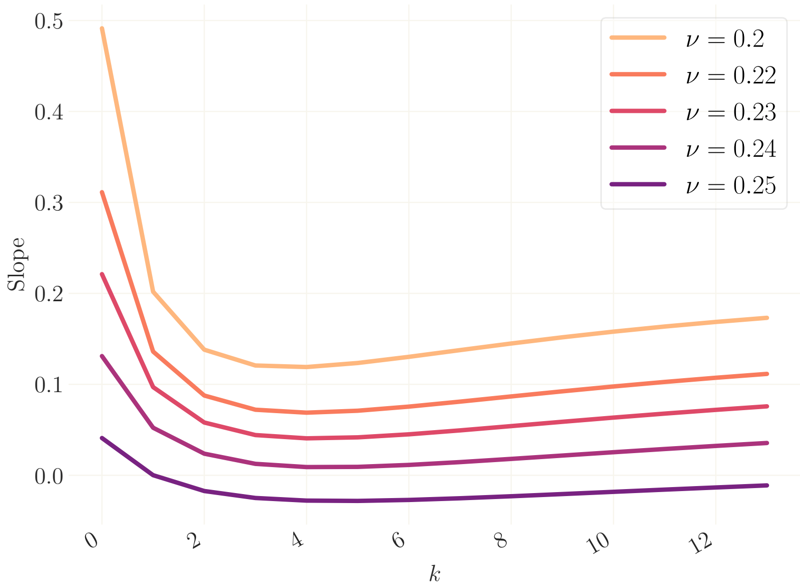


Figure C.1: Theoretical Phillips Curve slope $A_\pi(k)/A_y(k)$. This exercise ignores the static effect of the ELB, that is captured by $a_\pi(k)$ and $a_y(k)$.

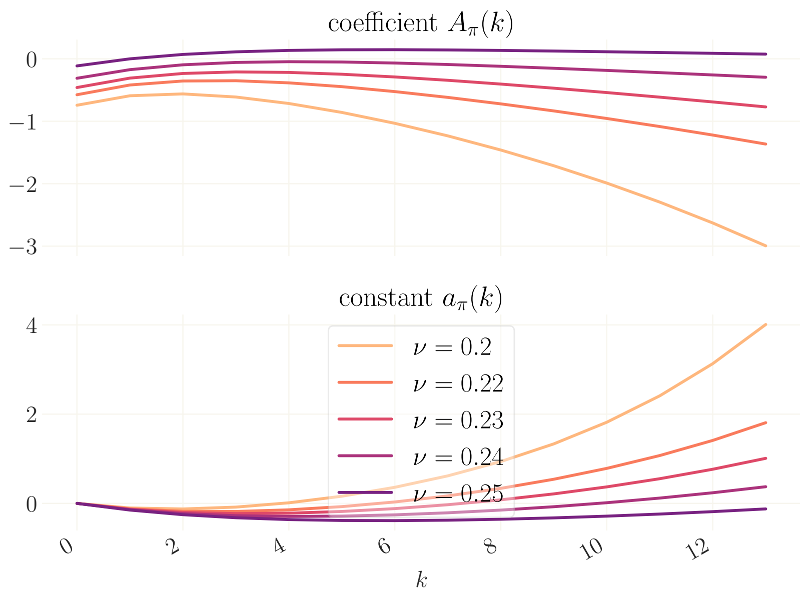


Figure C.2: Expected ELB Duration and Impact Response

# Appendix D Additional Figures

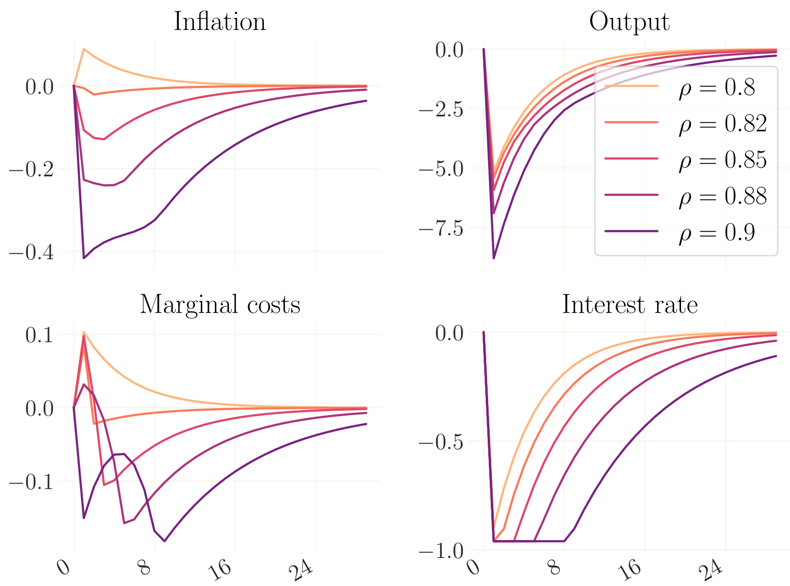


Figure D.3: Impulse responses to 2% risk premium shocks for different values of $\rho$, given $\nu = 0.24$.