The Hockey Stick Phillips Curve and the Zero Lower Bound

Preliminary results, comments are welcome. Latest version:

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Gregor Boehl[a,*], Philipp Lieberknecht[b]

[a] *University of Bonn*
[b] *Deutsche Bundesbank*

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Abstract

We argue that the recent observed disconnect between inflation and economic activity can be explained by the interplay between the zero lower bound (ZLB) and the costs of external financing. In normal times, variations in the credit spreads and the safe interest rate balance out and costs for employing production factors dictate firms' marginal costs and their price setting. However, at the ZLB the safe interest rate is constrained and higher spreads can more than offset the effect of lower production factor costs. As a consequence, financial shocks at the ZLB induce only moderate inflation responses. The resulting Phillips curve features a hockey stick shape: it exhibits the usual positive slope away from the ZLB but flattens considerably for negative output gaps at the ZLB. Short-lived forward guidance shocks may induce weak inflationary or even deflationary effects via the same mechanism, thereby attenuating the forward guidance puzzle.

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1 Introduction

"What is the relationship between inflation and economic activity?" Given the fundamental role of these two macroeconomic concepts, it seems almost absurd that this question has recently puzzled the profession. After the Global Financial Crisis of 2007/2008 and the associated financial turmoil, inflation seemed disconnected from economic activity, leading to puzzles of "missing disinflation" and "missing inflation" (Ball and

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*Corresponding author
Email addresses: gboehl@uni-bonn.de, philipp.lieberknecht@bundesbank.de

Mazumder, 2011; Coibion and Gorodnichenko, 2015; Lindé and Trabandt, 2019). These observations sparked considerable interest in analyzing the determinants of the resulting seemingly flat Phillips curve. While the explanations put forward are numerous and manifold, we found one key contributing factor yet to be missing: the zero lower bound (ZLB) on nominal interest rates, which was reached by several central banks around the globe in recent years, coincidental with the observed inflation puzzles.[1]

In this paper, we show how the interplay of the ZLB and financial frictions re-shapes the relationship between inflation and economic activity. From recent research, it is understood that financial distortions can be crucial for firms' price setting behavior and, thereby, for inflation dynamics (Gilchrist et al., 2017; Lieberknecht, 2019). We argue that during normal times, firms' marginal cost are dominated by the – procyclical – costs of employing production factors, which hence determines their price setting. In the presence of financial frictions, marginal costs further contain the costs of external financing. These consist of the real safe interest rate and, caused by financial frictions, a countercyclical credit spread, where the two balance in normal times. However, if the nominal rate is constrained by ZLB, higher credit spreads can more than offset the effect of lower production factor costs. As a result, financial shocks at the ZLB induce only moderate deflation responses, and can in extreme cases even be inflationary. Taking the ZLB into account, the resulting observational Phillips curve[2] is thus shaped like a hockey stick: It features the usual positive slope in normal times, while being flat at the ZLB (for considerably negative output gaps).

We show these results using a small-scale New Keynesian DSGE model featuring an explicit role for external financing. In the model, financial frictions result from the combination of a working capital channel in the spirit of Ravenna and Walsh (2006) and a costly state verification problem à la Townsend (1979) and Bernanke et al. (1999). Workers need to be paid before production, generating external financing needs for the entrepreneurs operating the firms. The costs of external finance consist of the real interest rate plus a risk premium, which depends positively on entrepreneurs' (countercyclical) leverage. We focus on the effects of financial shocks in the form of risk premium shocks in the spirit of Smets and Wouters (2007). These shocks have a high explanatory power for the joint movement of consumption, investment and inflation in the 2007/2008 recession and its aftermath (Gust et al., 2017; Kulish et al., 2017; Boehl and Strobel, 2020; Boehl et al., 2020). As an exogenous increase of the risk premium, they generate a wedge between the central bank interest rate and the household's return on assets, thereby affecting both Euler equation and the firms' credit spread.

As our first contribution, we derive closed-form solutions for the macroeconomic dynamics following financial shocks, for normal times and the case of a binding ZLB. In particular, we show that a longer expected ZLB duration can be associated with a weaker deflationary response, or even an increase in inflation. We derive the necessary conditions under which this scenario is possible and argue that they are fairly regular: We

[1]Throughout this paper, we refer to the concept of a lower bound on nominal interest rates at zero. Our results equally hold when allowing for an effective lower bound (ELB) above zero, or in negative territory.

[2]This refers to the realized values for inflation and output gap, i.e. the *observed* or *empirical* Phillips curve. As discussed below, this is not equivalent to the New Keynesian Phillips curve describing firms' price setting behavior.

show that this case may occur for large financial shocks whenever the elasticity of the credit spread with respect to entrepreneur leverage is sufficiently high. The closed-form solutions furthermore highlight that *Neo-Fisherian* effects – in the form of an overall increase of inflation following contractionary financial shocks – are possible. They are in particular likely if the ZLB is expected to bind for an extended period of time.

As a second contribution, we numerically investigate the role of the elasticity of the credit spread to entrepreneur leverage, the financial shock size and its persistence within our model framework. The simulated observational Phillips curve, i.e. the realized values of inflation and output gap that would be observed in general equilibrium, features a striking hockey stick shape: For normal times with positive or mildly negative output gaps, it exhibits a conventional positive slope in output gap - inflation space, whereas the slope is considerably flat for significantly negative output gaps when the ZLB is binding.

Lastly, our third contribution is to discuss the implications for monetary policy. We argue that the hockey stick Phillips curve does not constitute a threat to policymakers per se: At the ZLB, (further) contractionary financial shocks do not lead to an additional substantial decline of inflation. However, designing appropriate monetary policy at the ZLB in times of financial tensions is challenging. In particular, monetary policy shocks and financial shocks generate very similar macroeconomic dynamics in our framework. As a consequence, half-hearted forward guidance shocks with relative low persistence can even be deflationary: their short-term effect of further decreasing (future expected) refinancing costs may dominate their long-term effect of increases in the price level from stimulating consumption. We hence also provide an explanation for the forward guidance puzzle (Carlstrom et al., 2015; Del Negro et al., 2015a; Kiley, 2016) and suggest that any forward guidance measures must be undertaken with vigor.

This paper is related to the literature on the Phillips curve and missing (dis-)inflation in recent years. As first argued by Ball and Mazumder (2011) and subsequently confirmed for many advanced economies by Friedrich (2016), inflation did not fall as much as expected given the depth of the recession in the aftermath of the Global Financial Crisis. In subsequent years, however, inflation was lower than expected given the economic recovery. The explanations put forward are manifold and encompass anchored household or firm expectations (Ball and Mazumder, 2018; Coibion and Gorodnichenko, 2015), various measures of economic slack (Gordon, 2013; Watson, 2014; Krueger et al., 2014; Faccini and Melosi, 2019), supply shocks and wage rigidities (Daly and Hobijn, 2014; Lindé and Trabandt, 2019), optimal monetary policy (McLeay and Tenreyro, 2020) or global factors (Bobeica and Jarociński, 2019; Forbes, 2019). Compared to this literature, our paper provides a complementary explanation for inflation dynamics: The ZLB affects the cyclicality of marginal costs via the costs of external financing, thereby leading to a disconnect between economic activity and inflation.

Our paper is also related to a second strand of the literature investigating most recent inflation dynamics through the lens of New Keynesian DSGE models, notably Christiano et al. (2015), Del Negro et al. (2015b) and Gilchrist et al. (2017). In line with our paper, these contributions show that adding financial frictions to these models helps to explain the missing disinflation puzzle in the US in the aftermath of the Global Financial Crisis. Closely related to our work, Gilchrist et al. (2017) explain inflation dynamics via financial distortions, i.e. larger credit spreads in recessions. While our paper shares this argument, we provide additional insights that a binding ZLB amplifies the effects of financial frictions such that credit spreads may even dominate inflation dynamics. This is in line with insights by Bianchi and Melosi (2017) and Boehl and Strobel (2020) wo find that accounting for the ZLB substantially improves the empirical fit of estimated DSGE models. Our analytical results also complement the numerical analyses conducted in these papers.

Lastly, our paper is related to the literature on Neo-Fisherianism in the sense that the short-run causality between variations in the policy rate and the response of inflation may be ambiguous or state-dependent. As such, Gabaix (2016) deems Neo-Fisherianism uncontroversial in the long run, a finding backed by García-Schmidt and Woodford (2019). Modern proponents challenging this view ("When is the long run?") are Cochrane (2011, 2016, 2017) and García-Schmidt and Woodford (2019). The latter find that in a perfect-foresight world, credible (long-run) changes in long-run targets can have immediate effects. In contrast, our paper shares the view of Gerke and Hauzenberger (2017) that the above effect rather is an artifact of equilibrium selection instead of a "classic" macroeconomic effect. Although it is clear that a New-Keynesian model with the ZLB produces a multiplicity of equilibrium paths (Benhabib et al., 2001), it is unclear how many of these paths can be stable equilibria. We contribute to this literature by showing that Neo-Fisherian effects exist even when expectations are anchored to the long-term interest rate, i.e. when assuming the "classic" terminal conditions.

The rest of the paper is structured as follows. Section 2 outlines the New Keynesian DSGE model with financial frictions and discusses the components of marginal costs in this framework. In Section 3, we derive closed-form solutions for macroeconomic dynamics following financial shocks. Section 4 complements by showing numerical solutions and analyzing the resulting observational Phillips curve. In Section 5, we investigate the implications for monetary policy at the ZLB. Section 6 concludes.

2 Model

Our analysis is based on a small-scale New Keynesian DSGE model featuring an explicit role for external financing via financial frictions. The model setup is based on our earlier work in Boehl (2017) and Lieberknecht (2019). For details on the model, we refer to these references. We assume that production is subject to a working capital channel (or cost channel) as in Ravenna and Walsh (2006). A distinct role for equity finance is motivated via a costly state verification problem in the spirit of Townsend (1979) and Bernanke et al. (1999). To maintain analytical tractability, firms are divided into a wholesale and retail sector. Entrepreneurs operating wholesale firms borrow money from the financial intermediary to finance production, and their shares are traded at the financial markets exchange. Their (homogeneous) good is sold to the monopolistic retail sector where diversification takes place and the resulting diversified goods are sold to a representative household, who consumes and supplies labor in a perfectly competitive labor market. A monetary authority sets the nominal interest rate, which is subject to a lower bound.

2.1 Households

Households maximize the expected present value of life-time utility by deciding over consumption of a composite good C_t and time devoted to the labour market H_t. For each supplied unit of labour they receive the real wage W_t. Households can deposit monetary savings D_t at the financial intermediary, for which they receive the gross nominal interest rate R_t in the next period. The final consumption good is composed of differentiated retail products and is sold in a market with monopolistic competition. The composite good and its respective aggregate price index are given by standard CES aggregators.

The households problem is completely standard and optimization yields the usual inter-temporal Euler equation and an intra-temporal labor supply equation

$$C_t^{-\sigma} = \beta E_t \left[\frac{R_t}{\Pi_{t+1}} U_t C_{t+1}^{-\sigma} \right], \tag{1}$$

$$H_t^{\eta} = W_t C_t^{-\sigma}, \tag{2}$$

where Π_t is gross inflation. In the spirit of Smets and Wouters (2007), we understand U_t as a premium on the risk-free interest rate that reflects the state of the financial system. This type of shock has the highest explanatory power regarding the post-2000 macroeconomic dynamics across all standard shocks, and can explain a large share of the joint dynamics of consumption, investment and inflation following the 2007/2008 financial crisis (Gust et al., 2017; Kulish et al., 2017; Boehl and Strobel, 2020; Boehl et al., 2020). We label U_t as the *financial shock* in the following. The parameters σ, η and β are the inverse elasticity of intertemporal substitution, the inverse Frisch elasticity of labor supply and the discount rate, respectively.

2.2 Wholesale and Retail Firms

The wholesale sector consists of a continuum of firms indexed by j. Each firm is operated by a risk-neutral entrepreneur. Labor is the only production factor, and the CRS production function for the homogeneous good is given by

$$Y_{j,t} = \omega_{j,t} H_{j,t}, \tag{3}$$

where $Y_{j,t}$ is output produced by firm j and $\omega_{j,t}$ is a firm-specific idiosyncratic productivity shock. Similar to Ravenna and Walsh (2006), it is assumed that workers have to be paid before production takes place, while profits are realized at the end of the period. This working capital channel (also labeled the *cost channel*) motivates a positive role for external finance.

In the following, we focus on the economic intuition behind this idea and refer the interested reader to Boehl (2017) for formal details. The amount of external finance $L_{j,t}$ demanded by firm j is given by j's desired working capital $W_t H_{j,t}$ minus its equity $N_{j,t}$:

$$L_{j,t} = W_t H_{j,t} - N_{j,t}. \tag{4}$$

As in Bernanke et al. (1999), we follow a costly state verification (CSV) approach along the lines of Townsend (1979). The idea behind the CSV approach is that the realization of the idiosyncratic productivity shock is private information of the entrepreneur. As a consequence, banks can only observe produced output when paying monitoring costs. The contract that solves this CSV problem specifies that the interest rate on a loan obtained by an entrepreneur from the intermediary $R_{j,t}^L$ contains an endogenous risk premium on the prevailing real interest rate.[3] This is because banks anticipate the

[3] In line with Bernanke et al. (1999), this solution assumes that all bargaining power accrues to the

possibility that the monitoring fee has to be payed. The risk premium or credit spread depends on the individual firm's leverage,

$$R_{j,t}^{L} = z\left(\frac{W_t H_{j,t}}{N_{j,t}}\right)\frac{R_t}{E_t[\Pi_{t+1}]}U_t, \tag{5}$$

with $z'(\cdot) > 0$. Intuitively, when the leverage ratio falls, the premium on external finance falls because more collateral is provided such that the loan becomes less risky. Banks only monitor firms if the entrepreneur defaults and seize the remaining output as collateral. It can be shown that all entrepreneurs take identical choices in equilibrium, such that Equation (5) also holds in the aggregate:

$$R_t^{L} = z\left(\frac{W_t H_t}{N_t}\right)\frac{R_t}{E_t[\Pi_{t+1}]}U_t, \tag{6}$$

Since the wholesale sector is assumed to be perfectly competitive, wholesale firms are price takers. Denote by X_t the gross markup that retailers charge over wholesale goods. Equivalently, X_t^{-1} is the relative price of one unit of wholesale goods, which needs to equal marginal costs MC_t. In aggregate, no-arbitrage requires the rate of return on working capital to equal the rate on external funding. It follows that marginal costs are given by

$$MC_t = X_t^{-1} = W_t R_t^L = W_t\, z\left(\frac{W_t H_t}{N_t}\right)\frac{R_t}{E_t[\Pi_{t+1}]}U_t. \tag{7}$$

We follow the simplified version of Lieberknecht (2019) with respect to equity financing. We assume that entrepreneurs can issue equity in the stock market, which is bought by risk-neutral financial traders associated with the financial intermediaries. Imposing no arbitrage on financial markets and noting that in equilibrium, entrepreneurs must be indifferent between external finance and equity finance, the return on assets satisfies

$$E_t[R_{t+1}^A] = R_t^L \tag{8}$$

Invoking rule-of-thumb behavior from financial traders, it can be shown that the evolution of equity is given by

$$N_t = \Psi\left(Y_t\right), \tag{9}$$

with $\Psi'(\cdot) > 0$, such that equity financing is procyclical with respect to output, which captures the key notion of standard financial accelerator models à la Bernanke et al. (1999).

After wholesale goods have been produced, retailers buy the homogeneous good $Y_{j,t}$ on the wholesale market. After differentiation, they sell it in the monopolistically competitive consumer market. As commonly found in the literature, firms' price setting decisions are subject to nominal rigidities à la Calvo (1983): Each retailer is able to change its price in any given period with probability $1-\theta$. The price setting problem of retailers is completely standard and gives rise to a classic New Keynesian Phillips Curve, expressed in terms of the markup X_t or marginal costs MC_t.[4]

entrepreneur.

[4]See Bernanke et al. (1999) for details on this particular solution.

2.3 *The Central Bank*

The central bank follows a standard contemporaneous monetary policy rule. The monetary policy rule reads

$$\frac{R_t^n}{R^n} = \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} exp(v_t), \tag{10}$$

where R_t^n is the notional gross nominal interest rate. It is generally equivalent to the gross nominal interest rate on deposits R_t, but the latter is subject to a ZLB constraint and cannot fall below $\bar{R}$:

$$R_t = \max\left\{\bar{R}, R_t^n\right\}, \tag{11}$$

The monetary policy rule is deliberately kept simple in order to guarantee that the model can be solved in closed form. v_t is a monetary policy shock evolving as

$$v_t = \rho_r v_{t-1} + \epsilon_{r,t}. \tag{12}$$

Note that when the ZLB constraint in Equation (11) binds, v_t can be understood as a forward guidance shock as it prolongs the expected duration of the ZLB.

2.4 *Understanding the Components of Marginal Costs*

For the sake of being able to present our results in closed form, we linearize the equations characterizing the equilibrium around an efficient steady state.[5] The full set of equilibrium equations is shown in the Appendix. In the following, we let small-case letters denote variables in log-deviations from the steady state.

In the financial accelerator framework, financial frictions originate in the firm sector and therefore primarily affect the supply side of the economy. In contrast, households behave as in the standard framework. This serves to isolate the effects of financial frictions on marginal costs and inflation dynamics. The role of financial frictions for marginal costs and inflation dynamics is thus best understood by considering the linearized New Keynesian Phillips curve. In our framework, the latter can be cast in the familiar textbook form

$$\pi_t = \kappa\, mc_t + \beta E_t[\pi_{t+1}] \tag{13}$$

with $\kappa = \frac{(1-\theta\beta)(1-\theta)}{\theta}$.

Hence, financial frictions do not alter the price setting behavior of firms per se, as prices are tied to marginal costs and expectations of future inflation. However, financial frictions determine and affect the components of *marginal costs.* Linearizing Equation (7), marginal costs can be written as

$$mc_t = w_t + (r_t - E_t[\pi_{t+1}]) + s_t, \tag{14}$$

[5] Steady state subsidies from the government (financed by lump-sum taxes) can correct for the inefficiencies arising from monopolistic competition and the presence of financial frictions. See Lieberknecht (2019) for details.

where s_t denotes the linearized credit spread $z(\cdot)$. This highlights that marginal costs consist of three components: First, the costs of hiring production factors, i.e. labor, represented by the real wage. We call this component the *real* marginal costs in the following. The second and third term are the costs of external finance, which consist of the risk-free real interest rate and an endogenously determined external finance premium. The former represents a pure (financing) cost channel in the spirit of Ravenna and Walsh (2006), whereas the latter constitutes the credit spread arising from informational asymmetries between borrowers and lenders. These costs of external finance are absent from the standard NK model, where marginal costs consisting solely of real wages.[6]

We may obtain a slightly different representation of marginal costs by replacing real wages and the credit spread. To this end, denote the elasticity of the credit spread with respect to entrepreneur leverage as $z'(\cdot) = \nu$ and the elasticity of equity with respect to output as $\Psi'(\cdot) = \psi$. Then, marginal costs can be written as

$$mc_t = \gamma\, y_t + (r_t - E_t[\pi_{t+1}]) + u_t, \tag{15}$$

with

$$\gamma \equiv \sigma + \eta - \nu(\psi - 1 - \sigma - \eta) \tag{16}$$

denoting the elasticity of marginal costs with respect to output. This representation of marginal costs in Equation (15) highlights that financial frictions alter the behavior of marginal costs along the business cycle via those three components. The real marginal cost component (the term $\sigma + \eta$ in γ) is procyclical. As output increases, expanding production requires firms to offer a higher real wage in order to incentivise higher labor supply from workers. The cyclicality of the pure cost channel (the real interest rate in Equation (15)) is ambiguous and depends on the source of aggregate fluctuations, as this determines the endogenous nominal interest rate setting by the central bank. The financial shocks u_t can be categorized as a particular form of demand shocks, as they trigger a co-movement of output and inflation. As the central bank's response to inflation is dominant over the response to output for most conventional monetary policy rules, the cost channel is thus procyclical following financial shocks shocks.[7]

Leverage lev_t is (in linearized form) given by

$$lev_t = w_t + h_t - n_t, \tag{17}$$

where we assume $n_t = \psi y_t$ as above. In a financial accelerator economy, the credit spread (also known as the external finance premium) is countercyclical (Bernanke et al., 1999). Entrepreneur leverage is only countercyclical if the procyclicality of net worth outweighs the procyclicality of the loan value. Given Equation (17), this means that entrepreneur net worth must be more procyclical than the wage bill. Using the household's intratemporal optimality condition and the net worth evolution, we can rewrite

[6] We abstract from aggregate productivity shocks. In the standard NK model (and in our framework), such shocks alter real marginal costs by affecting effective labor units.

[7] For technology shocks and other pure supply-side shocks that raise output while being deflationary, the cost channel is countercyclical.

the above equation as

$$lev_t = -(\psi - 1 - \sigma - \eta)y_t. \tag{18}$$

For any form of "demand-side" disturbances like our financial shock or a monetary policy shocks, the key necessary and sufficient condition for leverage to be countercyclical is thus that the term in brackets is larger than zero. This implies the following parameter restriction:

Assumption 1. *The elasticity of net worth with respect to output satisfies*

$$\psi > 1 + \sigma + \eta. \tag{19}$$

This condition assures that the credit spread is a positive function of entrepreneur leverage (Lieberknecht, 2019). In other words, the elasticity of the external finance premium with respect to leverage ν is positive (and a function of steady state contract terms and entrepreneur balance sheet). This implies that under Assumption 1, the resulting countercyclical entrepreneur leverage leads to a countercyclical external finance premium.

The three components of marginal costs are thus characterized by opposing cyclicality over the business cycle. For financial shocks, real marginal costs and the pure financing costs are procyclical, whereas the external finance premium is countercyclical. Since firms' price setting is tightly connected to their marginal costs, this has important implications for inflation dynamics. In particular, the presence of financial frictions changes the relationship between inflation and output over the business cycle. The extent to which this occurs depends on the relative strength of the various components.

3 Financial Shocks at the Zero Lower Bound

In this section, we analyze how a binding ZLB on nominal interest rates affects the transmission of risk premium shocks in the economy. To this end, we derive closed-form general equilibrium solutions in normal times and at the ZLB. Contrasting the two cases highlights that macroeconomic dynamics at the ZLB may be fundamentally different compared to normal times.

3.1 Financial Shocks in Normal Times

We first consider the macroeconomic effects of financial shocks in normal times, i.e. in the absence of a binding ZLB on nominal interest rates. To this end, we exploit the analytic tractability of the framework, which allows to characterize the equilibrium in closed form. The whole model can be represented in three equations[8] as

$$\pi_t = \kappa\gamma\, y_t + (\beta - \kappa)E_t[\pi_{t+1}] + \kappa(r_t + u_t), \tag{20}$$

$$y_t = -\sigma^{-1}\left(r_t - E_t[\pi_{t+1}] + u_t\right) + E_t[y_{t+1}], \tag{21}$$

$$r_t = \max\left\{\phi_\pi \pi_t + \phi_y y_t + v_t, \bar{r}\right\}, \tag{22}$$

[8]See the Appendix for more details on this particular representation.

in addition to the exogenous processes for the financial shock u_t and the monetary policy shock v_t,

$$u_t = \rho\, u_{t-1} + \epsilon, \tag{23}$$

$$v_t = \rho_r\, v_{t-1} + \epsilon_{r,t}. \tag{24}$$

Equation (20) is the New Keynesian Phillips curve, relating inflation to output[9], nominal interest rates and expected inflation. This representation of the Phillips curve can be obtained by combining the definition of marginal costs together with the intratemporal household condition and the evolution of entrepreneur net worth. The slope with respect to output is given by

$$\kappa\gamma = \kappa(\sigma + \eta - \nu(\psi - 1 - \sigma - \eta)). \tag{25}$$

The first term $\kappa(\sigma + \eta)$ in Equation (25) captures the real marginal cost part, whereas the second part represents the endogenous evolution of the external finance premium. Based on Assumption 1, this second term is negative such that the countercyclical credit spread reduces the overall slope of the Phillips curve with respect to output, as compared to the standard NK model. The third and fourth term in the Phillips curve capture the financing cost channel and the purely exogenous markup effect that arises from financial shocks by increasing the credit spread.

Equation (21) is the linearized Euler equation, governing the intertemporal consumption allocation of households as a function of the real interest rate. Equation (22) is the Taylor rule defining how the central bank sets the (notional) interest rate as a reaction to inflation and output. We incorporate the ZLB constraint, specifying that the nominal interest rate can not be lower than $\bar{r}$. The latter two equations are, apart from the max-operator, completely standard and identical to the textbook NK model. Financial frictions thus manifest solely in the New Keynesian Phillips curve, highlighting again that the financial accelerator is a supply-side friction that directly affects inflation dynamics.

In a first step, we assume that the ZLB constraint is not binding. To solve the model, we use the method of undetermined coefficients and postulate that the equilibrium response of endogenous variables is a linear function of the exogenous financial shock. We summarize our results in the following Proposition:

Proposition 1. *The impact responses of inflation and output to a financial shock in normal times (without a binding ZLB on nominal interest rates) are given by:*

$$\pi_t = a_0\, u_t, \tag{26}$$

$$y_t = b_0\, u_t, \tag{27}$$

[9]Note that for financial shocks, the responses of output and the output gap are identical. This is because output in an efficient economy without nominal rigidities and financial frictions does not respond to financial shocks. We generally refer to output in the following, but the identity of the two concepts is of central importance for monetary policy.

where

$$a_0 = -\frac{\kappa\gamma - \kappa\sigma(1-\rho)}{(1-\beta\rho)(\sigma(1-\rho)+\phi_y) + \kappa\gamma(\phi_\pi - \rho) - \kappa\sigma(1-\rho)(\phi_\pi - 1)}, \tag{28}$$

$$b_0 = -\frac{1 + (\phi_\pi - \rho)a_0}{\sigma(1-\rho)+\phi_y}. \tag{29}$$

Proof. See Appendix. ■

In combination with Proposition 1, the following Lemma 1 shows – in analytic form – the conventional macroeconomic effects of financial shocks: Financial shocks are a particular form of demand shocks. A positive financial shock increases the wedge between the interest rate controlled by the central bank and the return on bonds held by households. This wedge reduces current consumption. Thus, a positive financial shock decreases overall output. Via the New Keynesian Phillips curve, inflation decreases as well.

Lemma 1. *The impact responses of inflation and output to a financial shock in normal times (without a binding ZLB on nominal interest rates) are negative, i.e.*

$$a_0 < 0, \tag{30}$$

$$b_0 < 0, \tag{31}$$

iff the elasticity of the credit spread to entrepreneur leverage satisfies

$$\nu < \frac{\eta + \rho\sigma}{\psi - 1 - \sigma - \eta}. \tag{32}$$

Proof. See Appendix. ■

The analytic solutions display precisely the different channels through which the financial shock operates and allows for a corresponding decomposition. In a_0, the first term in the numerator is the slope of the Phillips curve with respect to output, whereas the second term captures the exogenous markup effect of the financial shock. Following a positive financial shock, real marginal costs decrease, because less workers are needed for production given the decline in demand (the first part of $\kappa\gamma$). This reduces inflation. At the same time, the financial shock increases the costs of production via the external finance premium, as financial frictions in the firm sector intensify (the second part of $\kappa\gamma$ and the markup effect). This increase in the credit spread partially counteracts the decline in real marginal costs, weakening the overall disinflationary effect.

The cost channel is represented by the last term in the denominator in a_0. This term features a negative sign and is thus – ceteris paribus – disinflationary. Generally, if the central bank reacts stronger (weaker) to fluctuations in inflation and output, the denominator is larger (smaller), such that the overall response of inflation is smaller (larger). However, the lower nominal interest rates set by the central bank as a reaction to the overall decrease in inflation also lowers marginal costs directly, amplifying the disinflationary response. Thus, the cost channel weakens the overall stabilizing property of the central bank's interest rate policy.

Following financial shocks, the various components of marginal costs thus move in different directions. Whereas real marginal costs and the pure financial cost channel amplify the disinflationary response, the credit spread channel weakens it. As seen in Lemma 1, the overall inflation response in normal times is negative, as long as the elasticity of the credit spread to entrepreneur leverage is not excessively high. As shown in the Appendix, determinacy of the model requires the denominator in a_0 to be positive. Intuitively, the model is determinate if a stronger central bank reaction to deviations from steady state translates into lower deviations in general equilibrium. Lemma 1 is thus equivalent to the parameter restriction that guarantees a positive numerator, and thus $a_0 < 0$ (note the minus in front of the fraction). Under the parameter restriction in Lemma 1, the real marginal cost channel dominates the price setting of firms, whereas the effects of interest rate channel and credit spread channel approximately level out.

Nevertheless, the analytic solutions reveal that Neo-Fisherian effects – an overall increase of inflation following positive financial shocks – are in principle possible. This situation may occur if the credit spread channel dominates both real marginal costs and the pure cost channel because the credit spread sensitivity to leverage is excessively high.

Lemma 2. *The impact response of inflation to a financial shock in normal times (without a binding ZLB on nominal interest rates) is Neo-Fisherian whenever a_0 is positive, i.e. if the elasticity of the credit spread to entrepreneur leverage satisfies*

$$\nu > \frac{\eta + \rho\sigma}{\psi - 1 - \sigma - \eta}. \tag{33}$$

Proof. See Appendix. ■

Note that this results directly from the presence of financial frictions linking credit spreads to marginal costs. In the absence of financial frictions, the policy functions in Proposition 1 are unambiguously negative. While the hypothesis that inflation is Neo-Fisherian in normal times cannot be reject ex-ante (c.f. the discussion in Section 1), we want to focus on case in which our financial shocks is a classic demand shock to maintain the analogy to the Global Financial Crisis. In the following, we hence generally assume that Equation (33) is not satisfied such that a_0 remains negative:

Assumption 2. *The elasticity of the credit spread to entrepreneur leverage satisfies the Condition* (32) *from Lemma 1.*

This implies the natural case of an upwards sloping Phillips curve for financial shocks, i.e. a positive relationship between inflation and output.

3.2 Financial Shocks at the Zero Lower Bound

We now turn to the case of a binding ZLB on nominal interest rates, and analyze the macroeconomic dynamics following financial shocks. We assume that the ZLB currently binds, because an exogenous disturbance induced a contraction of the economy. This could, for example, be the financial shock above. We assume that such a shock endogenously brought the economy to the ZLB and makes private agents expect the ZLB to bind for a specific number of periods (often called the *ZLB spell duration*, e.g. Holden, 2019). In this section, we take this ZLB spell duration as given and do *not* adjust agents' expectations on the spell duration to any *additional* shocks, which we will discuss in

Section 4. This scenario hence focuses on *marginal* effects of (further) financial shocks at the ZLB. While this perspective abstracts from the mapping between shocks and the expected duration of the ZLB, it allows for a straightforward analytical comparison to the case of normal times. We analyze the overall effect of financial shocks using numerical methods in Section 4.

We use a guess-and-verify approach to solve for the equilibrium in closed form. The equilibrium response of inflation and output can be characterized by policy functions which are linear and recursive in the expected ZLB length. The following proposition summarizes the results:

Proposition 2. *Suppose that the ZLB on nominal interest rate is expected to bind for $k \geq 1$ periods. Then, the impact responses of inflation and output to a financial shock are given by:*

$$\pi_t = a_k \, u_t, \tag{34}$$

$$y_t = b_k \, u_t, \tag{35}$$

where

$$a_k = \kappa \left(1 - \gamma\sigma^{-1}\right) \left(1 + \frac{\bar{r}}{u_t}\right) + \rho \left(\beta - \kappa + \kappa\gamma\sigma^{-1}\right) a_{k-1} + \rho\kappa\gamma \, b_{k-1}, \tag{36}$$

$$b_k = -\sigma^{-1} \left(1 + \frac{\bar{r}}{u_t}\right) + \rho\sigma^{-1} \, a_{k-1} + \rho \, b_{k-1}. \tag{37}$$

Proof. See Appendix. ■

To interpret Proposition 2, recall that Assumption 2 guarantees that the policy functions a_0, b_0 are negative in normal times, such that inflation and output decrease following a positive financial shock. Now consider the inflation response for an expected ZLB duration of one quarter ($k = 1$), i.e. a_1. Both a_0 and b_0 are negative under Assumption 2, such that the second and third term in Equation (36) are negative. The term in front of a_0 is close to unity for persistent shocks and shows the ZLB's amplification property: The impact response of inflation increases (ceteris paribus) in the expected length of the ZLB. This reflects the inability of the central bank at the ZLB to counteract further contractionary shocks by means of additional conventional monetary stimulus. The resulting upward pressure on real interest rates depresses consumption, and accordingly overall output.

However, there is an opposing effect on the overall inflation response, captured by the first term in Equation (36). This term can be positive, such that there is potential for a policy function for inflation that is concave in the expected ZLB duration. In other words, it is possible that the disinflationary effect following positive financial shocks is *lower* if the ZLB is expected to bind for a longer period of time. A necessary condition for a concave inflation policy function is that $1 > \gamma\sigma^{-1}$, which is equivalent to the following Lemma.

Lemma 3. *The inflation policy function is concave if the elasticity of the credit spread* *with respect to entrepreneur leverage satisfies that*

$$\nu > \frac{\eta}{\psi - 1 - \sigma - \eta}. \tag{38}$$

It thus follows that the overall response of inflation following inflationary shocks depends crucially on the elasticity of the credit spread with respect to entrepreneur leverage ν. This can also easily be seen by inspecting the solution for a_k, from which one can derive that

$$\frac{\partial a_k}{\partial \nu} = \frac{\partial a_k}{\partial \gamma}\frac{\partial \gamma}{\partial \nu} > 0. \tag{39}$$

The first term in Equation (36) depends negatively on γ. The second depends positively on γ, but following the recursion brings up a_0, which we assumed to be negative. The last term is positive in γ as well, while $b_{k-1} < 0$ for all reasonable calibrations. The effect of an increase in ν can hence be traced back unambiguously: A higher elasticity of the credit spread with respect to entrepreneur leverage c.p. increases the inflationary effect of financial shocks.

Intuitively, a concave policy function hence requires that the credit spread channel (the left-hand side in Equation (38)) dominates both the real marginal cost channel and the financial cost channel (the right-hand side in Equation (38)). As outlined above, the external finance premium is inflationary following contractionary financial shocks: The credit spread increases both exogenously and endogenously, such that marginal costs increase ceteris paribus. In contrast, real marginal costs decline and lead to a disinflationaray effect, amplified by the central bank reaction operating through the direct cost channel. We hence impose on the elasticity of the credit spread with respect to entrepreneur leverage ν that:

Assumption 3. *The elasticity of the credit spread with respect to entrepreneur leverage satisfies Condition* (38) *from Lemma 3.*

If the elasticity of the credit spread with respect to entrepreneur leverage ν is high, the credit channel is strong. In the case of a contractionary shock, this implies that credit spreads increase substantially. If ν is sufficiently large, the credit channel can dominate the price setting of firms, increasing inflation ceteris paribus. In other words, financial frictions need to be sufficiently pronounced, such that the external finance premium dominates. Note that Assumption 3 is weaker than the counterpart in Assumption 2.

A further requirement for a concave policy function is that financial shocks are sufficiently large. This can be seen by inspecting the term $(1 + \frac{\bar{r}}{u_t})$, which is only positive if the following Assumption holds:

Assumption 4. *The financial shock size satisfies*

$$u_t > -\bar{r} = \beta^{-\sigma} - 1. \tag{40}$$

In the case of white-noise financial shocks ($\rho = 0$), Assumptions 3 and 4 are also jointly sufficient for a concave policy function of inflation in the expected ZLB length.In this case, private agents know that the macroeconomic effects of a contemporaneous financial shock vanish in the next period. Forward-looking expectations about inflation and output are thus zero, and current inflation dynamics are solely determined by the relative strengths of the real marginal costs and the external finance premium channels.

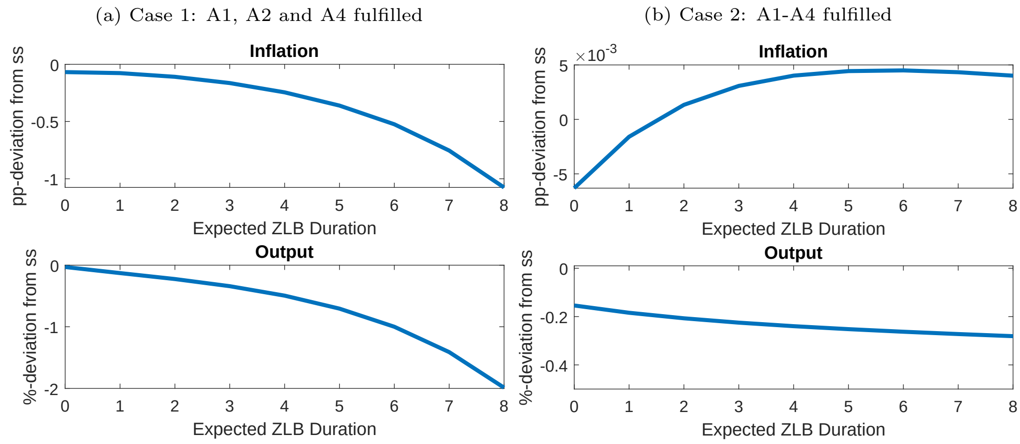

Figure 1: Expected ZLB Duration and Impact Response

Figure 1 displays the policy functions a_k and b_k under two illustrative calibrations. In the first case, the parameters satisfy Assumptions 1, 2 and 4 (such that the external finance premium is countercyclical, financial shocks have conventional effects in normal times and the shock is relatively large). In the second case, the calibration additionally satisfies Assumption 3. As seen in Figure 1, in the first case, the policy functions for inflation and output are strictly decreasing in the expected ZLB duration; a longer expected ZLB duration implies a stronger macroeconomic effect of additional financial shocks. In the second case, however, the policy function for inflation is concave, peaking at an expected ZLB duration of six quarters in positive (Neo-Fisherian) territory. In other words, if the ZLB is expected to bind for a longer period of time, the overall inflation response may even turn positive. This shows that inflation dynamics following financial shocks may be fundamentally different at the ZLB compared to normal times.

4 Numerical Results and the Hockey Stick Phillips Curve

In this section we extend our closed-form solutions by a numerical analysis of the full general equilibrium rational expectations solution. While we derive our analytical solutions conditional on the expected ZLB spell k, we now treat it as endogenous. We employ numerical solution methods to present impulse responses to financial shocks and trace out the corresponding observational Phillips curve.

4.1 Calibration and Solution Method

Throughout this section, we fix the model's structural parameters to standard values taken from Woodford (2003), and adjust them to the most recent estimates (up until 2019) from Boehl and Strobel (2020). We set $\beta = 0.99$, representing the standard view of a quarterly model. We calibrate $\sigma = 1$, which is a common assumption in line with a balanced growth path, and is also backed by Boehl and Strobel (2020). Following the

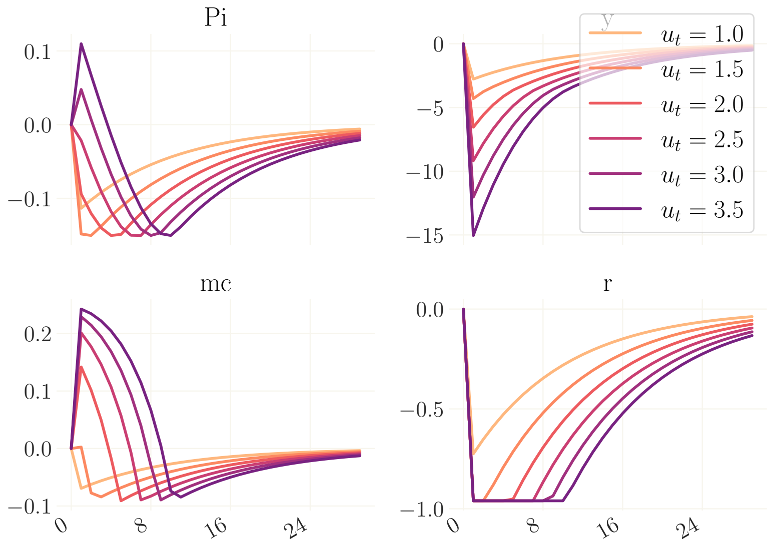

Figure 2: Impulse responses to different financial shocks for $\nu = 0.25$.

same line of reasoning, we set $\eta = 0.5$. We let the fraction of non-adjusting price setters ζ be 0.66, which the commonly found textbook value of 0.66. This is conspicuously lower than the larger estimates from Smets and Wouters (2007) and Boehl and Strobel (2020) as we aim to avoid assuming a flat New Keynesian Phillips curve ex-ante. In particular, the observed flat slope of the post-2008 Phillips curve is an endogenous results of this paper.

For the parameters pertaining to the financial frictions, we fix $\psi = 8$ such that the output effects of financial shocks are amplified by approx. 20% relative to the standard NK model, which is roughly in line with the amplification degree documented by Bernanke et al. (1999). In the following we regard ν as a free parameter, and conduct corresponding comparative exercises.[10]

Regarding monetary policy parameters, we set ϕ_π to 1.5 (a commonly used standard prior), and ϕ_y to 0.2. The latter value is relatively high compared to the standard prior of 0.125, but again in line with Boehl and Strobel (2020). As the authors argue, this reflects the strong reaction of the Fed to output during the ZLB episode from 2009–2015, during which inflation was close to its target value but output remained persistently depressed. Lastly, we set $\rho = 0.9$ which reflects a lasting, quite persistent shock which resembles a post-2009 scenario.

The analytical solutions shown in the previous section hold for the impact period

[10] As shown by Lieberknecht (2019), the financial friction parameters are generally non-linear functions of steady state contract and entrepreneur balance sheet values. In turn, these depend on aggregate (quarterly) default probabilities, the variance of entrepreneurs' productivity and banks' monitoring costs. We prefer to calibrate the financial friction parameters directly to allow for straightforward comparative exercises.

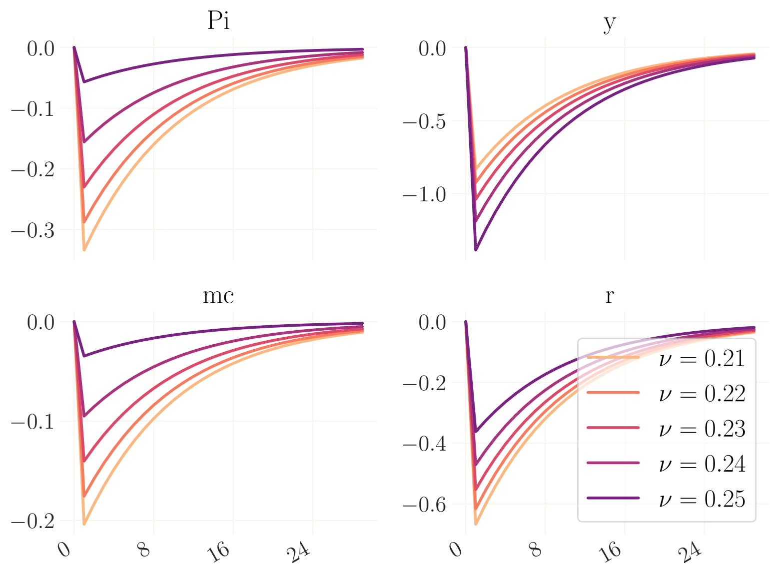

Figure 3: Impulse responses to 0.5% financial shocks for different values of ν. The shock is not strong enough to cause a binding ZLB, which results in conventional inflation dynamics for most values of ν.

when the shock occurs, under the assumption of a pre-specified expected duration of the ELB k. However, in general and in the absence of special policy measures such as forward guidance, k is an equilibrium outcome to be determined endogenously at each point in time, given the contemporaneous exogenous disturbances that causes the ELB constraint to bind. To solve the model at the ELB, we use the numerical solution method proposed by Boehl (2020). A brief description of the solution method is outlined in Appendix A.3.

4.2 Impulse Responses to Financial Shocks

Figure 2 displays impulse responses following contractionary financial shocks of differing size. For the impact responses, these correspond to the analytical policy functions in Proposition 1 and Proposition 2. As seen from the impulse responses, varying the size of the financial shock triples the contractionary effect on output, whereas the disinflationary effect only increases about 50%. As the shock size increases, the initial response of inflation shifts upwards, in line with the analytical insight from Assumption 4. For a value of u_t larger than 2.5, the initial response of inflation becomes positive. Note that, since the responses of endogenous variables is a simple linear map of u_t, and u_t decreases each period by $(1-\rho)$, the lines are actually the same but shifted outwards by a larger initial shock.

Figure 3 shows impulse responses for different values of the elasticity of the credit spread with respect to entrepreneur leverage ν. In this figure, we consider a weak financial shock that is insufficiently strong to cause a binding ZLB. As a result, the dynamics look conventional, with inflation (and marginal costs) falling in response to the shock. An exception is the extreme case for $\nu = 0.26$ (not shown in the Figure), which –

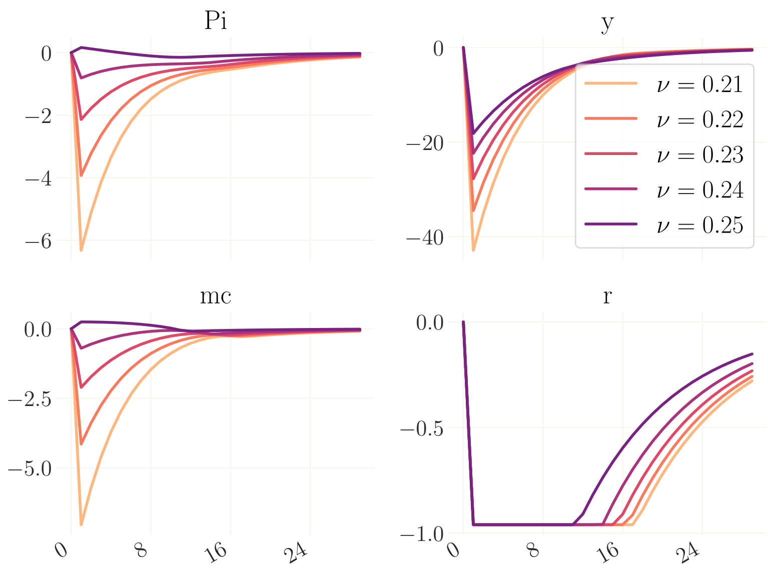

Figure 4: Impulse responses to 3% financial shocks for different values of ν. The shocks are strong enough to cause a binding ZLB, which results in unconventional inflation dynamics via the financial cost channel.

independently of whether or not the ZLB binds – raises inflation independently from whether or not the ZLB binds. This extreme case may be interpreted as the *Neo-Fisherian* parameterization outlined in the previous section: financial costs dominate the firms cost structure, and hence financial shocks translate directly to higher prices. It is also equivalent to the case where Assumption 3 and Lemma 1 are not satisfied.

In Figure 4, we consider a large financial shock, that pushes the economy to the ZLB, for the same values of ν as in Figure 3. As seen from the graphs, the binding ZLB has an elevating effect on marginal costs, which dampens inflation. For $\nu = 0.25$, inflation actually increases, whereas it shows regular dynamics in the absence of the ZLB (see Figure 3). This corresponds to the standard case outlined in the previous section: The elasticity of the credit spread with respect to entrepreneur leverage is large enough to generate a concave inflation policy function, but not excessively large such that Neo-Fisherian solutions emerge in normal times.

As Proposition 2 suggests, the persistence of financial shocks ρ is a central parameter for the strength of the repellent effect as well, both at the ZLB and for the extreme Neo-Fisherian case. The lower ρ is, the stronger are the repellent effects. This effect can be understood by looking at Equation (36): a lower ρ also implies a stronger discounting and hence a less dominant effect of the anticipated course of the financial shock.For example, with $\rho = 0.7$, a ν of 0.2 is sufficient to get similar Neo-Fisherian effects at the ZLB as for $\rho = 0.9$ and $\nu = 0.26$. This observation may be important for policy considerations, as we discuss in the next section. We illustrate this in Figure A.3 in Appendix A.4. Hence, even for very low values of ν, one can always create a Neo-Fisherian effect in response to financial shocks simply by choosing an appropriately low ρ.

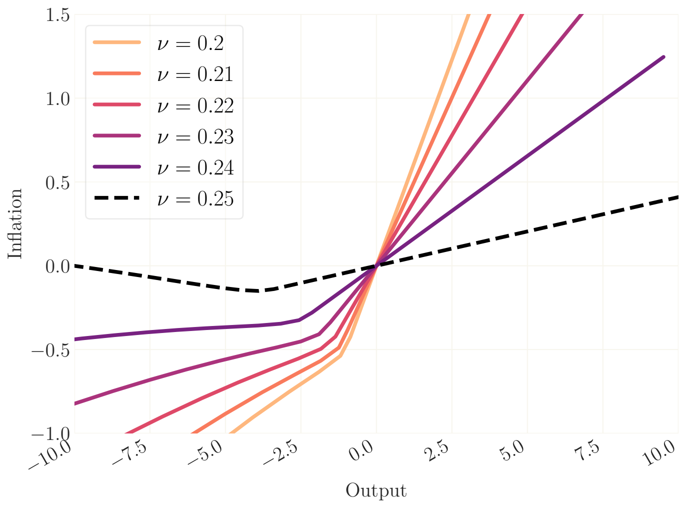

Figure 5: Observed Phillips Curve for an economy facing financial shocks. For each value of ν, we simulate the model for u_t on the interval $[-4, 4]$ and plot the respective combination of π_t and y_t.

4.3 The Observational Hockey Stick Phillips Curve

Figure 5 illustrates our main finding. The figure plots the impulse responses to financial shocks projected into $\{y_t, \pi_t\}$-space. We interpret this as the *observational Phillips Curve*, i.e. the realized values of inflation and output (gap) that would be observed in general equilibrium.[11] This is in contrast to the theoretical New Keynesian Phillips curve – as shown in Equation (13) and (20) – which merely represents firms' price setting under the assumption of nominal price rigidities. The most remarkable observation in Figure 5 is the striking hockey stick shape of the observational Phillips curve. For positive values of output, the observed slope of the Phillips curve is reasonable positive, in line with standard theory. However, the observational Phillips curve flattens out at the ZLB, for substantially negative values of output (or large financial shocks). For $\nu = 0.24$ the observed slope in the region of -3% output is almost zero, while having a reasonable slope in the origin.

In other words, an economic observer aiming to infer the slope of the Phillips curve in times of a binding ZLB and financial frictions would inherently conclude that the Phillips curve is dead. This observation, however, results from the previously discussed credit spread channel, which may dominate firms' price setting at the ZLB. In contrast to the observational Phillips curve, the New Keynesian Phillips curve is well and alive. This means that the relationship between firms' prices and marginal costs, governed by the Calvo parameter, is intact. Note again that our calibration avoids pre-assuming a flat New Keynesian Phillips curve, with the Calvo parameter $\zeta = 0.66$ implying that firms change prices approximately every 3 quarters.

[11] Note again that the output response following financial shocks is identical to the output gap response, see Footnote 9. As such, the figure can equivalently be interpret as showing the output gap - inflation space.

As the elasticity of the credit spread to leverage ν increases, the Phillips Curve becomes flatter in both normal times (with active monetary policy) and when the economy is at the ZLB: The hockey stick rotates in the origin. For a value of $\nu = 0.25$ we observe that the credit spread effect at the ZLB is strong enough that inflation actually increases with output, while the Phillips curve is still upwards sloping in normal times. An even higher ν of 0.26 finally shifts the system towards a fully Neo-Fisherian regime, where the Phillips curve is downward sloping on the full domain of u_t (not shown in Figure 5).

5 Implications for Monetary Policy

We now turn to the implications of our findings for central banks. For monetary policymakers, the changing transmission of financial shocks at the ZLB raises a number of difficulties and might require a different design of monetary policy actions.

5.1 Interpreting the Observational Phillips Curve

Our result about the flat observational Phillips curve at the ZLB means that the correct identification of the relationship between inflation and output is challenging. This is because policymakers need to infer the structural relationship between inflation and output (i.e. the structural New Keynesian Phillips slope and determinants of firms' price setting) using only observed equilibrium values. At the ZLB, this requires estimates of contemporaneous macroeconomic shocks, private sector expectation of the ZLB length and the currently prevailing degree of financial frictions. Acquiring this level of information in real time seems hardly possible in practice.

Abstracting from this challenge, the flat observational Phillips curve does not necessarily constitute a threat to policymakers per se. At the ZLB, (further) contractionary risk shocks do not lead to a substantial decline of inflation. Given a strong mandate to stabilize inflation, a lower deflationary pressure is equivalent to a lower sense of urgency for monetary policy to counteract. This also means that central banks might not necessarily be forced to resort to unconventional monetary policy instruments at the ZLB. In fact, as Figures 2 and 4 show, a stronger financial cost channel – a higher ν – rather leads to an attenuated fall in output.

However, one could argue that the source of inflation is important as well, raising further difficulties for monetary policy. In our theoretical framework, the lower deflationary pressure at the ZLB following financial shocks stems from higher credit spreads. If credit spreads are major determinants of inflation at the ZLB, this also implies that central banks should be predominantly concerned with financial conditions. In such a situation, reducing financial distress directly via appropriate monetary policy operations on money markets might be the most efficient way to steer inflation. Unfortunately, central banks might find themselves in a catch-22 situation. On the one hand, high credit spreads might reflect substantial distress in the financial sector, thereby constituting a concern from a financial stability perspective. On the other hand, lower credit spreads induced by looser monetary and financial conditions increase the deflationary pressure. Therefore, a financial recovery might not necessarily be associated with (a revival of) inflation. As such, disentangling the role of real marginal costs and credit spreads for overall inflation seems important to design appropriate monetary (and macroprudential) policies.

5.2 Monetary Policy Shocks at the ZLB

The difficulties of interpreting the observational Phillips Curve and identifying the sources of inflation translate into delicate decisions about the appropriate design of monetary policy at the ZLB. To make matters worse, the effects of monetary policy itself are also affected by the presence of financial frictions and the ZLB. We analyze this aspect by first looking at monetary policy shocks at the ZLB.

Lemma 4. *In normal times (awy from the ZLB), monetary policy shocks v_t generate identical macroeconomic dynamics as financial shocks u_t, given the same persistence. They are only distinguishable via the response of the interest rate.*

Lemma 5. *At the ZLB, monetary policy shocks v_t and financial shocks u_t are not distinguishable, given the same persistence.*

The intuition behind Lemma 4 is straightforward. Inspecting the model representation in three equations, as shown in Section 3.1, reveals that monetary policy shocks appear in the same places as financial shocks. Therefore, in this framework, monetary policy shocks are observationally equivalent to financial shocks and generate identical macroeconomic dynamics. As a consequence, all results from the previous sections concerning financial shocks in normal times are valid for monetary policy shocks as well. Notably, this includes the closed-form solutions and the possibility of Neo-Fisherian effects of monetary policy shocks in normal times for extreme calibrations. It also follows immediately that the central bank can, in principle, offset financial shocks perfectly.

At the ZLB, however, monetary policy shocks govern the expectations on the future interest rate path, acting like explicit forward guidance by the central bank. Figures 6 and 7 show the effects of such forward guidance shocks at the ZLB. Both figures reveal that forward guidance at the ZLB may be associated with Neo-Fisherian effects. Intuitively, forward guidance shocks that prolong the expected ZLB duration have two effects that go in the same directions. First, the effect that expected rates are lower, which transmits to the economy via the standard Euler channel and the various channels on marginal costs. Second, agents expect that the inversion of the policy function will remain active for more periods. This depends crucially on the forward guidance persistence and the degree of financial frictions. We summarize these considerations in the following lemma:

Lemma 6. *At the ZLB, forward guidance shocks v_t may be associated with Neo-Fisherian effects, such that expansionary forward guidance is disinflationary iff*

$$\rho_r < \rho. \tag{41}$$

Note that the condition in Lemma 6 is a necessary, but not a sufficient condition. Assume a combination (ρ, ν) for which a given shock u_t is deflationary. As the mechanics behind forward guidance and financial shocks are equal, we learn from Equation (36) in Proposition 2 that a smaller ρ (or here: ρ_r) can reduce the weight on the (negative terminal) second and third term. In that sense, a decrease in ρ has a similar effect as an increase in ν. We show this effect in Figure A.3 in the Appendix.

While it is safe to assume a high persistence of the financial shock, the persistence of the forward guidance shock is a policy parameter that can in principal be chosen by the central bank. However, it also depends on how successful the central bank is in its

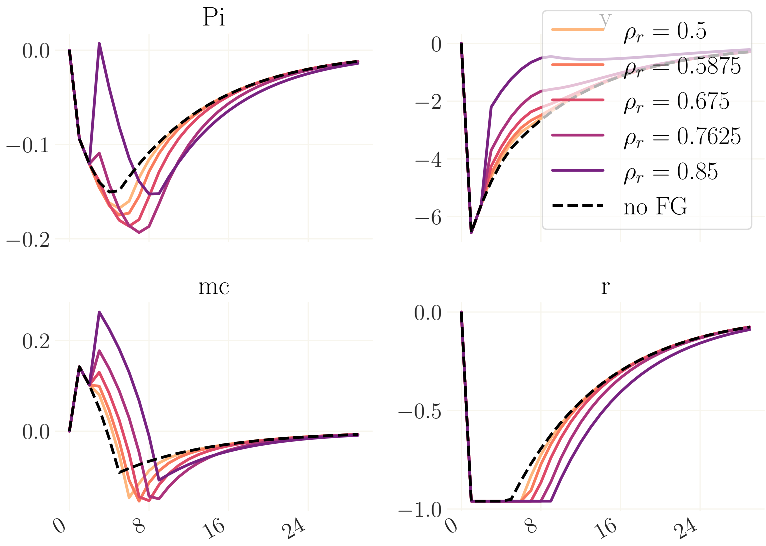

Figure 6: Dashed line: impulse responses to a 2% financial shock for $\nu = 0.25$. Colored lines are the same financial shock combined with a forward guidance shock in period 3. Different colors correspond to different persistences of the forward guidance shock.

communication strategies. As illustrated in Figure 6, a monetary policy shock with low persistence can hence trigger negative inflation responses because the short-run effect of decreasing financial costs dominates the medium-term effect that works through the household Euler Equation. As such, half-hearted forward guidance may be associated with undesirable macroeconomic dynamics.

5.3 Monetary Policy Rules at the ZLB

After investigating monetary policy shocks, we now turn to the systematic behavior of central banks. While at first glance, it may seem that these rules are irrelevant at the ZLB, they are in fact crucial for macroeconomic dynamics. This is because rational private agents take the monetary policy rule into account when forming expectations about future variables and the remaining ZLB duration. As such, choosing an appropriate monetary policy rule is of central importance for central banks at the ZLB as well.

From a policy-making perspective, the minimum requirement that any appropriate rule should satisfy is that it guarantees a determinate equilibrium.

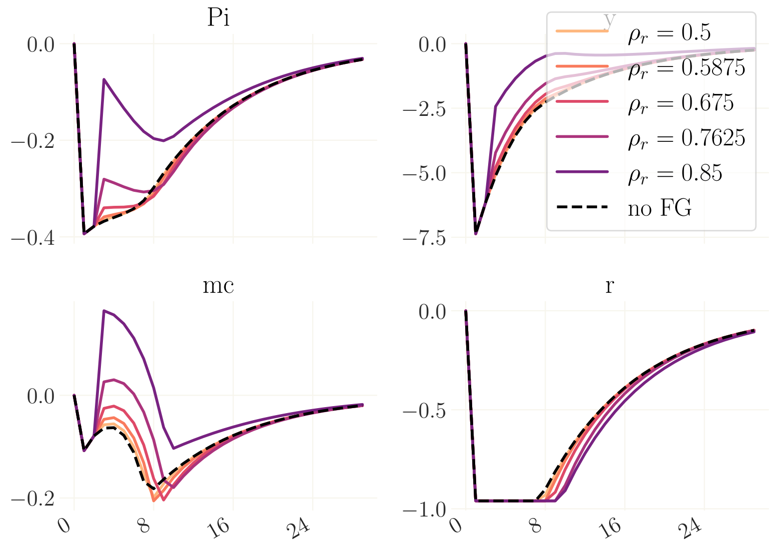

Figure 7: Dashed line: impulse responses to a 2% financial shock for $\nu = 0.24$. Colored lines are the same financial shock combined with a forward guidance shock in period 3. Different colors correspond to different persistences of the forward guidance shock.

Proposition 3. *The policy parameters in the central bank's monetary policy rule must satisfy the following conditions to guarantee a determinate solution:*

$$\phi_\pi + \frac{1-\beta}{\kappa\gamma}\,\phi_y > 1, \tag{42}$$

$$\kappa(\sigma^{-1}\gamma - 1)\,\phi_\pi + \sigma^{-1}\phi_y > \beta - 1 - \kappa \tag{43}$$

Proof. See Appendix. ■

Equation (42) may be interpreted as a modified Taylor principle for a financial accelerator economy. If the central bank decides to react to inflation only ($\phi_y = 0$), the associated coefficient ϕ_π needs to be larger than unity, as in Taylor (1993). If the central bank reacts to output as well ($\phi_y > 0$), determinacy requires the weighted sum of policy coefficients to be larger than unity. Compared to a standard New Keynesian framework, the key difference is that the presence of financial frictions affects the degree of substitutability between reacting to inflation and to output. Under Assumption 1, the slope of the New Keynesian Phillips curve with respect to output (the term $\kappa\gamma$) is lower due to the countercyclical credit spread. At first glance, it thus seems that policy responses to output can *substitute* more effectively for policy responses to inflation.

However, Equation (43) may constitute additional complications for the design of monetary policy rules. To see this, suppose that $(\sigma^{-1}\gamma - 1) < 0$. Note that this is exactly the condition for a concave policy function of inflation at the ZLB, i.e. Assumption 3. For our benchmark calibration, this occurs for relatively low values of $\nu > \frac{1}{11}$. In this case, Equation (43) implies that the responses to inflation and output are *complements* for some combinations of $\{\phi_\pi, \phi_y\}$, or equivalently constitutes a lower bound restriction for the response of output. In other words, a stronger reaction to inflation must be accompanied by a corresponding stronger reaction to output. This clashes with the modified Taylor rule that exhibits the conventional substitutability.

Figure 8 displays this result graphically. As the elasticity of the credit spread with respect to entrepreneur leverage ν increases, a higher value for ϕ_y is necessary to keep the model determined for high values of ϕ_π. For example, in the case of $\nu = 0.2$, $\phi_\pi > 1.76$ requires that $\phi_y > 0$.

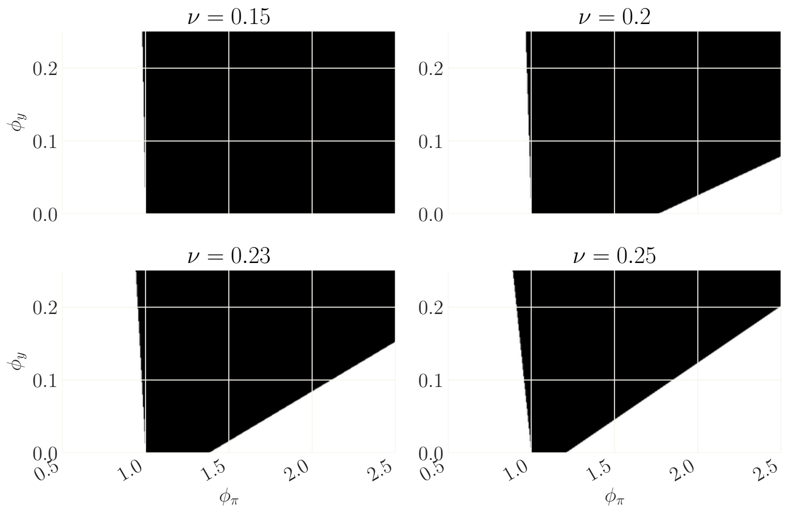

Figure 8: Determinancy regions for different values of ν.

Intuitively, abstracting from financial frictions, inflation can be stabilized by raising nominal interest rates appropriately. Higher nominal interest rates amount to higher real interest rates, decreasing consumption and output. As a consequence, real marginal costs fall, and inflation decreases. Whether the hike of nominal interest rates constitutes a reaction to (positive) deviations of inflation or output is irrelevant. In the presence of financial frictions, however, an interest rate hike as a reaction to output has the additional effect of increasing marginal costs and thus inflation. Depending on the specific characteristics of the economy, the central bank might find itself in a knife edge scenario where the appropriate window for policy responses to output deviations is quite small.

Overall, the key message emerging from this section is that the conduct of monetary policy in the presence of financial frictions and a binding ZLB may prove difficult. While the hockey stick Phillips curve blurs the relationship between inflation and output at the ZLB, conventional monetary policy wisdoms are abolished: Short-lived monetary policy shocks may be associated with Neo-Fisherian inflation effects, and the determinacy conditions may place rather tight restrictions on appropriate monetary policy rules.

6 Conclusion

This paper argues that a binding zero lower bound (ZLB) on nominal interest rates may contribute to an observational disconnect between inflation and economic activity. At the ZLB, the costs of external financing in the form of credit spreads can dominate firms' price setting and thereby generate inflationary pressure. As a result, the Phillips curve features a considerably flatter slope when the ZLB binds compared to normal times. The resulting observational Phillips curve is shaped like a hockey stick.

Our results translate into strong implications on the conduct of forward guidance, and provide a potential solution to the forward guidance puzzle: similar to financial shocks, the effects of forward guidance can be decomposed in short-run deflationary effects via the firms' refinancing cost channel, and a medium-term inflationary effect via real marginal costs. For rather short-lived forward guidance impulses, the first deflationary effect may dominate and forward guidance can in fact lower inflation. Accordingly, only shocks with a high expected persistence succeed in fostering inflation and growth.

We view the combination of financial frictions and the ZLB as an additional building block in the quest to explore the recent inflation puzzles. Our theory is complementary to the existing explanations put forward in the related literature. A challenge for future research is to discriminate between these various explanations empirically to assess their relative quantitative importance. This would convey useful insights to policymakers, potentially with a view to enabling better informed (real-time) macroeconomic assessments and (monetary) policy decisions.

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Appendix A Appendix

Appendix A.1 Equilibrium Equations

This section lists the full set of equations defining equilibrium. On the household side, we have the intertemporal Euler equation and the intra-temporal labor-consumption trade-off, Equations (1) and (2) in the main text:

$$C_t^{-\sigma} = \beta E_t \left[\frac{R_t}{\Pi_{t+1}} U_t C_{t+1}^{-\sigma} \right], \tag{A.1}$$

$$H_t^{\eta} = W_t C_t^{-\sigma}. \tag{A.2}$$

On the firm side, we have the the aggregate production function, which is obtained by aggregating over the individual production functions displayed in Equation (3):

$$Y_t = \frac{H_t}{v_t^p} \tag{A.3}$$

where v_t^p is a measure of price dispersion defined below. Marginal costs are given by Equation (7):

$$MC_t = W_t R_t^L \tag{A.4}$$

The price setting behavior by firms is defined by the following equations, which are standard for Calvo (1983) pricing and make use of two auxiliary variables f_t^1 and f_t^2:

$$f_t^1 = \frac{\varepsilon - 1}{\varepsilon} f_t^2 \tag{A.5}$$

$$f_t^1 = C_t^{-\sigma} MC_t Y_t + \beta \zeta E_t \left[\Pi_{t+1}^{\varepsilon} f_{t+1}^1 \right] \tag{A.6}$$

$$f_t^2 = C_t^{-\sigma} \Pi_t^* Y_t + \beta \zeta E_t \left[\left(\frac{1}{\Pi_{t+1}} \right)^{1-\varepsilon} \left(\frac{\Pi_t^*}{\Pi_{t+1}^*} \right) f_{t+1}^2 \right] \tag{A.7}$$

$$1 = \zeta \left(\frac{1}{\Pi_t} \right)^{1-\varepsilon} + (1-\zeta) \left(\Pi_t^* \right)^{1-\varepsilon} \tag{A.8}$$

$$v_t^p = \zeta \Pi_t^{\varepsilon} v_{t-1}^p + (1-\zeta) \left(\Pi_t^* \right)^{-\varepsilon} \tag{A.9}$$

The interest rate specified in the credit contract is defined by Equation (6):

$$R_t^L = z \left(\frac{W_t H_t}{N_t} \right) \frac{R_t}{E_t[\Pi_{t+1}]} U_t \tag{A.10}$$

Entrepreneur net worth evolves according to Equation (9):

$$N_t = \Psi \left(Y_t \right), \tag{A.11}$$

The central bank operates according to a monetary policy rule shown in Equation (10)

$$\frac{R_t^n}{R^n} = \left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left(\frac{Y_t}{Y} \right)^{\phi_y} exp(v_t), \tag{A.12}$$

The zero lower bound (ZLB) constraint is given by Equation (11):

$$R_t = \max\left\{\bar{R}, R_t^n\right\} \tag{A.13}$$

Finally, the aggregate resource constraint is

$$Y_t = C_t \tag{A.14}$$

These 14 conditions define the equilibrium for 14 endogenous variables $(C_t, Y_t, H_t, \Pi_t, \Pi_t^*, W_t, R_t, R_t^L, R_t^n, N_t, MC_t, f_t^1, f_t^2, v_t^p)$, together with the evolution of the two exogenous shocks:

$$log(U_t) = \rho\, log(U_{t-1}) + \epsilon_t \tag{A.15}$$
$$v_t = \rho_r\, v_{t-1} + \epsilon_t r, t. \tag{A.16}$$

The linearized equilibrium conditions are as follows:

$$\begin{aligned}
c_t &= -\sigma^{-1}\left(r_t + u_t - E_t\pi_{t+1}\right) + E_t[c_{t+1}], && \text{(A.17)}\\
w_t &= \eta h_t + \sigma c_t, && \text{(A.18)}\\
y_t &= h_t, && \text{(A.19)}\\
mc_t &= w_t + r_t^L, && \text{(A.20)}\\
\pi_t &= \kappa mc_t + \beta E_t[\pi_{t+1}], && \text{(A.21)}\\
r_t^L &= r_t - E_t[\pi_{t+1}] + \nu(w_t + h_t - n_t) + u_t, && \text{(A.22)}\\
n_t &= \psi y_t, && \text{(A.23)}\\
r_t^n &= \phi_\pi \pi_t + \phi_y y_t + v_t, && \text{(A.24)}\\
r_t &= \max\left\{\bar{r}, r_t^n\right\}, && \text{(A.25)}\\
y_t &= c_t, && \text{(A.26)}\\
u_t &= \rho\, u_{t-1} + \epsilon_t, && \text{(A.27)}\\
v_t &= \rho_r\, v_{t-1} + \epsilon_{r,t}, && \text{(A.28)}\\
& && \text{(A.29)}
\end{aligned}$$

where lower-case variables denote log-deviations from steady state.

The three-equation representation shown in Section 3.1 can be obtained by combining Equations (A.18)-(A.23) into one single Phillips curve and using the resource constraint Equation A.26 to eliminate c_t.

Appendix A.2 Proofs

Proposition 1. *The impact responses of inflation and output to a financial shock in normal times (without a binding ZLB on nominal interest rates) are given by:*

$$\pi_t = a_0\, u_t, \tag{A.30}$$
$$y_t = b_0\, u_t, \tag{A.31}$$

where

$$a_0 = -\frac{\kappa\gamma - \kappa\sigma(1-\rho)}{(1-\beta\rho)(\sigma(1-\rho)+\phi_y) + \kappa\gamma(\phi_\pi - \rho) - \kappa\sigma(1-\rho)(\phi_\pi - 1)}, \tag{A.32}$$
$$b_0 = -\frac{1 + (\phi_\pi - \rho)a_0}{\sigma(1-\rho)+\phi_y}. \tag{A.33}$$

Proof. The proof relies on the method of undetermined coefficients. We guess that the solution is given by $\pi_t = a_0\, u_t$ and $y_t = b_0\, u_t$. Using this guess, the system of equation can be written as

$$(1 - \kappa\phi_\pi - \rho(\beta - \kappa))a_0 u_t = \kappa u_t + \kappa(\gamma + \phi_y)b_0 u_t, \tag{A.34}$$
$$(1 + \phi_y\sigma^{-1} - \rho)b_0 u_t = -\sigma^{-1}(\phi_\pi - \rho)a_0 u_t - \sigma^{-1}u_t, \tag{A.35}$$

where we replaced the nominal interest rate using the (unconstrained) Taylor rule. Note that expectations of future variables can be replaced by using the law of motion for the financial shocks under rational expectations. The solution is obtained by dividing both equations by u_t, substituting for b_0 in the first equation using the second equation and rearranging. ■

Lemma 1. *The impact responses of inflation and output to a financial shock in normal times (without a binding ZLB on nominal interest rates) are negative, i.e.*

$$a_0 < 0, \tag{A.36}$$
$$b_0 < 0, \tag{A.37}$$

iff the elasticity of the credit spread to entrepreneur leverage satisfies

$$\nu < \frac{\eta + \rho\sigma}{\psi - 1 - \sigma - \eta}. \tag{A.38}$$

Proof. The proof consists of three parts. First, we show that the model's determinacy conditions imply that the denominator of a_0 is positive. Second, the sign of a_0 then depends on its numerator, which is equivalent to the parameter restriction in the Lemma. Third, the sign of b_0 follows from a_0.

First, let us consider the determinacy conditions. The forward looking components of our model can be expressed as

$$\mathbf{M}\mathbf{x}_t = E_t[\mathbf{x}_{t+1}], \tag{A.39}$$

with $\mathbf{x}_t = (y_t, \pi_t)'$. To arrive at this formulation, we can rewrite Equations (21) and (22) (ignoring exogenous innovations and the ELB) as

$$(1+\sigma^{-1}\phi_y)y_t = -\sigma^{-1}\left(\phi_\pi \pi_t - E_t[\pi_{t+1}]\right) + E_t[y_{t+1}], \tag{A.40}$$
$$(1-\kappa\phi_\pi)\pi_t = \kappa(\gamma+\phi_y)y_t + \beta_\kappa E_t[\pi_{t+1}], \tag{A.41}$$

where we define $\beta_\kappa = \beta - \kappa$ for convenience. Then, we can rewrite

$$\mathbf{A}\mathbf{x}_t = \mathbf{B}\mathbf{x}_{t+1}, \tag{A.42}$$
$$\begin{bmatrix} 1+\sigma^{-1}\phi_y & \sigma^{-1}\phi_\pi \\ -\kappa(\gamma+\phi_y) & 1-\kappa\phi_\pi \end{bmatrix} \mathbf{x}_t = \begin{bmatrix} 1 & \sigma^{-1} \\ 0 & \beta_\kappa \end{bmatrix} \mathbf{x}_{t+1}. \tag{A.43}$$

It is straightforward that

$$\mathbf{B}^{-1} = \frac{1}{\beta_\kappa} \begin{bmatrix} \beta_\kappa & -\sigma^{-1} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\beta_\kappa^{-1}\sigma^{-1} \\ 0 & \beta_\kappa^{-1} \end{bmatrix}, \tag{A.44}$$

and hence

$$\mathbf{M} = \mathbf{A}\mathbf{B}^{-1} = \begin{bmatrix} 1+\sigma^{-1}\phi_y & \sigma^{-1}\phi_\pi \\ -\kappa(\gamma+\phi_y) & 1-\kappa\phi_\pi \end{bmatrix} \begin{bmatrix} 1 & -\beta_\kappa^{-1}\sigma^{-1} \\ 0 & \beta_\kappa^{-1} \end{bmatrix}, \tag{A.45}$$
$$= \begin{bmatrix} 1+\sigma^{-1}\phi_y & -\beta_\kappa^{-1}\sigma^{-1}(1+\sigma^{-1}\phi_y - \phi_\pi) \\ -\kappa(\gamma+\phi_y) & \beta_\kappa^{-1}\sigma^{-1}\kappa(\gamma+\phi_y) + \beta_\kappa^{-1}(1-\kappa\phi_\pi) \end{bmatrix}, \tag{A.46}$$
$$= \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix}. \tag{A.47}$$

The eigenvalues of the system are given by $|\mathbf{M} - \lambda\mathbf{I}| = \lambda^2 + p\lambda + q$, where

$$p = -(m_1+m_4) = -\left(1+\sigma^{-1}\phi_y + \beta_\kappa^{-1}\sigma^{-1}\kappa(\gamma+\phi_y) + \beta_\kappa^{-1}(1-\kappa\phi_\pi)\right) \tag{A.48}$$

is the negative of the trace and

$$q = m_1m_4 - m_2m_3 = \beta_\kappa^{-1}(1+\sigma^{-1}\phi_y - \kappa\phi_\pi + \sigma^{-1}\phi_\pi\kappa\gamma) \tag{A.49}$$

is the determinant. As there are no endogenous states, determinancy under conditions by Blanchard and Kahn (1980) requires the modulus of both eigenvalues of $\mathbf{M}$ to be larger than zero. We can find a representation of the absolute value of these eigenvalues in terms of the elements of $\mathbf{M}$ as

$$|\lambda_{1,2}^r| = \begin{cases} -p/2 + \sqrt{p^2/4 - q} > 1 \\ -p/2 - \sqrt{p^2/4 - q} > 1 \end{cases} \qquad \text{if } p^2/4 \geq q, \tag{A.50}$$
$$|\lambda_{1,2}^i| = \sqrt{p^2/2 - q} > 1 \qquad \text{if } p^2/4 < q. \tag{A.51}$$

$|\lambda_{1,2}^r|$ are the real eigenvalues if the respective condition for the square root is satisfied, $|\lambda_{1,2}^i|$ are corresponding imaginary eigenvalues otherwise. Using the condition in Equation (A.50) in the second case implies that $-p/2 > 1$, or equivalently

$$p < -2. \tag{A.52}$$

Rearranging the second case in Equation (A.50) also implies

$$1 + p + q > 0. \tag{A.53}$$

Together with Equation (A.52), this implies

$$q > 1. \tag{A.54}$$

Equation (A.54) is also a necessary condition for the case of imaginary eigenvalues. Similarly, one can show that Equation (A.52) and Equation (A.53) imply that Equation (A.51) holds. Therefore, Equations (A.52)-(A.54) are jointly sufficient for both eigenvalues to be larger than one in modulus.

In our model, the three necessary condition $1 + p + q > 0$, $p < -2$ and $q > 1$ thus read

$$\phi_\pi + \frac{1-\beta}{\kappa\gamma}\phi_y > 1, \tag{A.55}$$

$$\sigma^{-1}\phi_y + \beta_\kappa^{-1}\sigma^{-1}(\kappa\gamma + \kappa\phi_y) + \beta_\kappa^{-1}(1 - \kappa\phi_\pi) > 1, \tag{A.56}$$

$$1 + \sigma^{-1}(\kappa\gamma\phi_\pi + \phi_y) - \kappa\phi_\pi > \beta_\kappa. \tag{A.57}$$

As a second step, we can use these determinacy conditions to derive a sign for the denominator of a_0. Let us suppose that the denominator is positive, i.e.

$$(1 - \beta\rho)(\sigma(1-\rho) + \phi_y) + \kappa\gamma(\phi_\pi - \rho) - \kappa\sigma(1-\rho)(\phi_\pi - 1) > 0. \tag{A.58}$$

This can be rearranged to

$$\left(\phi_\pi + \frac{1-\beta}{\kappa\gamma}\phi_y - 1\right) + \frac{1-\rho}{\kappa\gamma}\left(\kappa\gamma + \beta\phi_y + \sigma(1 - \beta\rho - \kappa\rho - \kappa\phi_\pi)\right) > 0. \tag{A.59}$$

The first term in large brackets is positive, which can be seen directly from the necessary condition in Equation (A.55). After some algebraic manipulations, one can show that Equation (A.56) implies that the second term in brackets is also positive. This shows that the denominator of a_0 is positive.

With the denominator being positive, the sign of a_0 depends on the numerator, including the minus in front of the fraction. The condition for $a_0 < 0$ is thus

$$\kappa\gamma - \kappa\sigma(1-\rho) > 0. \tag{A.60}$$

Using the definition of γ, this is equivalent to

$$\sigma + \eta - \nu(\psi - 1 - \sigma - \eta) > \sigma(1-\rho). \tag{A.61}$$

Rearranging yields the parameter restriction in terms of the elasticity of the credit spread to entrepreneur leverage.

As a last step, the sign of b_0 can be determined given the solution for a_0. The denominator of b_0 is positive for conventional parameters, such that the sign is determined by the numerator, including the minus. Inserting a_0, this is given by

$$-1 + (\phi_\pi - \rho)\frac{\kappa\gamma - \kappa\sigma(1-\rho)}{Z}, \tag{A.62}$$

where Z denotes the denominator of a_0. After some algebraic manipulations, this is equivalent to

$$-Z^{-1}\Big((1-\beta\rho)(\sigma(1-\rho) + \phi_y) + (1-\rho)^2\kappa\sigma\Big), \tag{A.63}$$

which is unambiguously negative for $0 \leq \rho \leq 1$. ∎

Lemma 2. *The impact response of inflation to a financial shock in normal times (without a binding ZLB on nominal interest rates) is Neo-Fisherian whenever a_0 is positive, i.e. if the elasticity of the credit spread to entrepreneur leverage satisfies*

$$\nu > \frac{\eta + \rho\sigma}{\psi - 1 - \sigma - \eta}. \tag{A.64}$$

Proof. This is the converse case of Lemma 1. As argued in the corresponding proof, determinacy of the model requires the denominator of a_0 to be positive. The condition for $a_0 > 0$ is hence that the numerator (including the minus in front of the fraction) is positive. This is equivalent to

$$\sigma(1-\rho) > \gamma. \tag{A.65}$$

Using the definition of γ to obtain

$$\sigma(1-\rho) > \sigma + \eta - \nu(\psi - 1 - \sigma - \eta) \tag{A.66}$$

and rearranging yields the desired result. ∎

Proposition 2. *Suppose that the ZLB on nominal interest rate is expected to bind for $k \geq 1$ periods. Then, the impact responses of inflation and output to a financial shock are given by:*

$$\pi_t = a_k\, u_t, \tag{A.67}$$
$$y_t = b_k\, u_t, \tag{A.68}$$

where

$$a_k = \kappa\left(1 - \gamma\sigma^{-1}\right)\left(1 + \frac{\bar{r}}{u_t}\right) + \rho\left(\beta - \kappa + \kappa\gamma\sigma^{-1}\right)\, a_{k-1} + \rho\kappa\gamma\, b_{k-1}, \tag{A.69}$$
$$b_k = -\sigma^{-1}\left(1 + \frac{\bar{r}}{u_t}\right) + \rho\sigma^{-1}\, a_{k-1} + \rho\, b_{k-1}. \tag{A.70}$$

Proof. Similar to Proposition 1, the proof relies on the method of undetermined coefficients. Suppose that the ZLB on nominal interest is expected to bind for $k \geq 1$ periods. Denoting the corresponding policy functions for by a_k and b_k, respectively, we can rewrite the system of equations as

$$a_k\, u_t = \kappa\gamma\, b_k u_t + \kappa(\bar{r} + u_t) + (\beta - \kappa)\rho\, a_{k-1}, \tag{A.71}$$

$$b_k\, u_t = -\sigma^{-1}(\bar{r} + u_t) + \rho\sigma^{-1} a_{k-1} + \rho\, b_{k-1}, \tag{A.72}$$

where the central bank interest rate is replaced by the ZLB value. Note that expectations of future variables can be replaced by the corresponding policy functions for the case of an expected ZLB duration of $k-1$ under rational expectations, using the law of motion for the financial shocks. The solution is obtained by dividing both equations by u_t, substituting for b_k in the first equation using the second equation and rearranging. ∎

Proposition 3. *The policy parameters in the central bank's monetary policy rule must satisfy the following conditions to guarantee a determinate solution:*

$$\phi_\pi + \frac{1-\beta}{\kappa\gamma}\,\phi_y > 1, \tag{A.73}$$

$$\kappa(\sigma^{-1}\gamma - 1)\,\phi_\pi + \sigma^{-1}\phi_y > \beta - 1 - \kappa \tag{A.74}$$

Proof. The first equation follows directly from the condition $1 + p + q > 0$, which is required to satisfy the Blanchard and Kahn (1980) conditions. This is Equation (A.55) in the proof for Proposition 1. The second equation can be obtained by rearranging the condition $q > 1$, which is Equation (A.57) above. ∎

Appendix A.3 Solution method

To outline the procedure, a different representation of the policy functions than in the previous section is used. The analytic solutions of the previous section are expressed in terms recursive policy functions in terms of u_t. A different, non-recursive way of presenting these policy functions is suggested in Boehl (2020). The simplicity of our model allows to ease the notation therein and express our model with $\mathbf{x}_t = (\pi_t, y_t)'$ in matrix form as

$$\mathbf{x}_t + \mathbf{c}\max\{\mathbf{d}\mathbf{x}_t, \bar{r}\} = \mathbf{N}E_t\mathbf{x}_{t+1} + \mathbf{c}u_t, \tag{A.75}$$

where $\mathbf{N}$ is the system matrix of the constrained system, $\mathbf{c}$ contains the coefficients that determine how $\mathbf{x}_t$ is affected by r_t (and thereby also by u_t) and $\mathbf{d}$ contains the parameters of the monetary policy rule. $\bar{r} < 0$ is the actual model-implied lower bound of r_t.

The analytic solutions of the previous section are expressed in as a recursion in terms of u_t, which allows neatly for analytical insights. Boehl (2020) proposes a non-recursive representation. Assume again that the economy is at the ELB for k periods. Then

$$\mathbf{x}_t + \mathbf{c}\bar{r} = \mathbf{N}E_t\mathbf{x}_{t+1} + \mathbf{c}u_t, \tag{A.76}$$

$$E_t\mathbf{x}_{t+1} + \mathbf{c}\bar{r} = \mathbf{N}E_t\mathbf{x}_{t+2} + \mathbf{c}u_{t+1}, \tag{A.77}$$

$$\dots$$

$$E_t\mathbf{x}_{t+k-1} + \mathbf{c}\bar{r} = \mathbf{N}E_t\mathbf{x}_{t+k} + \mathbf{c}u_{t+k-1}, \tag{A.78}$$

$$E_t\mathbf{x}_{t+k} = \mathbf{A}(0)u_{t+k}. \tag{A.79}$$

Recursively inserting (A.79) into (A.78) yields, acknowledging that $E_t u_{t+s} = \rho^s u_t$,

$$\mathbf{x}_t = \mathbf{N}^k\mathbf{A}(0)\rho^k u_t + \sum_{i=0}^{k-1}\mathbf{N}^i\mathbf{c}\rho^i u_t - \sum_{i=0}^{k-1}\mathbf{N}^i\mathbf{b}\bar{r}, \tag{A.80}$$

$$= \mathbf{A}(k)u_t + \mathbf{a}(k)\bar{r}. \tag{A.81}$$

Rewriting (A.80) yields

$$\pi_t = A_\pi(k)u_t + a_\pi(k)\bar{r}, \tag{A.82}$$

$$y_t = A_y(k)u_t + a_y(k)\bar{r}. \tag{A.83}$$

In verbal terms, this implies that depending on the expected number of periods at the ELB k, we can express the vector of controls $\mathbf{x}_t$ as a linear map $A_j(k)$ of u_t and the (constant) vector $a_j(k)$. Both terms are nonlinear functions of k defined on $\mathbb{N}_0$. A mode detailed description of these policy functions, including a graphic illustration, can be found in Appendix A.3. More clearly: given k, the policy function is simply a two dimensional linear projection of the scalar u_t.

Definition 1 recapitulates the conditions for k to be an equilibrium value under the assumption that each shock causes the ELB to hold instantly without any transition period.

Definition 1. *For each period t, an equilibrium value of $k \in \mathbb{N}_0$ must satisfy that the ELB binds in expectations exactly until period $t + k$. Hence,*

$$\mathbf{d}\mathbf{x}_t > \bar{r} \implies k = 0, \tag{A.84}$$

while for $k > 0$ it must hold that

$$\mathbf{d}E_t\mathbf{x}_{t+k} > \bar{r}, \tag{A.85}$$

and

$$\mathbf{d}E_t\mathbf{x}_{t+k-1} \leq \bar{r}. \tag{A.86}$$

The parsimonious nature of our model allows that, for each u_t, a k can simply be found by iterating over $k \in \mathbb{N}_0$ (where, naturally, k is likely to be small). More sophisticated iteration schemes for a general formulation of the dynamic system can be found in Boehl (2020).

To provide some quantitative impression given our model, for $\nu = 0.2$, a 1% risk premium shock will cause the ELB to initially bind for $k = 2$ periods, a 2% shock will cause $k = 9$ and a 3% shock an endogenous duration of $k = 12$ periods. Naturally, these numbers are subject to change, given the actual calibration of ν.

In Figure A.1 we show the reduced-form slope of the Phillips Curve, based only on the dynamic effect in response to the risk premium shock. The figure confirms that the slope is considerably high if away from the ELB, but drops once the ELB is reached and remains consistently low as the number of expected durations at the ELB increases.

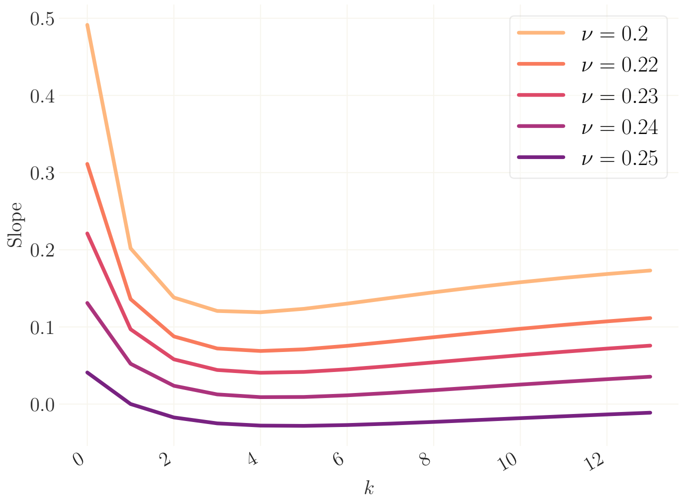

Figure A.1: Theoretical Phillips Curve slope $A_\pi(k)/A_y(k)$. This exercise ignores the static effect of the ELB, that is captured by $a_\pi(k)$ and $a_y(k)$.

Figure A.2 plots the non-recursive policy functions for π_t. For a more moderate value of ν of 0.2, the mapping $A_\pi(k)$ from $u_t \rightarrow \pi_t$ decreases with k while the linear part $a_\pi(k)$ increases in about the same fashion. As higher shocks are necessary to cause a higher k, the dynamic effect of the shock dominates the static effect and inflation falls. For $\nu = 0.22$, $A_\pi(k)$ becomes more convex, meaning that the coefficient that translates financial shocks to inflation increases for low expected durations. This effect is not offset by the static effect of a longer anticipated ELB period, which leads to a more muted inflation response. For a value of $\nu = 0.24$, the dynamic response approaches zero while for $\nu = 0.25$, $A_\pi(k)$ turns positve for values of k larger than two. As the static effect is again too weak to counteract this leads to an increase of inflation on impact, as it is captured in Figure 2.

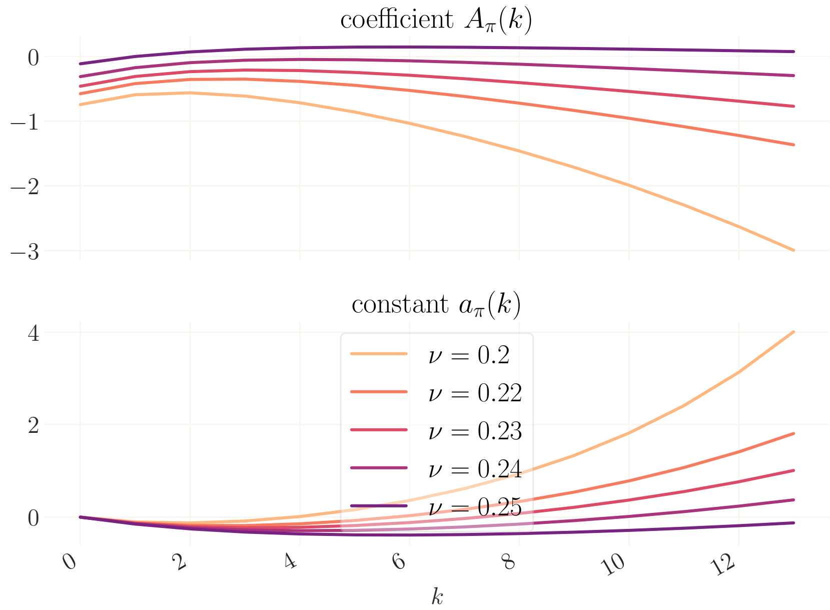

Figure A.2: Expected ZLB Duration and Impact Response

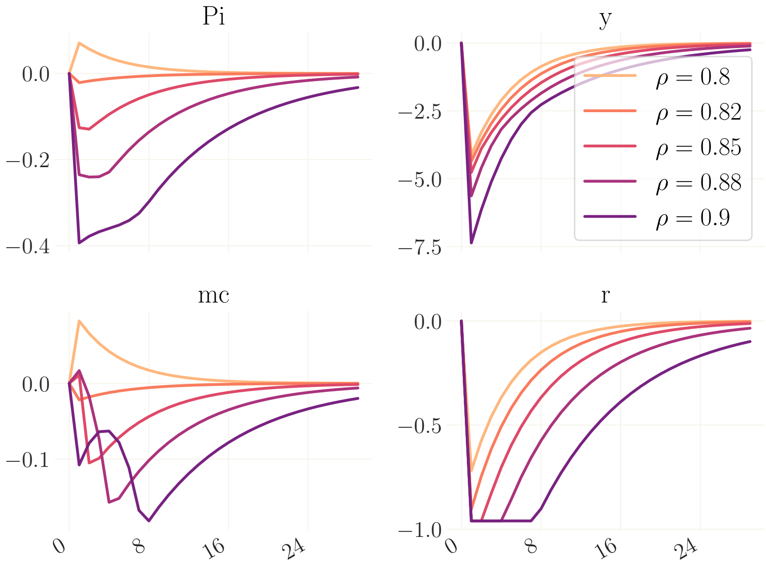

Figure A.3: Impulse responses to 2% risk premium shocks for different values of ρ, given $\nu = 0.24$.