# Monetary Policy and Speculative Asset Markets

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**Abstract**

I study monetary policy in an estimated financial New-Keynesian model extended by behavioral expectation formation in the asset market. Credit frictions create a feedback between asset markets and the macroeconomy, and behaviorally motivated speculation can amplify fundamental swings in asset prices, that potentially cause endogenous, nonfundamental bubbles and bursts. Booms in asset prices improve firms financing conditions and are therefore deflationary. These features significantly improve the power of the model to replicate empirical key moments. A monetary policy that targets asset prices can dampen financial cycles and reduce volatility in asset markets (dampening effect). This comes at the cost of creating an additional channel through which asset price fluctuations transmit to macroeconomic fundamentals (spillover effect). I find that unless financial markets are severely overheated, the undesirable fluctuations in inflation and output caused by the spillover effect more than outweigh the benefits of the dampening effect.

*Keywords:* Monetary policy, nonlinear dynamics, asset price targeting, credit constraints, bifurcation analysis

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## 1 Introduction

Despite historically low interest rates throughout the last decade, inflation rates in the United States and the Euro area remained persistently low. At the same time, asset prices have expanded substantially and continuously (Borio et al., 2018; Rungcharoenkitkul et al., 2019). A debate that repeatedly gained momentum is whether central banks should *lean against asset prices* to dampen financial cycles and mitigate potential spillovers to the economy.[1] Does expansionary monetary policy fuel financial markets while at the same time leaving inflation and output largely unaffected? If so, this could lead to overheated financial markets which in turn create a potential threat to the economy, and which again would call for monetary accommodation.

A situation in which price stability and financial stability require opposite actions poses an inherent challenge for monetary policy. Most policy proposals either aim to dampen (nonfundamental) asset price fluctuations, or seek to suppress the transmission of financial cycles to the macroeconomy. However, if both targets are interconnected – hence if macroeconomic volatility feeds back to asset prices and vice versa – both proposals are at most second best.

The debate on the role of asset prices for monetary policy also overlooks a second important point: most of the analysis is based on models with fully rational agents. Either, it is assumed that nonfundamental volatility in asset prices is stochastic (e.g., Bernanke and Gertler, 2000), or sophisticated equilibrium selection strategies are applied to establish rational sunspot bubbles (Galí, 2014; Miao and Wang, 2018). Both approaches can yield misleading policy implications. Modeling asset price fluctuations as purely stochastic might ignore potential feedback effects from the economy to asset prices. In particular, such a stochastic approach ignores that macroeconomic conditions may be the driver of an overly-optimistic or pessimistic evaluation of asset prices. In contrast, the assumption of "rational bubbles" in the spirit of Galí (2014) entails some counterintuitive implications on the co-movement

[1] For example Poole (1970); Cecchetti (2000) and Borio and Lowe (2002).

of interest rates and asset prices.

This paper addresses these points using a microfounded financial New Keynesian model in which asset prices can have macroeconomic effects through credit frictions. Much in line with the bulk of the literature on financial frictions (Christiano et al., 2010, 2014; Davis and Taylor, 2019), a boom in asset prices improves the financial conditions of firms and acts similar to a supply shock. If firms use loans from a financial intermediary to leverage their profits, and pledge their equity as collateral, borrowing conditions and finance costs depend on the value of firms' equity. If the value of equity is reflected in asset prices, an exogenous increase in asset prices reduces financing costs and is preceded by a decline in inflation.[2]

I extend this model with a behavioral framework of asset trading. Recent studies on survey evidence of market expectations stress the relevance of nonfundamental expectations for asset pricing. Greenwood and Shleifer (2014); Adam et al. (2017, 2018) highlight the strong empirical correlation between current and expected returns. This observation is at odds with rational expectations, which suggests an obverse relationship. Similarly, evidence from the behavioral laboratory (Hommes et al., 2005; Hommes, 2011; Assenza et al., 2013) documents that trading patterns in experimental asset markets can be summarized by simple, heterogeneous forecasting rules instead of complex cognitive mechanisms. In line with these findings, Adam et al. (2016); Winkler (2019) show that introducing bounded rationality into otherwise standard models significantly improves their empirical performance. This result is backed by empirical evidence from vector autoregression models (e.g. Abbate et al., 2016). These findings challenge the conventional view on a monetary policy that targets asset prices: nonfundamental fluctuations in asset markets that are driven by behavioral motives are no longer exogenous but may be a response to variations in macroeconomic aggregates. Then, episodes of (overly) expansion-

[2] Another potential channel is a wealth effect that works through aggregate demand (Bernanke and Gertler, 2000): increasing stock prices raise the nominal value of assets held by households, which amplifies consumer demand. Such effect is ruled out in a representative agent framework where seller and buyer are identical and changes in asset prices level out to zero in aggregate.

ary monetary policy can indeed stimulate optimistic and extrapolative behavior that then evolves independently, and, by itself, can become a driver of economic dynamics.

The behavioral model of financial market interaction used in this paper draws on the behavioral finance literature, in particular on Brock and Hommes (1998); LeBaron (2006); Hommes (2006). This type of model is motivated by first principles and supported by the evidence from the laboratory as well as the empirical facts on asset market expectations. Crucially, this behavioral extension allows for extreme cases such as herding behavior to be captured, and features expectation dynamics that can be self-reinforcing. This can cause large and persistent swings in asset prices that are not backed by economic fundamentals. Very strong behavioral beliefs can result in endogenous asset price dynamics in the sense that they show cyclic or chaotic patterns even absent any structural shocks.

The model presented here reproduces endogenous recurring financial crises as an equilibrium outcome. These crises emerge from dislocations in asset markets that are closely tied to the outcomes of macro-markets. For example, a feedback loop can arise if monetary policy responds reduces the interest rate in response to falling prices. As a consequence to lower interest rates, asset prices will rise. Myopic participants in the asset market may extrapolate this increase in asset prices into the future, additionally boosting asset prices. In my model, an increase in asset prices triggers a decrease in firms' marginal financing costs, ultimately causing deflationary pressure. However, if inflation declines in response to rising asset prices, the central bank may again be prompted to further reduce the interest rate. A self-enforcing cycle emerges that leads to financial "bubbles". The degrees of freedom that arise from introducing boundedly rational behavior are limited by assuming the expectation of markets other than financial markets are rational conditional on the outcome of asset markets.

In line with the results from Winkler (2019), I find that incorporating a behavioral model of speculation into a model with credit friction allows for a row of empirical key moments to be captured, not only regarding the real and monetary sides of the economy but also regarding the dynamics of asset prices and asset price expectations. The parameter values obtained from estimating the model strongly suggest that asset prices play a notable role in firms' price-setting decisions. At the same time, the parameter estimates imply a high degree of trend extrapolation in the asset market, which is aligned with the empirical literature on expectations formation.

I find that a central bank that seeks to reduce volatility in inflation tends to amplify fluctuations in asset prices for two reasons. First, in my model asset prices are primarily driven by interest rate variations. If the central bank reacts strongly to inflation, the volatility of interest rates increases, reflecting in higher asset price volatility. Second, my model predicts that asset price booms are deflationary. Hence, the central bank responds to asset price increases by reducing interest rates, which amounts to an additional boost for asset prices. Therefore, a monetary policy that temporally reacts in an overly expansionary manner to deflationary pressures can by itself stimulate asset price hikes, thereby constituting a source of financial cycles. Potentially, this cycle not only causes deflationary pressure as a second-round effect but comprises the risk of future financial crises.

Because interest rates are a major determinant of asset prices, a monetary policy that targets asset prices can dampen the excess volatility of financial markets. However, such policy constitutes an additional link between asset prices and the macroeconomy – interest rates respond to asset prices and, thereby, affect all other markets. I document that in general equilibrium, this spillover effect is stronger than the dampening effect and overall, an asset price targeting strategy comes with an increase in the volatility of inflation and output. Instead, policymakers can decide not to fight asset price volatility, but to limit the spillovers. However, the scope for such a policy is also limited: reducing the response of inflation to nonfundamental asset price fluctuations amplifies the spillover to output, and vice versa. This casts doubt on the usefulness of a monetary policy strategy that "leans against asset prices".

These results contrast with Adam and Woodford (2018), who find a positive role for monetary policy in targeting housing prices if expectations are not fully rational. The differences in the results are possibly because housing prices more strongly affect the demand side of the economy, and, hence, are inflationary. My analysis goes beyond the work of Winkler (2019) because the interaction of boundedly rational markets and monetary policy in my model allows to replicate fully endogenous financial crisis patterns that slowly build up and then burst suddenly, thereby spreading hazards throughout the economy. I explicitly study the role of monetary policy in preventing such endogenous crises – or at least its role in dampening their harm. In such cases dampening the feedback from monetary policy to asset markets is indeed favorable; however, the scope of such a policy is again limited by the amplification of the spillover effect of fluctuations from asset markets to the macroeconomy.

This work adds to a larger literature that studies the macroeconomics of asset prices. The bulk of this literature mainly focuses on rational expectations.[3] Miao et al. (2012) and Miao et al. (2016), building on a Bayesian model with rational stock price bubbles, find that the feedback between asset prices and asset price expectations plays a key role in the formation of a stock price bubble. They report that about 20% of the variance of GDP can be explained through fluctuations in stock prices. The work of Galí (2014) most prominently represents a series of studies on rational bubbles and monetary policy. A key assumption is, that the nonfundamental part of a bubble grows *proportionally to* the policy rate, which does not align with the common intuition that bubbles emerge as a consequence of overly expansionary monetary policy. Notably, Assenmacher and Gerlach (2008) find that asset prices react almost instantaneously to monetary surprises, and provide evidence in support of the conventional view that bubbles build up when monetary policy is rather loose. Furthermore, the authors find that shocks to asset prices can affect both, GDP and credit volume and that stock price fluctuations can explain about 10% to 15% of the variance of GDP.

[3] For a more complete survey see Allen et al. (2011).

The remainder of this paper is structured as follows. Section 2 introduces the macroeconomic model. I discuss theoretical insights in Section 3 and present simulation results in Section 4. Section 5 provides an in-debt policy analysis. Section 6 concludes.

## 2 Model

I propose a small-scale New Keynesian model with credit frictions as in Bernanke et al. (1999, henceforth BGG). The model features households, two types of firms, a financial intermediary, a central bank and an asset market with boundedly rational traders. Credit frictions affect firms through a cost channel, and provide a medium through which asset price fluctuations caused by boundedly rational traders influence macroeconomic fundamentals.

Firms are divided into retail firms and wholesale firms, the latter being operated by risk-neutral entrepreneurs. Following Ravenna and Walsh (2006), wages must be paid before production takes place, implying that wholesale firms have a demand for external financing. These firms borrow from financial intermediaries that face a costly state verification problem and, thus, require a risk premium to insure against the risk of defaulting firms.

The model deviates from full rationality in two dimensions. First, traders in the asset market are assumed to be boundedly rational. Their presence can lead to speculative dynamics and amplify the effects of macroeconomic variables in response to economic shocks.[4] Second, all other agents – households and firms – are assumed to form *conditionally model consistent rational expectations*: they are unaware of the presence of boundedly rational traders and behave *as if* all agents in the model are rational. To ensure consistency, these "rational" agents cannot directly observe the state of aggregate exogenous shocks; otherwise, they could learn that asset prices are not fully rational. This section first characterizes the

[4]One important advantage of using a parsimonious model over a larger model as for example in Christiano et al. (2005) and Smets and Wouters (2007) is that it allows to isolate and analyze the underlying key economic mechanisms as e.g. in Sims and Wu (2019).

model environment and presents the equations that define the general equilibrium (Subsections 2.1 to 2.4).[5] Subsection 2.6 then presents in detail the interaction of conditionally rational and boundedly rational agents.

### *2.1 Households*

Households are ex-ante identical and maximize the expected present value of future utility derived from consumption and leisure. $C_t$ denotes the composite consumption good traded at prices $P_t$, and $H_t$ are labour hours supplied by households in a competitive labor market at real wage rate $W_t$. Furthermore, households can deposit monetary savings $D_t$ with a financial intermediary for which, in equilibrium, they receive the gross nominal interest rate $R_{t+1}$ in the next period. The maximization problem for household $i$ is:

$$\max E_t \sum_{s=t}^{\infty} \beta^{s-t} \zeta_{i,t} \left( \ln C_{i,s} - \frac{H_{i,s}^{1+\eta}}{1+\eta} \right) \tag{1}$$

s.t. the budget constraint

$$C_{i,t} + D_{i,t} \leq W_t H_{i,t} + R_t \frac{P_{t-1}}{P_t} D_{i,t-1} + \mu \int_0^{\bar{\omega}_t} \omega H_t / X_t dF(\omega) \; \forall t = 1, 2, ... \tag{2}$$

where $\eta$ is a measure of the disutility of labor. The term $\mu \int_0^{\bar{\omega}_t} \omega H_t / X_t dF(\omega)$ represents ”audition costs” that incur when a firm defaults and are assumed to be equally distributed among households. Subsection 2.2 and Appendix A provide details. Throughout, I use $E_t$ as the expectations operator for conditionally model consistent rational expectations, that are treated in detail in Section 2.6.

Each household $i$ is subject to an idiosyncratic preference shock $\zeta_{i,t}$. Define $d_{i,t} = \log\left(\frac{\zeta_{i,t}}{\zeta_{i,t+1}}\right)$ and assume that $d_{i,t} = \rho_d d_{i,t-1} + \epsilon_{i,t}$ follows an AR(1) process. Assumption 1 is key for the formation of conditionally rational expectations, justifying the co-existence of rational and boundedly rational agents:

[5] A full exposition of all model equations and the derivation of their log-linearized equivalents can be found in Appendix C.

**Assumption 1.** *Aggregate shock processes are unobservable to individual agents. Each agent $i$ can only observe his individual realization $d_{i,t}$ but neither the aggregate $d_t$ nor the stochastic innovations $\epsilon_{-i,t}$ of any other agent.*

This assumption implies that, although each individual household reacts to their realization of their shock, they are unable to identify the sources of aggregate economic fluctuations. In particular, households are unable to distinguish between endogenous variations in asset prices and variations caused by exogenous disturbances.

To allow for straightforward aggregation, assume that all shock innovations $\epsilon_{i,t} = \epsilon_t^d$ are ex-ante identical. This assumption implies that all households make identical consumption-savings decisions and the household sector can be viewed as a representative household with the aggregate preference shock

$$d_t = \rho_d d_{t-1} + \epsilon_t^d \tag{3}$$

also following an AR(1) structure.

The maximization problem of households yields a usual Euler equation and labor supply equation. Aggregation yields

$$\exp(d_t)C_t^{-1} = E_t\left\{\beta R_{t+1}\frac{P_t}{P_{t+1}}C_{t+1}^{-1}\right\}, \tag{4}$$

$$H_t^\eta = \frac{W_t}{C_t}. \tag{5}$$

Finally, assume that the transversality condition is satisfied and households' budget constraints hold with equality.

### *2.2 Wholesale firms and retailers*

Firms are separated into wholesale and retail firms for reasons of analytical tractability. Each one wholesaler and one retailer form a firm consortium that pools its resources and profits. This – arguably artificial – separation of wholesale and retail firms is not strictly necessary but simplifies the exposition. In particular, it allows for a standard derivation of the New Keynesian Phillips curve. Firms are indexed by $j$, and each wholesale firm sells its product under perfect competition to the retail firm in its consortium. The retail firms diversifies these goods, and they are sold under monopolistic competition. Each wholesaler is operated by a risk-neutral entrepreneur and choses equity $N_{j,t}$ such that the expected discounted sum of dividends over equity is maximized. They produce the same type of goods $Y_{j,t}$ using a CRS technology with labor as the only production factor

$$Y_{j,t} = \omega_{j,t} A_t H_{j,t}, \tag{6}$$

where $\omega_{j,t}$ is a firm-specific idiosyncratic productivity shock and $A_t$ is an aggregate technology shock, the log of which follows an AR(1) process:

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_t^a. \tag{7}$$

Similar to Ravenna and Walsh (2006), assume that workers are paid before production takes place. The timing of events is as follows. At the beginning of each period, aggregate shocks realize. Then, wholesalers choose their labor inputs and agree on a loan contract with financial intermediaries. Firm-specific shocks $\omega_{j,t}$ realize after workers are paid. Then, goods are produced, diversified and sold in the final goods market. Finally, each consortium decides on its dividend payments $\Theta_{j,t+1}$ which are distributed at the beginning of the next period.[6]

Shares of the firms' equity are sold to financial intermediaries. Firms know that intermediaries are risk-neutral and assume that they act under rational expectations. Hence, each firm $j$ believes that the following no-arbitrage condition for the

[6] I assume that dividend payments can be also negative. In this case, a firm obtains funding resources from their shareholders who are willing to increase the firm's equity if the net present value of future dividends given the additional investment is positive. Using this assumption ensures a closed form solution of the model. See e.g. Martin and Ventura (2010) for a similar approach.

real price $S_{j,t}$ of their shares holds:

$$S_{j,t} = E_t \left\{ \frac{\Theta_{j,t+1} + S_{j,t+1}}{R_{t+1}\frac{P_t}{P_{t+1}}} \right\}. \tag{8}$$

Equation (8) states that the real share price must equal the return on investment in shares, that is, the sum of the expected dividend $\Theta_{j,t+1}$ and the expected selling price in the next period $S_{j,t+1}$, discounted by the expected return on the investment in the risk-free bond $R_{t+1}\frac{P_t}{P_{t+1}}$. In other words, the gross return from investing one monetary unit into firm $i$'s shares would be equal the real risk free rate. Importantly, this also implies that $S_{j,t}$ would be a recursive representation of the present value of firms' expected future dividends $S_{j,t} = E_t \sum_{s=1}^{\infty} \frac{\Theta_{j,t+s}}{\prod_{k=1}^{s} R_{t+k}\frac{P_{t+k-1}}{P_{t+k}}}$. This relationship is important because it implies that maximizing discounted dividends over equity is equivalent to maximizing the inverse book-to-market ratio, that is, the price of shares over equity.

Note that Equation (8) is the law of motion for $S_{j,t}$ as perceived by conditionally rational firms and households. However, because asset pricing is assumed to *not* be fully rational ex-post, Equation (8) does *not* hold in general equilibrium. This statement is revisited in greater detail in Subsection 2.6, which introduces co-existing boundedly rational agents.

Denote by $X_t$ the gross markup that retailers charge over the price of wholesale goods. Hence, $X_t^{-1}$ can be interpreted as the relative price that a wholesale firm receives for one unit of his goods, i.e. the price of a good in wholesale relative to its price in retail. The aggregate return on working capital across firms (i.e. the return from investing one monetary unit in labor) can be expressed in terms of the markup as $A_t(X_t W_t)^{-1}$.

The amount of external funds $B_{j,t}$ demanded by firm $j$ is given by the difference between $j$'s working capital $W_t H_{j,t}$ and her equity $N_{j,t}$:

$$B_{j,t} = W_t H_{j,t} - N_{j,t}. \tag{9}$$

As in BGG, I assume that external financing is subject to a costly state verification problem, whereby banks cannot observe the state of the entrepreneur unless they pay a monitoring cost. Firms pledge the proceedings of their production as collateral and, to minimize agency costs, the bank only pays the monitoring fee when the entrepreneur defaults to seize his collateral. To compensate for possibly incurring monitoring costs, the optimal financial contract includes a risk premium over the risk-free interest rate.

The risk premium increases with the leverage ratio $\frac{N_{j,t}}{W_t H_{j,t}}$ – that is, the ratio of equity to total working capital – because default becomes more likely and, hence, loans are more risky. Appendix A provides the full optimization problem of entrepreneurs. As it is shown there, optimality requires that the inverse of the book-to-market ratio, given by $N_{j,t}/S_{j,t}$, in terms of the leverage ratio satisfies

$$\frac{S_{j,t}}{N_{j,t}} = z_1 \left( \frac{N_{j,t}}{W_t H_{j,t}} \right). \tag{10}$$

where $z_1$ is a decreasing function, i.e. $z_1' < 0$. Respectively, the market value of a firm over its book value (that is, the value of its equity) increases if leverage decreases. Intuitively, this condition must hold because firms could otherwise increase their return on equity by adjusting the borrowing volume relative to their equity.

As also shown in Appendix A, no-arbitrage requires that the rate of return on working capital in aggregate equals the (real) rate on external funding:

$$\frac{A_t}{X_t W_t} = z_2 \left( \frac{N_{j,t}}{W_t H_{j,t}} \right) R_{t+1} \frac{P_t}{P_{t+1}}, \tag{11}$$

which again can be expressed as a function $z_2$ with $z_2' < 0$ of firms' leverage. Denote the derivatives of $z_1$ and $z_2$ in the steady state by $z_1' = -\nu_1$ and $z_2' = -\nu_2$ respectively. Then, after log-linearizing and combining Equations (10) and (11), the log-deviation of the markup over wholesale prices, $x_t$, can be represented as

$$x_t = \nu s_t - \psi y_t - (r_{t+1} - \pi_{t+1}) + \frac{(1+\nu)(1+\eta)}{1+2\nu} a_t, \tag{12}$$

where lowercase letters denote log-deviations from the steady state, $\nu = \frac{\nu_1}{1-\nu_2}$ is the elasticity of the markup with respect to asset prices and $\psi = (1+\nu)(1+\eta) + \nu$ is the elasticity of the markup with respect to output.

Because wholesale firms are perfectly competitive, they set identical prices and, hence, offer the same expected return on equity. Consequently, the stock market evaluations of all firms' shares are identical and determine the level of firms' equity. Higher asset prices lower the cost of borrowing that in turn reduces marginal costs and, finally, the final goods price.

Retailers take prices as given, buy the wholesale output and differentiate the output at no cost. Differentiation allows each retailer to have some degree of market power. Similar to households, assume that retailers can only observe the total productivity level $\omega_{j,t} A_i$ of their associated wholesaler, but cannot distinguish between the two factors. Hence, the aggregated technology level is unobservable to the individual retailer when setting prices. Finally, households purchase and consume CES aggregates of these retail goods. To motivate price inertia, assume that retail firms are subject to nominal rigidities as in Calvo (1983). The solution to the price setting problem of the retailers is the New Keynesian Phillips curve, expressed in terms of the markup $X_t$.[7]

Equation (12) is a key equation of this model. In terms of deviations from the steady state, it holds that marginal costs for retailers are the reciprocal of the markup $x_t$ charged on wholesale goods. Marginal costs can be subdivided in two components: real factor costs, represented by $\psi y_t + \frac{1+\eta}{1+\nu} a_t$, and financing costs captured by $\nu s_t - (r_{t+1} - \pi_{t+1})$. Real factor costs cover the costs of employing labor that are increasing in wages $W_t$ and labor hours $H_t$. Financing costs include

[7]See Bernanke et al. (1999) for details on this particular solution.

the risk free real rate $(r_{t+1} - \pi_{t+1})$ and a risk spread that depends on asset prices $s_t$. The role of real factor costs for retailers' price setting is standard, as is its propagation through the economy. A decrease in the risk free real rate leads to a one-to-one decline in marginal costs, that, through the Phillips curve, is reflected by a decline in the price level. Respectively, an increase in asset prices $s_t$ eases the financial conditions of wholesale firms and, thereby, reduces the risk spread. This reduction is reflected in a decrease in marginal costs for retailers and, via the Phillips curve, causes inflation to fall.

### *2.3 Financial intermediation*

Assume a continuum of financial intermediaries indexed by $k$ that operate under perfect competition. Intermediaries have access to central bank liquidity, which is provided at the nominal reserve rate $R_{t+1}$. Households provide deposits to financial intermediaries for which they also receive $R_{t+1}$. Each intermediary obtains a share of deposits $D_{k,t}$ that can be either invested in the financial market, or given to firms as credit. Hence, the balance sheet of $k$'s is given by $D_{k,t} = S_t J_{k,t} + B_{k,t}$, where shares $J_{k,t}$ are assumed to be provided in fixed unit supply, $\sum_k J_{k,t} = 1$.

The investment decision of intermediary $k$'s is carried out by a trader $k$ which is assumed to be *boundedly rational.* Expectations are heterogeneous across traders. Their setup is treated in full detail in Section 4.2. Although rational firms, assuming that intermediaries are also rational, believe that Equation (8) holds, the intermediaries actual demand for assets is given by aggregating over the different expectations $\hat{E}_{k,t} S_{t+1}$. Up to a first order approximation, this yields

$$S_t = \hat{E}_t \left\{ \frac{\Theta_{t+1} + S_{t+1}}{R_{t+1} \frac{P_t}{P_{t+1}}} \right\}, \tag{13}$$

where $\hat{E}_t$ denotes the aggregate expectation over the different type-$k$-traders. Note that this equations is the same equation as Equation (8), but with the $\hat{E}_t$ operator. In general equilibrium only (13) holds whereas (8) is the law of motion as perceived by firms and households.

## 2.4 The central bank

The central bank follows a simple monetary policy rule, according to which the nominal interest rate responds to inflation and – possibly – to the output gap and/or to asset prices. The case in which monetary policy reacts to asset prices is referred to as asset price targeting (APT) or "leaning against asset prices". The linearized monetary policy rule is

$$r_{t+1} = \phi_\pi \pi_t + \phi_y \hat{y}_t + \phi_s s_t, \tag{14}$$

where $\hat{y}_t = y_t - y_t^{f,RE}$ is the output gap in period $t$ with $y_t^{f,RE} = a_t$ denoting output under flexible prices and rational expectations.[8]

Note that I consider a monetary policy rule that responds to the *level* of asset prices, instead of asset price inflation. Importantly, the level of asset prices directly influences macroeconomic aggregates and, conversely, it is also the level of asset prices that is determined by future interest rates and, potentially, traders' behavioral sentiments. The central bank cannot ex-ante distinguish whether fluctuations in asset prices reflect (expected) changes in fundamentals or whether they arise as a form of speculative "bubble". Hence, the central bank must *always* react to changes in asset prices independent of the source of these fluctuations.

The central bank maximizes households' welfare $\mathcal{W}_t$ which, up to a second order approximation and in terms of discounted expected utility, is given by

$$\mathcal{W}_t = E_t \sum_{s=0}^{\infty} \beta^s \left( \frac{U_{t+s} - U}{U_c C} \right) \propto E_t \sum_{s=0}^{\infty} \beta^s \mathcal{L}_{t+s}. \tag{15}$$

Here, $\mathcal{L}_t$ denotes the central bank's loss function at time $t$ as defined by

$$\mathcal{L}_t = \pi_t^2 + \lambda \hat{y}_t^2. \tag{16}$$

[8]The representation for the allocation under rational expectations follows from the combination of log-utility with a CES production function when prices are fully flexible and letting $\omega_j \to 1$. Assuming the usual government subsidy to offset the steady state markup, this allocation is first-best efficient and all real prices and quantities are independent of monetary policy.

Then, the relative weight on the output gap can be expressed as $\lambda = \frac{\kappa(1+\eta)}{\epsilon}$.[9] It follows that the central bank has no motivation to care for fluctuations in asset price per-se, but only as much as they are relevant for variations in inflation and the output gap. Note that it is assumed that speculative behavior on the asset market is assumed to still result in a stationarity fundamental steady state and that the government issues steady state subsidies to correct for the distortions arising from nominal rigidities and financial frictions. Similar to Caines and Winkler (2019), this representation is independent of whether or not financial markets are fully rational as long as the assumption on a stationary steady state is satisfied.

For simplicity, I abstract from any further economic activity of the treasury.

### *2.5 General equilibrium*

In general equilibrium, the linearized economy is characterised by the following set of equations, where lowercase letters denote the log-deviation of a variable from its steady-state value:

$$\pi_t = \beta E_t \pi_{t+1} - \kappa x_t, \tag{17}$$

$$y_t = E_t y_{t+1} - (r_{t+1} - E_t \pi_{t+1}) + d_t, \tag{18}$$

$$x_t = \nu s_t - \psi y_t - (r_{t+1} - E_t \pi_{t+1}) + \frac{1+\eta}{1+\bar{\nu}} a_t, \tag{19}$$

$$r_{t+1} = \phi_\pi \pi_t + \phi_y \hat{y}_t + \phi_s s_t, \tag{20}$$

$$s_t = \beta \hat{E}_t s_{t+1} - (r_{t+1} - E_t \pi_{t+1}). \tag{21}$$

Equation (17) is the New Keynesian Phillips curve which is derived from staggered price setting. Inflation is dynamically linked to the markup (the inverse of marginal costs), with $\kappa$ being the slope of the Phillips curve. Equation (18) is the dynamic IS-curve. The connection between the textbook model and the financial sector is represented by Equation (19), which describes an explicit role for asset

[9] $\epsilon = 10$ is the elasticity of substitution between final goods. Details of the proof can be found in Lieberknecht (2019) for an adaptation of this model.

prices $s_t$. Equation (20) is the linearized monetary policy rule. The linearized aggregate no-arbitrage-equation for asset prices, Equation (21), arises from Equation (13), with $\hat{E}_t$ as the aggregate and not necessarily rational expectations operator left to be specified.[10] The law of motion of the two shock processes is given by the Equations (3) and (7).

Crucially, the above equations hold in general equilibrium, but are not known to any agent. The following subsection relaxes the assumption of full rationality and establishes a law of motion for the model with rational and boundedly rational traders. The presence of bounded rationality is commonly seen as a necessary condition for speculative dynamics. To understand why, let speculation be defined as trading activity where traders seek profits from short-term fluctuations in asset prices. In a rational world with full and symmetric information, such speculative profits would be impossible because all traders perfectly anticipate any price changes. For this reason I use the terms *boundedly rational* and *speculative* interchangeably throughout this paper.

### *2.6 Coexistence of rational and boundedly rational agents*

Define the rational benchmark as the case in which *all* agents are rational, hence when $\hat{E}_t s_{t+1} = E_t s_{t+1}$. In this case Equations (17) – (21) can be represented as a 3-dimensional system

$$\mathbf{M}\mathbf{x}_t = \mathbf{P}E_t\mathbf{x}_{t+1} + \mathbf{v}_t, \tag{22}$$

with $\mathbf{x}_t = \{\pi_t, y_t, s_t\}$ and $\mathbf{v}_t = \{a_t, d_t\}$. The components of $\mathbf{M}$ and $\mathbf{P}$ are described in Appendix B.

One possible strategy for introducing boundedly rational expectations in asset prices is to deviate from complete rationality in all markets equally, such as in Evans and Honkapohja (2003) or De Grauwe (2011). However, fully abstracting

[10]Note that when log-linearizing (13) the steady state deviation of $\Theta_t$ is prefixed by the coefficient $1 - \beta$, which renders this term negligible small given any realistic volatility of dividends.

from rational expectations makes it difficult to single out and analyze the role of speculative asset markets for the model dynamics.

At the same time, the work of Greenwood and Shleifer (2014); Adam et al. (2017, 2018) on expectation formation on asset markets lends a strong motivation of boundedly rational asset markets. However, the survey data used in these studies hardly justify speculative behavior in aggregate good markets. Further, a nontrivial problem with models of boundedly rational expectations is that they come with additional degrees of freedom that typically cannot be disciplined easily. This problem is closely related to the critique of the *wilderness of bounded rationality*. Therefore, I introduce boundedly rational expectations in the asset market *only*.

However, the interaction of boundedly and fully rational agents can also be highly nontrivial. Using a reduced form asset pricing model, Boehl and Hommes (2021) show that adding fully rational agents to a market with sentiment trading dynamics can even amplify speculative dynamics because rational agents perfectly predict the behavior of sentiment traders. The complexity of rational expectations equilibria with quasi-periodic and potentially chaotic dynamics in their framework requires the authors to employ advanced numerical methods. For the sake of analytical tractability, I instead assume that those agents that are assumed to be rational form *conditional model consistent rational* expectations: their expectations are consistent with the realized values of inflation, output and asset prices.[11] At the same time, their expectations are consistent with all model equations but *conditional* on all other agents are also being rational. Therefore, conditional model consistent rational expectations indeed are fully rational in the absence of any other nonrational agents.

Let the information set of rational agents be $\mathcal{I}_t := \{y_t, \pi_t, s_t\} = \mathbf{x}_t$, which takes into account that by Assumption 1 the aggregate state of the exogenous processes is not known to any agent. The following is assumed additionally.

[11] A similar approach is chosen in Adam and Woodford (2018) who consider nonrational beliefs in the housing market in an otherwise rational expectations model conditional on the outcome of the housing market.

**Assumption 2.** *The distribution of agent types is unobservable to any of the agents.*

This assumption is necessary as otherwise rational agents would be able to use the information on the distribution of agent types to infer on potential biases in asset prices. If agents (in particular financial intermediaries) knew that asset prices do not reflect a correct evaluation of the firms value, this would add another layer of complexity to the firms pricing decision. Hence, Assumptions 1 and 2 together ensure that rational agents are unable to distinguish between exogenous shocks and speculation dynamics that cause fluctuations in asset prices.

Inflation expectations are formed by firms and households, which are assumed to be (conditionally) rational. They are hence homogeneous and can be denoted by $E_t[\pi_{t+1}|\mathcal{I}_t]$ without any subscript. Expectations on output $E_t[y_{t+1}|\mathcal{I}_t]$ are formed by households and, accordingly, are also homogeneous and rational. Rational agents form expectations on asset prices only in so far, as they matter implicitly for inflation and output. Because neither the Phillips curve nor the Euler equation depends directly on expectations on asset prices, they do not show up in any equation. As asset prices are determined in the boundedly rational asset market (represented in Equation (21)), only the boundedly rational expectations on asset prices, $\hat{E}_t s_{t+1}$ show up explicitly. It follows that the actual law of motion (ALM) of the system is given by

$$\mathbf{M}\mathbf{x}_t = \mathbf{P} \begin{vmatrix} E_t\left[\pi_{t+1}|\mathcal{I}_t\right] \\ E_t\left[y_{t+1}|\mathcal{I}_t\right] \\ \hat{E}_t s_{t+1} \end{vmatrix} + \mathbf{v_t}. \tag{23}$$

We are now concerned with solving for $E_t[\pi_{t+1}|\mathcal{I}_t]$ and $E_t[y_{t+1}|\mathcal{I}_t]$. Define the current estimate (or, nowcast) of the states of the aggregate exogenous processes conditional on $\mathcal{I}_t$ as

$$\hat{\mathbf{v}}_\mathbf{t} = E_t[\mathbf{v_t}|\mathcal{I}_t]. \tag{24}$$

Combining this with (22), the perceived law of motion (PLM) for rational agents is

$$\begin{vmatrix} \mathbf{M} & \mathbf{0_v} \\ \mathbf{0_x} & \boldsymbol{\rho} \end{vmatrix} \begin{vmatrix} \mathbf{x_t} \\ \hat{\mathbf{v}}_t \end{vmatrix} = \begin{vmatrix} \mathbf{P} & \mathbf{0_v} \\ \mathbf{0_x} & \mathbf{I} \end{vmatrix} E_t \begin{vmatrix} \mathbf{x_{t+1}} \\ \hat{\mathbf{v}}_{t+1} \end{vmatrix}, \tag{25}$$

where $\boldsymbol{\rho}$ is a diagonal matrix of the autocorrelation parameters of exogenous processes. This simply expresses System (22) in terms of the estimated states of the exogenous processes instead of the respective true states. Denote the (linear) rational expectations solution of (25) by $\boldsymbol{\Omega}$.[12] Model consistency of rational expectations requires that

$$\begin{vmatrix} \pi_t \\ y_t \end{vmatrix} = \boldsymbol{\Omega}\hat{\mathbf{v}}_t \qquad \text{and} \qquad \begin{vmatrix} E_t[\pi_{t+1}|\mathcal{I}_t] \\ E_t[y_{t+1}|\mathcal{I}_t] \end{vmatrix} = \boldsymbol{\Omega}\boldsymbol{\rho}\hat{\mathbf{v}}_t. \tag{26}$$

It follows directly that, even without explicitly solving for $\hat{\mathbf{v}}_\mathbf{t}$, the relationships in (26) can be used to express the conditional expectations on inflation and output in the next period in terms of inflation and output in the current period:

$$\begin{vmatrix} E_t[\pi_{t+1}|\mathcal{I}_t] \\ E_t[y_{t+1}|\mathcal{I}_t] \end{vmatrix} = \boldsymbol{\Omega}\boldsymbol{\rho}\boldsymbol{\Omega}^{\mathbf{-1}} \begin{vmatrix} \pi_t \\ y_t \end{vmatrix}. \tag{27}$$

Plugging this result in (23), we can represent the ALM in terms of the true exogenous states, $\mathbf{v_t}$, and traders' expectations on asset prices, $\hat{E}_t s_{t+1}$.[13] Define this mapping as $\boldsymbol{\Psi} : (\boldsymbol{\Phi}, \boldsymbol{\phi}) \rightarrow \mathbb{R}_{3\times 3}$, where $\boldsymbol{\Phi}$ is the set of model parameters $(\beta, \nu, \psi, \kappa, \rho_a, \rho_d)$ and $\boldsymbol{\phi}$ the monetary policy parameters $(\phi_\phi, \phi_s)$:

$$\begin{vmatrix} \pi_t \\ y_t \\ s_t \end{vmatrix} = \boldsymbol{\Psi} \begin{vmatrix} d_t \\ a_t \\ \hat{E}_t s_{t+1} \end{vmatrix}, \tag{28}$$

The system in (28) represents the solution for inflation, output and asset prices as a function of the actual exogenous shocks and boundedly rational beliefs $\hat{E}_t s_{t+1}$.

[12] The derivation of $\boldsymbol{\Omega}$ is provided in Appendix B.

[13] Note that nonsingularity of $\boldsymbol{\Omega}$ is implied. Singularity of $\boldsymbol{\Omega}$ would mean that either $\pi$ or $y_t$ need to be zero independently of $\mathbf{v}_t$, or either $a_t$ or $d_t$ to have no effect on $\mathbf{x}_t$.

## 3 Theoretical insights

The deep model parameters are calibrated to values commonly found in the literature. The quarterly household discount rate $\beta$ is set to 0.99, and the autocorrelation parameters of the structural shocks are $\rho_a = 0.9$ and $\rho_d = 0.7$, respectively. Consistent with Woodford (2003), I set the inverse Frisch labor supply elasticity $\eta$ to 0.3 and firms' probability $\theta$ to change prices in a given period to 0.66. Hence, the slope of the Phillips curve is $\kappa = (1-\theta)(1-\beta\theta)/\theta \approx 0.179$. The value for the elasticity of substitution between varieties is set to $\epsilon = 10$ Furthermore, for the baseline scenario, I choose the monetary policy response coefficients to be $\phi_\pi = 1.5$ and $\phi_s = 0$, implying that the central bank does not target asset prices. In Section 4, I provide estimates of key parameters, including an estimate of the derivative of the external financing premium in steady state $z'(\cdot) = -\tilde{\nu}$. I find that $\tilde{\nu} = 0.09$ such that $\psi = \frac{1+\eta+\tilde{\nu}}{1-\tilde{\nu}} \approx 1.52$ and $\nu = \frac{\tilde{\nu}}{1-\tilde{\nu}} \approx 0.099$.

Given these parameter settings, the weight on the output gap in the central bank loss function $\lambda = 0.023$ is very low, implying that the central bank cares much more about fluctuations in inflation than in output (or the output gap). This finding is standard (e.g. Woodford, 2003) across the broader class of small-scale New Keynesian models. Hence, the main concern of the central bank regarding asset prices should be to mitigate the spillover effects of asset price fluctuations on inflation.

The focus of this paper is to study the effects of asset price movements on welfare. The effects of conventional demand and supply shocks on inflation and output have been studied extensively in the literature. Therefore, in the following I focus on two channels: first, the pass-through of movements in asset prices to inflation and output dynamics, and second, the role of asset prices in amplifying the effects of conventional shocks. In the model presented in Equations (17) to (21) asset prices affect inflation and output only through marginal costs, which determine the firms' price-setting decision via the Phillips Curve. Hence, any exogenous increase in asset prices will have the effects of a positive supply shock, which lowers inflation and, if monetary policy responds more than one-to-one to inflation, raises output. This means that divine coincidence does not hold for asset price "shocks" and a stronger response to fluctuations in inflation will cause additional variations in output.

Equations (17) to (21) also illustrate why a monetary policy that responds to the output gap is not efficient for handling the macroeconomic effects of asset price fluctuations: imagine asset prices $s_t$ increase for some exogenous reason. Ceteris paribus, inflation will decrease, which triggers monetary policy to lower the nominal interest rate more than one-to-one. In turn, this reduction leads to a fall in the real rate which boosts aggregate demand through the Euler equation. If now $\phi_y > 0$, the central bank will have to raise nominal rates, which in equilibrium causes $r_{t+1}$ to fall less than implied by $\phi_\pi$ alone. Hence, in response to fluctuations in asset prices, the effects of $\phi_y > 0$ and $\phi_\pi$ cancel out. For this reason I will set $\phi_y = 0$ for the rest of this paper and focus the analysis on the effects of a monetary policy that directly targets movements in asset prices.[14]

In the absence of any exogenous disturbances, that is when $\mathbf{v}_t = \mathbf{0}$, the law-of-motion in (26) reduces to

$$\mathbf{x}_t = \mathbf{\Psi}_{\cdot,\mathbf{3}} \hat{E}_t s_{t+1} \qquad \text{and} \qquad s_t = \mathbf{\Psi}_{\mathbf{3},\mathbf{3}} \hat{E}_t s_{t+1}, \tag{29}$$

where $\mathbf{\Psi}_{3,3}$ can be interpreted as the root of the process that determines $s_t$.

Using this calibration, $\mathbf{\Psi}_{3,3}$ is close to a unit-root. This means that the expectations of the different agents in the asset market are strategic complementarities: an optimistic deviation of asset price expectations increases current asset prices. Higher asset prices reduce firms' borrowing cost, which in turn leads firms to lower aggregate prices. The induced fall in inflation implies a nontrivial role of monetary policy for asset prices as, in response, the central bank will cut nominal interest rates, thereby further fueling the boost in asset prices.

[14]The role of $\phi_y > 0$ for conventional shocks is discussed extensively in the literature and hence not a focus of this paper.

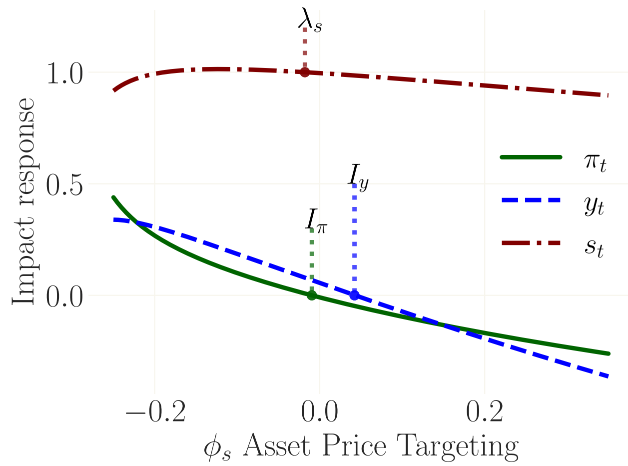


Figure 1: Direct ceteris paribus effect of a 1% change in asset price expectations on output, inflation and asset prices. The effect is shown as a function of the central bank policy coefficient to asset prices. Responses in deviations from steady state.

A large literature stresses the role of strategic complementaries for asset pricing (c.f. Bulow et al., 1985; Cooper and John, 1988). This work has shown that strategic complementarities in expectation formation can lead to large feedback and amplification effects and the associated dynamics are prone to instability.[15] Such systems can exhibit large swings and bubbles where the probability to observe a bubble increases as the characteristic root of the system approaches unity. Based on this argumentation, one can also interpret $\mathbf{\Psi_{3,3}}$ as a measure of the likelihood of bubbly episodes. Hence, a policy that aims at preventing financial bubbles should aim to reduce the magnitude of $\mathbf{\Psi_{3,3}}$. If such a policy is either unavailable or undesirable, the second best solution would be to minimize $\mathbf{\Psi_{1,3}}$ and $\mathbf{\Psi_{2,3}}$, thereby attenuating the spillover effect of asset price fluctuations on real activity.

Figure 1 shows the relationship between $\mathbf{\Psi_{\cdot,3}}$ and the monetary policy response coefficient to asset prices $\phi_s$. $\mathbf{\Psi_{1,3}}$, $\mathbf{\Psi_{2,3}}$ and $\mathbf{\Psi_{3,3}}$ can be interpreted as the general equilibrium response of $\pi_t$, $y_t$ and $s_t$, respectively, to a 1% change in asset price expectations. Negative values for $\phi_s$ imply that the central bank reduces nominal

[15]See Hommes (2011) for a review.

interest rates in response to higher asset prices. The marginal responses in endogenous variables are strong even for moderate values of $\phi_s$. $\lambda_s \approx -0.01$ marks the value of $\phi_s$ for which $\mathbf{\Psi_{3,3}} = 1$. $\mathbf{\Psi_{3,3}}$ is monotonically decreasing for positive values of $\phi_s$, suggesting that a moderate degree of asset price targeting can indeed reduce the amplification and feedback effects from higher expected asset prices to macroeconomic variables.

Note that setting $\phi_s = I_y \approx 0.05$ will entirely offset any impact of variations in asset price expectations on output (that is $\mathbf{\Psi_{2,3}} = 0$ for $\phi_s = I_y$). A monetary policy rule with $\phi_s > I_y$ triggers a decline in output in response to overly optimistic asset price expectations. Similarly, the equilibrium effect on inflation is zero when $\phi_s = I_\pi \approx -.02$. In this case, the central bank reduces the nominal rates in response to higher asset prices. Lower interest rates further reduce firms' borrowing cost, but also stimulate demand, causing a net-increase in output. In $I_\pi$, the latter (positive) effect and the direct (negative) effect through higher asset prices and lower interest rates are perfectly balanced.

In summary, when nonfundamental fluctuations in asset prices cause spillovers to macroeconomic fundamentals, the central bank faces a trade-off. By carefully targeting asset prices, policy makers can reduce the impact of asset price dynamics on inflation, but must accept an increase in dynamic feedback, which is captured by $\mathbf{\Psi_{3,3}}$. Alternatively, the central bank can choose to dampen the dynamic feedback, but such policy bears the risk to increase the impact of asset price fluctuations both on inflation and output. The following section quantitatively asses this trade-off in a behavioral framework of expectation setting.

## 4 Quantitative results

This section first presents a row of stylized facts from the data used to fit the model. Further, the model setup is completed by introducing a realistic, behaviorally grounded process for the formation of asset price expectations. I then discuss the dynamic properties of the full model and provide estimates of the model's parameters.

### *4.1 Data and stylized facts*

Episodes with booms and busts in asset markets are a recurring phenomenon. In their database on financial crises, Laeven and Valencia (2013) find 124 systemic banking crises between 1970 to 2007. They report that such crises are often preceded by credit booms, with an average pre-crisis annual credit growth of about 8.3%. The loss in GDP of these crises is roughly 20 percent in average. Similar findings are reported by Miao and Wang (2018); Davis and Taylor (2019); Greenwood et al. (2020). In an analysis of the housing market and equity prices in industrialized economies during the postwar period, the IMF found that booms in both markets arise frequently (on average every 13–20 years) with entailed drops in prices averaging around 30% and 45% respectively. These busts are associated with losses in output that reflect declines in both, consumption and investment.

Table 1 summarizes the first and second moments of inflation, output and asset prices in Core-Europe. The data is obtained from the OECD, asset prices are represented by the MSCI-Europe index.[16] The data is quarterly and ranges from 1976Q1 to 2012Q2, hence a total of 158 observations is used. The time series are deflated by the consumer price index (prices given in 2005 EUR, the HP-Filter is applied to the log of each series with $\lambda = 1600$). I opt for data from the Euro Area instead of US data, because in the US, the effective lower-bound (often called the *zero lower-bound*, ZLB) on interest rates was binding following 2008Q4. It is well understood that the ZLB can heavily affect the economic dynamics (see e.g. Boehl and Strobel, 2020). Although it would be technically straightforward to consider the ZLB as an additional source of nonlinearities, it is not clear how this would improve the analysis that is in focus of this paper.

As my empirical analysis is based on the first two moments of the data, the choice of the data source is very robust to a broad class of alterations. Other sources will result in comparable stylized facts.[17] Note that the data sample does

[16] At the time of writing the series on inflation and output is available at `https://data.oecd.org/`. Asset prices are downloaded from `https://www.msci.com/indexes`.

[17] Compare e.g. with Winkler (2019) or Adam et al. (2017).

| | $\pi$ | $y$ | $s$ |
|---|---|---|---|
| $SD$ | 0.0092 | 0.0104 | 0.1407 |
| $\pi$ | 1 | -0.1734 | -0.3867 |
| $y$ | – | 1 | 0.6025 |
| $s$ | – | – | 1 |

Table 1: Standard deviations and cross correlations of inflation, output and real asset prices. Quarterly data for Core-Europe from 1976 to 2014.

not include a severe domestic financial crisis. The sample will therefore be used to calibrate the model to normal times, but not to crisis times. Let me summarize the following stylized facts from the data:

i) The standard deviation of asset prices is roughly one order of magnitude higher than the standard deviations of inflation and output.

ii) Inflation is (weakly) countercyclical.

iii) Asset prices and output are positively correlated.

iv) Asset prices and inflation are negatively correlated.

v) The negative correlation between asset prices and inflation is stronger than the correlation between output and inflation. Note that this suggests that the link between asset prices and macroeconomic activity should be motivated through the supply side rather than through the demand side, as it is the case for the presented model.

Note also that both, the US and the Euro Area have recently seen high asset prices together with low interest rates and low inflation. The next task is to specify an explicit mechanism of expectation formation for the asset market, and to fit the complete model to the presented data.

### *4.2 Expectation formation process*

I assume that asset price expectations follow the *Heterogeneous Agent Switching Model* (Brock and Hommes, 1998, henceforth the BH-model). In this behavioral model, agents switch endogenously between simple forecasting heuristics, depending on the performance of each heuristic. As trading strategies are complementaries (see Bulow et al., 1985), agents have an incentive to mimic successful strategies. If the share of traders that use a specific heuristic accumulates to a critical mass, they may be able to outperform a rational trading strategy due to the direct positive feedback of expectations on prices.

The choice of the BH-model is driven by a set of empirical behavioral properties. Other than in models with rational expectations, the heterogeneous agent switching model can replicate a positive correlation between returns and expected returns, and fat tails of the distribution of asset prices (see e.g. Greenwood and Shleifer, 2014; Adam et al., 2017, 2018). The intuitive mechanism of expectations formation also corresponds to the concept of *animal spirits* in Keynes (1937). Further, the model is substantiated by evidence from the behavioral laboratory (see cf. Hommes, 2006). The BH-model allows to explicitly model *endogenous* financial cycles and the endogenous nonlinear propagation of real shocks to the asset market. This propagation goes beyond simple spillover effects: while a decrease in the interest rate will boost asset prices, this can trigger complicated expectations dynamics and extrapolative behavior that result in speculative asset price booms. As Boehl and Hommes (2021) show, these financial cycles can even turn into rare-disaster type financial crisis. The endogenous nature of these dynamics is a central motivation here: it is arguably hard to enrich our understanding of financial cycles and crises by using models where these events are purely driven by exogenous shocks, as it is often assumed (Christiano et al., 2015; Del Negro et al., 2015; Boehl et al., ming).

Assume that asset traders are heterogeneous in their forecasting rules. Let there be $H > 1$ simple predictors of future prices and let each predictor $h = 1, 2, \dots, H$ be of the form $\hat{E}_{t,h}s_{t+1} = g_h s_{t-1} + b_h$. From Section 2.3 we know that intermediary $k$s balance sheet is $D_{k,t} = S_t J_{k,t} + B_{k,t}$. The return on one monetary unit is $R_{t+1}\frac{P_t}{P_{t+1}}$ for deposits and $\hat{E}_{k,t}\left\{\Theta_{t+1} + S_{t+1}\right\}$ for shares. Subjective no arbitrage requires $R_{t+1}\frac{P_t}{P_{t+1}}S_t = \hat{E}_{k,t}\left\{\Theta_{t+1} + S_{t+1}\right\}$. As in BH, I aggregate linearly over these individual demand functions, which holds up to a first order approximation. Let $n_{t,h}$ denote the fraction of traders using predictor $h$ at time $t$. The economy wide price for shares $S_t$ is then given by

$$R_{t+1}\frac{P_t}{P_{t+1}}S_t = E_t\Theta_{t+1} + \sum_h n_{t,h}\hat{E}_{t,h}S_{t+1}. \tag{30}$$

Assume further that traders take the real interest rate $R_{t+1}\frac{P_t}{P_{t+1}}$ as given.[18] Log-linearization yields a representation analog to Equation (21), i.e.

$$s_t = \beta\hat{E}_t s_{t+1} - (r_{t+1} - \pi_{t+1}) \qquad \text{with} \qquad \hat{E}_t s_{t+1} = \sum_h n_h \hat{E}_{h,t} s_{t+1}. \tag{31}$$

The fractions $n_{h,t}$ of each predictor are updated according to predictor $h$'s *measure of performance* $U_{h,t}$. In line with Brock and Hommes (1998) I utilize *realized past profits* as the performance measure because it is a good proxy for evolutionary fitness. This incorporates the idea that those strategies that were more successful in the past are more likely to be chosen. Hence,

$$U_{h,t} = (\beta s_t - s_{t-1})(\beta\hat{E}_{t-1,h}s_t - s_{t-1}). \tag{32}$$

The choice of the performance measure is a major determinant of the nonlinear properties of the system. An important feature of realized past profits as performance measure is, that this measure rewards the prediction of the correct sign of a price change, instead of rewarding an actually accurate estimate of the price.

The probability that an agent choses predictor $h$ is given by the *multinomial discrete choice model*

$$n_{h,t} = \frac{\exp\{\gamma_U U_{h,t-1}\}}{Z_{t-1}} \qquad \text{and} \qquad Z_{t-1} = \sum_{h=1}^{H}\exp\{\gamma_U U_{h,t-1}\}, \tag{33}$$

where $\gamma_U$ is called the *intensity of choice.*

Consider a simple 3-type model where agents are either fundamentalists –

[18] Alternatively, the $R_t\frac{P_t}{P_{t+1}}$ could be included explicitly in the performance measure. This does not fundamentally change the dynamics, but leads to a slight asymmetry of the resulting bifurcations.

traders that believe next-period's price will be the fundamental price – or trend extrapolators with a positive and a negative bias, respectively. Assume that the latter two share a common trend-following parameter $\gamma_s$ and that the positively/negative bias is symmetric by $\gamma_b$. The three types are then given by

$$\begin{aligned} \hat{E}_{t,1}s_{t+1} &= 0, \\ \hat{E}_{t,2}s_{t+1} &= \gamma_s s_{t-1} + \gamma_b, \\ \hat{E}_{t,3}s_{t+1} &= \gamma_s s_{t-1} - \gamma_b. \end{aligned} \tag{34}$$

This specification of the mechanism for expectation formation on asset prices closes the model. It consists of a linear part associated with the macroeconomy and the formation of conditionally rational expectations, which is represented by Equation (26), and a nonlinear part that contains the boundedly rational expectation formation which is given by the performance measure $U_{h,t}$ (Equations 31 and 32), the fractions $n_{h,t}$ and the normalization factor (Equation 33), and the predictors (Equation 34).

*4.3 Dynamic properties*

The presented model permits a wide range of dynamics. In the simplest case it nests the standard small-scale rational expectations New Keynesian model in the spirit of Woodford (2003) and Galí (2008), where all dynamics result from the linear propagation of exogenous shocks. In the absence of any shocks, such linear model is stationary. While linear dynamics entail some convenient attributes – e.g. computationally efficient solution techniques and the straightforward application of econometric estimation methods – it is not suitable to explore the potentially complex interaction of highly dynamic and potentially overheated financial markets with the macroeconomy. My detailed account of the feedback mechanism between inflation, monetary policy, and speculative financial markets can give rise to complicated, endogenous dynamics. For this reason it is useful to first study the dynamics of this feedback mechanism in the absence of exogenous shocks.

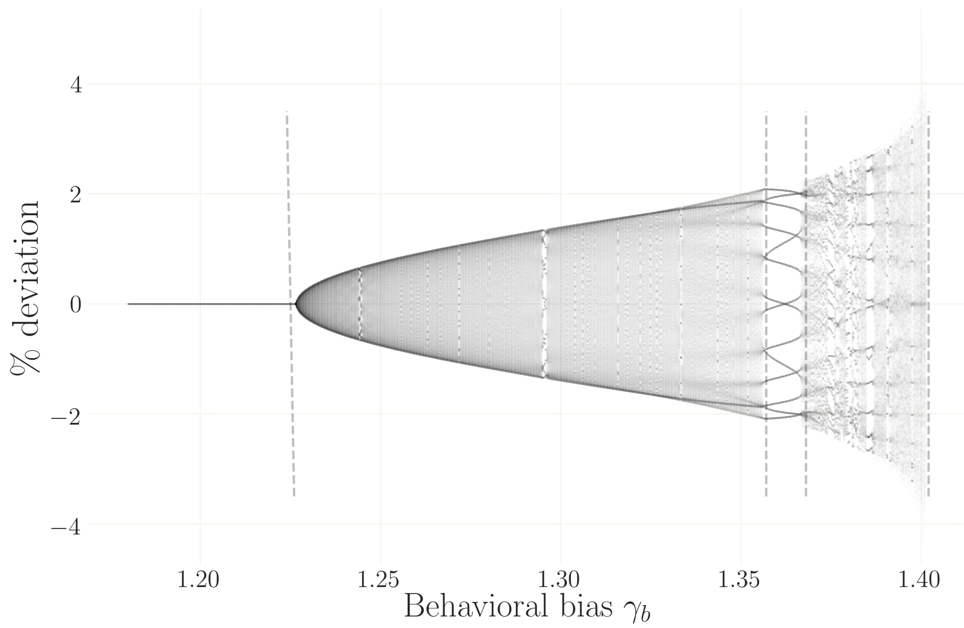


Figure 2: Bifurcation diagram for asset prices output w.r.t. bias parameter $\gamma_b$. A primary Hopf bifurcation at $\gamma_b \approx 1.22$ leads to periodic and quasi-periodic dynamics. All other parameters as in Table 2.

Nonlinear models, such as the one presented here, can give rise to endogenous fluctuations, or even chaotic dynamics: prices and aggregates can evolve even in the absence of any exogenous disturbances. They do not necessarily revert to a stationary steady state, even in the long run. The behavior of such systems is studied in the field of bifurcation theory (see e.g. Arnold et al., 2013). Figure 2 presents the bifurcation diagram of asset prices as a function of the parameter $\gamma_b$. For each parameter value on the $x$-axis, it displays all the points visited in the long-run and in the absence of any additional stochastic noise.[19]

The vertical grey lines separate four different regions of the parameter space. Each region differs in the *type* of dynamics. For values of $\gamma_b$ below 1.22 the fundamental steady state is stable and unique. In this region, the degree of the bias of sentiment traders is not large enough to impact on asset prices without exogenous

[19]For each value at the bifurcation parameter 11.000 iterations are run. A transition phase of 10.000 periods is omitted in the analysis.

shocks, and asset markets are dominated by fundamental beliefs. While exogenous impulses could lead to a temporal increase in the fraction of belief-biased agents – which would protract the response of the impulse – this effect is not strong enough to prevent prices from returning to the fundamental steady state in the long run.

A bias larger than 1.22 leads to limit cycles. As beliefs are to a large extent self-fulfilling, in an upswing the fraction of optimists will grow over time through positive self-enforcement. The natural limit of this feedback process – the amplitude – is reached when the price equals the bias $\gamma_b$. At this point, biased agents will not be able to extract any additional profits, and optimistic traders will reduce their portfolio holdings. By that, their profit shrinks and alternative strategies become relatively more attractive.[20] As alternative beliefs become more widespread, the fraction of positively biased traders will fall again and prices begin to tumble down.

For values larger than $\gamma_b \approx 1.37$ cycles become unstable and more erratic. The simulations suggest that the system is close to a *homoclinic orbit*[21]: the zero steady state is globally stable but locally unstable (Ott, 2002; Hommes, 2013). Long periods of stability can then be followed by asset price booms that are hard to predict, and which, through the credit-collateral channel, can be followed by abrupt, severe recessions. For even higher values of $\gamma_b$ dynamics become explosive because the fraction of fundamentalists is too small to successfully stabilize the system.

The bifurcation diagrams for $\gamma_s$ and $\gamma_U$ can be found in Appendix D. While amplitudes and periodicity of periodic and quasi-periodic dynamics can differ, the *types* of possible dynamics are the same as in the bifurcation map of $\gamma_b$.

[20] Figure D.10 illustrates graphically the different forces at work behind the financial limit cycles.

[21] A homoclinic orbit is a trajectory of a flow of a dynamical system which joins a saddle equilibrium point to itself. In the terminology of dynamic system theory, a homoclinic orbit is said to lie in the intersection of the stable manifold and the unstable manifold of an equilibrium, see c.f. Ott (2002).

### 4.4 *Fitting the model to the data*

The remaining free parameters are estimated using the simulated method of moments (SMM, McFadden, 1989). The underlying intuition behind SMM is to minimize some norm of the distance between the simulated and empirical moments. To prevent overfitting, I estimate five parameters to fit six moments. As with the generalized method of moments (Hansen, 1982), a weighting matrix is used to corrects for the quality of the moment estimates. The weighting matrix is estimated using the 2-step procedure. The simulated moments are retrieved from a batch of 100 simulated time series, each of the length of the original data.

SMM has the advantage that only the specified moments are targeted and not – as with likelihood-based methods – the complete time series. It is clear that a small-scale model as the one presented in Section 2 can not yield a satisfactory fit to economic times series, as it would be required in the context of likelihood based methods (see e.g. An and Schorfheide, 2007; Del Negro et al., 2007; Smets and Wouters, 2007). For this reason it is useful to only focus on the important moments of the data (Table 1) to discipline the model. Denote the set of estimated parameters by $\chi$ and the moments obtained by simulating the model given $\chi$ as $m(\chi)$. They are chosen to solve

$$\min_{\chi} \left(m(\chi) - \hat{m}\right)' W(\chi) \left(m(\chi) - \hat{m}\right), \tag{35}$$

where $\hat{m}$ are the moments obtained from the data and $W(\chi)$ is the weighting matrix.

As an additional reference point – in particular to provide a robustness check for the estimate of the central parameter $\nu$ – I also consider a rational expectations (RE) version of the model, where asset markets are homogeneous and fully rational but subject to an additional add-hoc exogenous AR(1) process on rational asset price expectations $E_t s_{t+1}$. This allows for exogenous fluctuations in asset prices in the RE-model and provides an equal number of (and comparable) degrees of freedom as in the model with speculative asset markets. Thus, for the rational model variant I add the exogenous state $v_t^s$ with

$$v_t^s = \rho_s v_{t-1}^s + \varepsilon_t^s, \qquad \varepsilon_t^s \sim N(0, \sigma_s) \tag{36}$$

to asset pricing equation (19).

For both models the elasticity of marginal costs to asset prices, $\nu$, and the standard deviations of the real shocks, $\sigma_d$ and $\sigma_a$ are estimated. Additionally, for the RE-model the autocorrelation coefficient and the standard deviation for the expectations shock, $\rho_s$ and $\sigma_s$, need to be fitted. For the nonlinear BH-model the parameters $\gamma_s$ and $\gamma_b$ are estimated and $\gamma_U$ set to unity. The latter is without lack of generality as the two former parameters already have sufficient degrees of freedom to allow for a rich portfolio of nonlinear dynamics (see Brock and Hommes, 1998).

| | $\gamma_s$ | $\gamma_b$ | $\rho_s$ | $\sigma_s$ | $\nu$ | $\sigma_a$ | $\sigma_d$ |
|---|---|---|---|---|---|---|---|
| BH-model | 0.998 | 1.229 | – | – | 0.082 | 0.001 | 0.004 |
| | (0.048) | (0.046) | | | (0.016) | (0.000) | (0.001) |
| RE-model | – | – | 0.735 | 0.030 | 0.106 | 0.001 | 0.005 |
| | | | (0.200) | (0.029) | (0.036) | (0.000) | (0.002) |

Table 2: SMM parameter estimates. Simulated moments obtained from 100 batches of simulated time series. Values in parenthesis give asymptotic standard errors. For the weighting matrix the 2-step procedure is used.

The parameter values obtained from SMM are displayed in Table 2. Independently of the specification, the procedure identifies a value of $\nu$ close to 0.1. Given that the standard deviation of asset prices is roughly ten times the standard deviation of inflation, this estimate of $\nu$ suggests a rather large impact of asset price fluctuations on the other variables. This estimate is also well aligned with the finding that a large share of the variance of GDP is explained by fluctuations in asset prices in Assenmacher and Gerlach (2008) and Miao et al. (2012) and provides a strong motivation to study the role of monetary policy intervention.

The standard deviations $\sigma_a$ and $\sigma_d$ suggest that demand-sided fluctuations are relatively more important than fluctuations originating from the supply side, a finding which is in line with recent estimates from the DSGE literature e.g. from Boehl and Strobel (2020). The estimate of $\gamma_s$ is almost unity, which corresponds with experimental results (Hommes, 2013) that report a high degree of trend extrapolation. Finally, relative to the high empirical volatility of asset prices, the estimate of the behavioral bias of $\gamma_b = 1.229$ can be seen as rather moderate.

| | **BH-Model** | | |
|---|---|---|---|
| | $\pi$ | $y$ | $s$ |
| $SD$ | 0.008 (.001) | 0.011 (.002) | 0.113 (.036) |
| $\pi$ | 1 | 0.175 (.151) | -0.298 (.086) |
| $y$ | – | 1 | 0.500 (.150) |
| $s$ | – | – | 1 |
| | **RE-Model** | | |
| | $\pi$ | $y$ | $s$ |
| $SD$ | 0.007 (.001) | 0.012 (.002) | 0.140 (.016) |
| $\pi$ | 1 | -0.130 (.161) | -0.443 (.102) |
| $y$ | – | 1 | 0.613 (.118) |
| $s$ | – | – | 1 |

Table 3: Standard deviations and cross correlations for the estimated models. Moments obtained from 100 batches of simulated time series. Values in parenthesis give asymptotic standard errors.

Table 3 shows the simulated moments for the two estimated models. For both models the moment estimates are close to the original moments of the data (Table 1). Relative to the RE-model, the endogenous amplification of asset prices through the BH-model reduces the correlation between asset prices and inflation. Likewise, endogenous amplification explains the high standard deviation of asset prices because it does not one-to-one feed back on inflation and output: an increase in asset prices dampens inflation through the marginal cost channel and the central bank lowers the interest rate which in turn stimulates demand. This ensures that the correlation between output and asset prices is relatively strong even in the absence of a strong procyclical dividend component.

Note that the weighting matrix puts low weights on the cross-correlation of inflation and output. This suggests that this measure varies notably across simulation batches and the moment is not well-identified. For this reason the cross-correlation of inflation and output is somewhat larger in the simulated estimated BH-model than in the data.

Let me summarize the findings from this section. The nonfundamental fluctuations in asset prices together with the link between asset prices and real activity allows to replicate key-moments of the data that are hard to reconcile in the absence of these additions.[22] That is, in the absence of asset price spillovers to the macroeconomy, asset prices would move one-to-one with the path of current and expected real interest rates. The next section will discuss the potential implications for monetary policy.

## 5 Asset price targeting

This section turns towards the quantitative policy implications of speculative asset markets. I first explore the role of monetary policy during normal times. Thereafter I take the model to the limit case of endogenous, financial crisis-type dynamics, and discuss the potential of an asset price targeting strategy in mitigating such crisis.

### *5.1 Asset price targeting in normal times*

Similar to the bifurcation diagram discussed in Section 4.3, Figure 3 shows the bifurcation diagram with respect to the policy parameter $\phi_s$ (using the estimated parameter values). Note that the parameters were estimated under the assumption of no asset price targeting ($\phi_s = 0$, gray dashed line in Figure 4). The figure suggests the existence of periodic movements in asset prices with a very small amplitude, which translate to similar cycles in inflation and output. Comparing these fluctuations to the standard deviations of the data suggests that the data is not driven by endogenous dynamics, but rather by the interplay of exogenous

[22] A large literature in financial economics documents various asset pricing "puzzles" such as the *Equity Premium Puzzle* (Mehra and Prescott, 1985, 2003) or the *Equity Volatility Puzzle* (Campbell, 2003; Shiller, 1981).

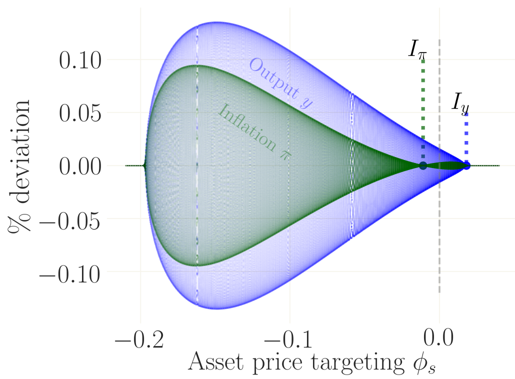


Figure 3: Bifurcation diagram for output (blue/light) and inflation (green/dark) with respect to the policy parameter$\phi_s$. Output and inflation inherit the financial cycles originating from the asset market. A primary Hopf bifurcation leads to periodic periodic dynamics. The system reverts to stationarity after a second Hopf bifurcation. All other parameters as in Table 2.

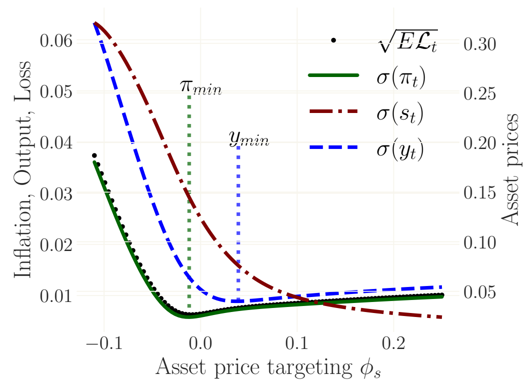


Figure 4: Standard deviations of stochastic simulations as a function of the asset price targeting policy parameter $\phi_s$. The economy is driven by exogenous shocks, but endogenously amplified by the nonlinear behavioral process in the asset market.

macroeconomic shocks and the endogenous amplification through the asset market. However, this still implies that speculative agents react endogenously to fluctuations in asset prices, which cannot rule out large nonfundamental swings in asset prices when combined with exogenous shocks.

Section 3 shows that an increase in $\phi_s$ can mitigate the direct impact of asset prices on output and reduce the positive feedback of asset price expectations. Figure 3 confirms that, in the absence of stochastic shocks, the central bank can mitigate endogenous speculative dynamics by carefully targeting asset prices. Increasing the interest rate in response to rising asset prices counteracts the decrease of the rate induced by inflation targeting. Relatively higher rates act as a natural dampener to the expectation feedback. For this reason the amplitude decreases with $\phi_s$. At the point $B(\lambda_s)$ of Figure 1, one of the eigenvalues of the system crosses the unit circle and the periodic solution turns into a stable fixed point. In the field of bifurcation such point is known as a (inverse) *supercritical Hopf-Bifurcation* (see e.g. Kuznetsov, 2013).

The previous exercise was helpful to understand the role of pure endogenous dynamics. Let us now turn to the more general case with stochastic simulations. Figure 4 shows the standard deviations of simulations as a function of the policy parameter $\phi_s$. Here, the exogenous processes for $a_t$ and $u_t$ are unmuted. The dynamics emerge as a combination of the i.i.d. noise and endogenous responses of the financial market to these shocks. The Figure indicates that

a) an increase of $\phi_s$ reduces endogenous asset price volatility,

b) an increase of $\phi_s$ up to $y_{min} \approx 0.018 \approx I_y$ dampens the transmission of asset price volatility to output, but increases the volatility of inflation,

c) increasing $\phi_s$ beyond a value of $y_{min}$ leads to additional fluctuations in both, inflation and output.

The "collateral damage" effect in (b) and (c) –i.e., the unwanted increase in the volatility of output and inflation– stems from the fact that a leaning against the wind policy necessarily also reacts to fluctuations in asset prices that are (efficient) general equilibrium responses to exogenous shocks. Take for example a negative productivity shock, which increases inflation and thereby triggers monetary policy to raise the interest rate. Together, this leads to the deflation of asset prices. An additional response of monetary policy to asset prices will hence induce economic costs in terms of an incremental increase in output and inflation. The logic for a demand shock works similarly.

In an economy in which asset prices are not a source of fluctuations, asset price targeting tends to increase volatility in real aggregates. This effect runs in the opposite direction of the stabilizing effect. The results from this section suggest that the optimal sensitivity of monetary policy to asset prices should be bounded by $\pi_{min} \approx -0.01 = I_\pi$ and $I_y$, which is a considerably small neighborhood around zero. Figure 4 also shows the square root of expected welfare, $\sqrt{E\mathcal{L}_t}$, as a function of $\phi_s$ (black dashed line). This measure has a strong overlap with the standard deviation of inflation (green line) because for the given parameterization, the central bank's weight on the output gap, $\lambda$, is very low. The minimum of this graph lies between

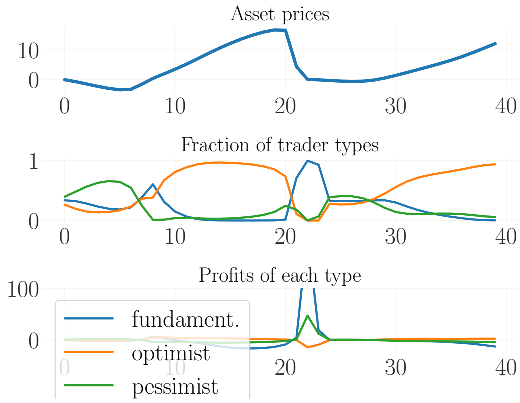


Figure 5: *Top panel:* time series of asset prices during the build-up and bust of a financial bubble. The dynamics are absent any additional shocks. Parameters: $\gamma_b = 2.5$, $\gamma_s = 0.93$ and $\gamma_U = 0.4$. $\nu = 0.09$ as in 2. *Center panel:* during the build-up, a large fraction of traders is optimistic. Optimism vanishes as the bubble peaks. During the crash fundamental beliefs dominate. *Bottom:* profits of each trader type. The fundamental predictor is unfavorable because it is costly.

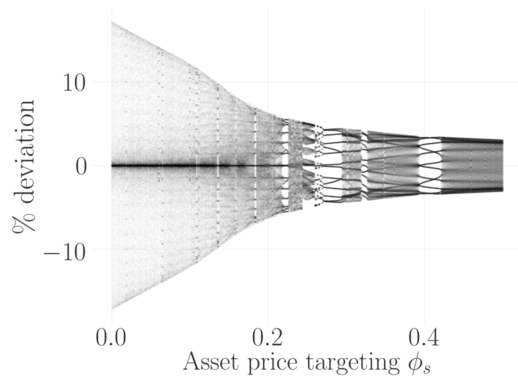


Figure 6: Bifurcation diagram for asset prices with respect to the policy parameter $\phi_s$. $\gamma_b = 2.5$, $\gamma_s = 0.93$ and $\gamma_U = 0.4$. $\nu = 0.09$ as in 2. For low values of $\phi_s$ the trajectories are close to a homoclinic orbit. For values of $\phi_s$ close to 0.5 the time series displays periodic and quasi-periodic behavior.

$I_\pi$ and zero, which constitutes an additional argument against asset price targeting.

The analysis in Galí (2014) is based on a framework that includes *rational bubbles*. Such rational bubbles grow proportionally to the interest rate. The author hence suggests to actually lower the policy rate when facing asset price bubbles. In contrast, my model suggests that such policy would lead to a notable increase in output and asset price volatility and, even for small values of the policy's sensitivity, an amplification of the spillovers from fluctuations in asset prices to the real economy.

### *5.2 Asset price targeting and endogenous financial crises*

The results from the previous subsection assign no positive role for asset price targeting. The central bank faces a trade-off between fragility of asset prices – the stabilization component of asset price targeting – and the additional volatility caused by such policy. Note that the parameter estimates from Table 2 reflect a sample, in which financial markets remain rather calm for the largest part. For

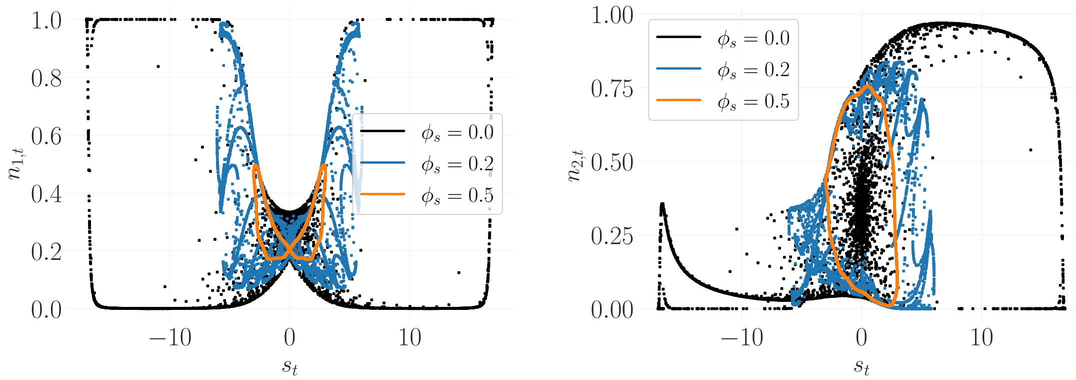


Figure 7: Phase diagrams of the endogenous dynamics. Parameters as in Figure 6. *Left:* asset prices and share of fundamentalists. *Right:* asset prices and share of optimistic traders.

this reason it is an intrusive result that the costs of additional volatility weight higher than the benefits from a stabilization of asset prices. An important question however is whether asset price targeting can help to prevent financial crisis and bubbles in times where financial markets are overheated.

My model straightforwardly allows to create endogenous financial crises. While it is natural to assume that the deep economic parameters –parameters such as $\eta$, $\nu$ or $\beta$– remain time-invariant, the behavioral parameters are likely subject to changes. Assume a regime where the bias of traders $\gamma_b$ temporally increases to 2.5. Set $\gamma_s = 0.93$ and $\gamma_U$ to 0.4. The choice of the latter is more moderate than in the benchmark, but necessary to prevent the system from explosive dynamics given the relatively high bias.[23] The resulting time series – again in the absence of any real shocks – are displayed in the top panel of Figure 5. After long episodes of stability, asset bubbles slowly build up through expectations dynamics (period 7 ff.); more and more traders switch to the positive-biased heuristic. At the same time both sentiment traders extrapolate the positive trend. This process is self enforcing until the effects of bias and extrapolation level out (period 20), and profits of the sentiment traders decrease. At this point a large share of traders switch to

[23]The bifurcation diagram for the intensity of choice can be found in Figure D.12 in Appendix Appendix D.

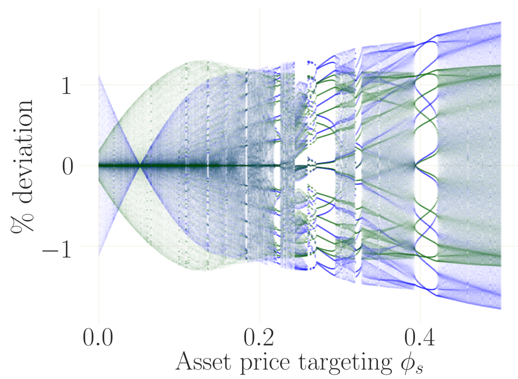


Figure 8: Bifurcation diagram for output (blue/light) and inflation (green/dark) with respect to the policy parameter $\phi_s$. Parameters as in Figure 6. Dynamics of inflation and output and output do not inherit the dampening effect of asset price targeting on asset prices. Instead, the amplitude of the dynamics increases with the aggressiveness of the policy.

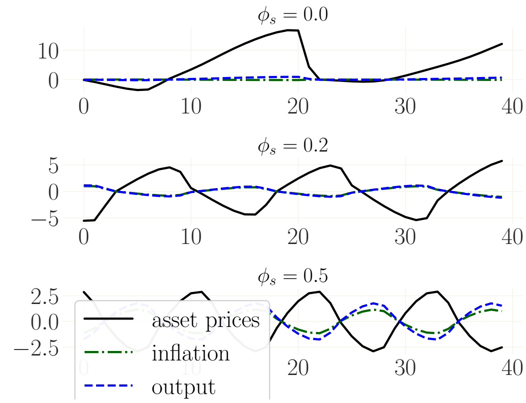


Figure 9: Time series of asset prices, inflation and output for different values of $\phi_s$. Endogenous dynamics in the absence of additional stochastic noise. $\gamma_b = 2.5$, $\gamma_s = 0.93$ and $\gamma_U = 0.4$. $\nu = 0.09$ as in 2. *Top panel:* the trajectory is close to an homoclinic orbit. After stable episodes, bubbles slowly emerge en then burst. *Center panel:* for $\phi_s = 0.2$ the time series exhibits quasi-periodic dynamics. *Bottom panel:* limit cycles for high values of $\phi_s$. Inflation and output are both countercyclical.

the fundamental trading strategy and prices collapse. During the asset bubble, refinancing conditions for firms improve which is reflected in a relative decrease in prices. The central bank responds with lowering interest rates, which stimulates consumption and, in general equilibrium, further fuels financial markets. Hence, the boom in asset prices is accompanied by an episode of high growth and low inflation.[24]

Can the central bank prevent such boom-bust cycle in asset prices? Figure 6 shows the bifurcation diagram of asset prices with respect to the policy parameter, taking the crisis setup developed in the previous paragraph as the starting point. An increase of the nominal interest rate in response to a surge in asset prices can indeed mitigate the cyclic movement in asset prices: when inflation decreases, the central bank wants to lower rates. If at the same time asset prices are high, the

[24]Note that since beliefs on asset prices are subjective, the real rate is not tied to the Euler equation for asset prices but rather, traders take the real rate as given.

central bank will lower rates by less, thereby partly cutting-off the feedback to financial markets. For the example chosen here, a value of $\phi_s$ of roughly 0.25 is sufficient to prevent trajectories close to an homoclinic orbit. Cyclical movements in asset prices remain, although with moderate amplitude. For values of $\phi_s > 0.25$ a higher feedback coefficient to asset prices does not have a notably strong effect on the amplitude of asset prices.

How does this dampening effect translate through the economy? Figure 8 shows the bifurcation diagram of output and inflation to the policy parameter. The Figure shows clearly that the reduction of endogenous fluctuations in asset prices does not pass on to the real economy. The increase in the interest rate in response to a boom in asset prices reduces the output response, which in turn further decreases inflation through its effect on wages. Hence, low values of $\phi_s$ decrease the volatility of output in response to financial cycles, but increase inflation volatility. This effect prevails until $\phi_s$ reaches the point $I_y$ from Figure 1. Here is the blind spot of the output response on asset price fluctuations: the interest rate response on inflation and asset prices cancel out exactly. An additional increase in interest rates again translates into more volatility in output, which – through the marginal cost channel – further raises inflation volatility.

Figure 7 shows the phase diagrams for different values of $\phi_s$. The left panel shows how a reduction of the dynamic feedback translates to the dynamics of the share of fundamentalists. For the case without any asset price targeting, the share of fundamentalists is zero for most times. As prices reach the peak-amplitude, all agents become fundamentalists and prices collapse. The right panel shows how the fraction of optimists evolves. As prices are higher, more traders are optimistic, which reflects in higher expectations on asset prices, which again drives up their level. At the peak, the fraction tumbles and prices collapse. With moderate/strong leaning against the wind, the reduction in expectations feedback reflects in more moderate cycles. At all times, a positive share of fundamentalists stabilize the market, while the periodic dynamics are mirrored in the periodic fluctuations in the share of optimistic agents.

The two bottom panels of Figure 9 picture the cases of $\phi_s = 0.2$ and $\phi_s = 0.5$. While in the middle panel, financial cycles are dampened, the output response to asset prices is already negative through the real-rate effect. For $\phi_s = 0.5$ the amplitude of the financial cycles is much reduced and close to 2%, but the spillover to output and inflation is rather extreme. This confirms the findings from the previous sections: while asset price targeting can indeed mitigate financial cycles, the respective spillover effects of such policy to inflation and output can be highly destructive.

## 6 Concluding remarks

This paper studies the role of a monetary policy that leans against asset prices in a model in which asset markets are governed by behavioral speculation. Credit frictions create a channel for spillover effects from asset prices to the macroeconomy. I document that a causal feedback between asset prices and real activity in combination with speculation in the asset market can help to replicate key moments in the data on inflation, output, and asset prices.

I find that a monetary policy that targets asset prices is not an welfare improvement. For the welfare of households, asset prices only matter in that they affect output and inflation. Although asset price targeting can indeed mitigate asset price fluctuations, at the same time such policy amplifies the transmission of these fluctuations to the macroeconomy. This argument favors a policy that does not target asset prices.

I further study the role of asset price targeting in the context of severe endogenous financial crises that arise from overheated financial markets. I document that such a policy can dampen the positive feedback between asset prices, macroeconomic fundamentals, and the interest rate. This finding suggests that carefully leaning against the wind may be advantageous if financial markets are severely overheated. However, the scope of such a policy is again bounded narrowly due to its potential to amplify the transmission of financial crises to the real economy.

Additionally, how how a central bank can safely identify whether financial markets are overheated remains unclear.

Policy institutions may be well-advised to handle tools such as asset price targeting with care because such instruments might add a structural link between asset prices and macroeconomic aggregates. An additional link implies the risk of other unforeseeable complications, independent of how closely asset prices and real activity are connected. This is particularly true because asset prices impact solely through signaling effects. This motivates other macroprudential policies that potentially restrict the degree of speculation or reduces speculative profits in financial markets (i.e. policies such as leverage requirements), and indicates that such a policy could contribute to overall economic stability. More research in this field is needed.

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## Appendix A Entrepreneurs' optimization problem

This section follows (Bernanke et al., 1999, BGG) closely, but instead of assuming idiosyncratic stochastic productivity of capital, I assume idiosyncratic risk in labor productivity. Additionally, the problem of optimal dividend payment (w.r.t. the optimal level of equity) is added. Deviating from BGG, I assume that the contract is defined in real terms with $Q_{t+1} = R_{t+1}\frac{P_t}{P_{t+1}}$. Firm $j$'s ex post gross return on one unit of labor, $\omega_j$, is i.i.d. across time with a continuous and at least once-differentiable CDF $F(\omega)$ over a nonnegative support and with an expected value of 1. I assume that the hazard rate $h(\omega) = \frac{dF(\omega)}{1-F(\omega)}$ is restricted to $h(\omega) = \frac{\partial(\omega h(\omega))}{\partial\omega} > 0$. The optimal loan contract between firms and financial intermediaries is then defined by a gross non-default loan rate, $Z_{j,t+1}$, and a threshold value $\bar{\omega}_{j,t}$ on the idiosyncratic shock $\omega_{j,t}$. For values of the idiosyncratic shock greater or equal than $\omega_{j,t}$, the entrepreneur will be able to repay the loan, otherwise he will default. $\bar{\omega}_{j,t}$ is then defined by

$$\bar{\omega}_{j,t} A_t Q^H_{t+1} H_{j,t} = Z_{j,t+1} B_{j,t}.$$

Dropping firms' subscripts, as in Bernanke et al. (1999) the optimal contract loan contract must then satisfy

$$\left\{[1 - F(\bar{\omega}_t)]\,\bar{\omega}_t + (1-\mu)\int_0^{\bar{\omega}_t} \omega dF(\omega)\right\} A_t H_t / X_t = Q_{t+1}(W_t H_t - N_t),$$

and the expected return to the wholesaler is (dropping time-subscript of $\omega_t$ for better readability)

$$E\left\{\int_{\bar{\omega}}^{\infty} \omega dF(\omega) - (1 - F(\bar{\omega}))\bar{\omega}\right\} A_t H_t / X_t.$$

Given constant returns to scale, the cutoff $\bar{\omega}$ determines the division of expected gross profits $A_t H_t / X_t$ between borrower and lender. Let me define

$$\mathfrak{F}(\bar{\omega}) = \int_0^{\bar{\omega}} \omega f(\omega) d\omega - \bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega) d\omega$$

to be the expected gross share of profits going to the lender with $\mathfrak{F}'(\bar{\omega}) = 1 - F(\bar{\omega})$ and $\mathfrak{F}''(\bar{\omega}) = -f(\bar{\omega})$. This implies strict concavity in the cutoff value. I define similarly the expected monitoring costs as

$$\mu \mathfrak{G}(\bar{\omega}) = \mu \int_0^{\bar{\omega}} f(\omega) d\omega,$$

with $\mu \mathfrak{G}'(\bar{\omega}) = \mu \omega f(\omega)$. See BGG for the proof that the following result is a nonrationing outcome. The individual firm's problem of choosing optimal equity can be solved by maximizing the discounted sum of profits over equity. Dividends in period $t$ are given by

$$\Theta_t = (1 - \mathfrak{F}(\bar{\omega}_t)) A_t H_t / X_t - N_{t+1} \tag{A.1}$$

The firms' optimization problem is thus

$$\begin{aligned} \max_{\{H_t\},\{\bar{\omega}_t\},\{N_t\},\{\lambda_t\}} E_t \sum_{s=t}^{\infty} N_t^{-1} \left( \prod_{l=t}^{s} Q_{l+1}^{-1} \right) \Big[ (1 - \mathfrak{F}(\bar{\omega}_s)) A_t H_s / X_s - N_{s+1} \Big] \\ -\lambda_s \Big( [\mathfrak{F}(\bar{\omega}_s) - \mu \mathfrak{G}(\bar{\omega}_s)] A_t H_s / X_s - Q_{s+1}(W_s H_s - N_s) \Big). \end{aligned} \tag{A.2}$$

The first-order conditions for this problem are

$$\bar{\omega} : -\mathfrak{F}'(\bar{\omega}_t)(N_t Q_{t+1})^{-1} - \lambda_t \left[\mathfrak{F}'(\bar{\omega}_t) - \mu \mathfrak{G}'(\bar{\omega}_t)\right] = 0 \tag{A.3}$$

$$H : (1 - \mathfrak{F}(\bar{\omega}_t))\, A_t (X_t N_t Q_{t+1})^{-1} - \lambda_t \left(\left[\mathfrak{F}(\bar{\omega}_t) - \mu \mathfrak{G}(\bar{\omega}_t)\right]/X_t - Q_{t+1} W_t\right) = 0 \tag{A.4}$$

$$N : -\frac{1}{N_t^2} E_t \sum_{s=t}^{\infty} \left(\prod_{l=t}^{s} Q_{l+1}^{-1}\right) \Big[ (1 - \mathfrak{F}(\bar{\omega}_s))\, A_t H_s / X_s - N_{s+1} \Big] - Q_{t+1} \lambda_t = 0 \tag{A.5}$$

$$\lambda : \left[\mathfrak{F}(\bar{\omega}_t) - \mu \mathfrak{G}(\bar{\omega}_t)\right] A_t H_t / X_t - Q_{t+1}(W_t H_t - N_t) = 0 \tag{A.6}$$

(A.3) reveals that $\lambda_t$ is a function of $\bar{\omega}$. Define

$$f_\lambda(\bar{\omega}_t) = \frac{\mathfrak{F}'(\bar{\omega}_t)}{\mathfrak{F}'(\bar{\omega}_t) - \mu \mathfrak{G}'(\bar{\omega})}. \tag{A.7}$$

Then, $\lambda_t N_t Q_{t+1} = f_\lambda(\bar{\omega}_t)$. Plugging (A.1) back into (A.5) yields

$$-\frac{1}{N_t^2} E_t \sum_{s=t}^{\infty} \left(\prod_{l=t}^{s} Q_{l+1}^{-1}\right) \Theta_s = Q_{t+1} \lambda_t, \tag{A.8}$$

where $E_t \sum_{s=t}^{\infty} \left(\prod_{l=t}^{s} Q_{l+1}^{-1}\right) \Theta_s$ is an expression for the discounted sum of expected dividends. This corresponds to the perceived law of motion for stock prices from Equation (8) in the paper, which states that any (conditionally) rational expects the term to equal $S_t$ (the price of a firm's share). Under the assumed information structure, (A.5) can thus be rewritten as

$$-\frac{S_t}{N_t^2} = Q_{t+1} \lambda_t \tag{A.9}$$

Combining this with the definition of $f_\lambda$ implies a relationship between the optimal choice of labor, prices and asset prices:

$$f_\lambda(\bar{\omega}_t) = \frac{S_t}{N_t}. \tag{A.10}$$

As in BGG, for a given $\bar{\omega}_t$ the FOC imply a unique level of working capital, and thereby a unique leverage ratio. Define hence

$$\Xi(\bar{\omega}_t) = \frac{N_t}{W_t H_t}, \tag{A.11}$$

and further $z_1 = f_\lambda \circ \Xi^{-1}$ to obtain

$$\frac{S_t}{N_t} = z_1 \left( \frac{N_t}{W_t H_t} \right), \tag{A.12}$$

which is Equation 10 from the paper.

To obtain Equation (11), plugging (A.6) into (A.4) and using the definition of $f_\lambda$ yields

$$(1 - \mathfrak{F}(\bar{\omega}_t))\, A_t / X_t = f_\lambda(\bar{\omega}_t) \frac{Q_{t+1} N_t}{H_t}. \tag{A.13}$$

Dividing both sides by $W_t$ and rearranging results in

$$\frac{A_t}{X_t W_t} = f_\omega(\bar{\omega}_t) \frac{N_t}{W_t H_t} Q_{t+1}, \tag{A.14}$$

with $f_\omega(\bar{\omega}_t) = \frac{f_\lambda(\bar{\omega}_t)}{1 - \mathfrak{F}(\bar{\omega}_t)}$. Again using $\Xi$ to define $z_2\left(\frac{N_t}{W_t H_t}\right) = (f_\omega \circ \Xi^{-1})\left(\frac{N_t}{W_t H_t}\right) \frac{N_t}{W_t H_t}$ results in

$$\frac{A_t}{X_t W_t} = z_2 \left( \frac{N_t}{W_t H_t} \right) Q_{t+1}, \tag{A.15}$$

which is Equation (11) from the paper.

## Appendix B Solving for the rational expectations equilibrium

The System in (22) reads as

$$\underbrace{\begin{bmatrix} 1-\kappa\phi_\pi & -\kappa\psi & \kappa(\nu-\phi_s) \\ \phi_\pi & 1 & \phi_s \\ \phi_\pi & 0 & 1+\phi_s \end{bmatrix}}_{\mathbf{M}} \underbrace{\begin{bmatrix} \pi_t \\ y_t \\ s_t \end{bmatrix}}_{\mathbf{x}_t} = \underbrace{\begin{bmatrix} \beta-\kappa & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1-\beta & \beta \end{bmatrix}}_{\mathbf{P}} \underbrace{\begin{bmatrix} E_t\pi_{t+1} \\ E_t y_{t+1} \\ E_t s_{t+1} \end{bmatrix}}_{E_t\mathbf{x}_{t+1}} + \mathbf{Q} \underbrace{\begin{bmatrix} v_t^\pi \\ v_t^y \\ 0 \end{bmatrix}}_{\boldsymbol{v}_t}. \tag{B.1}$$

For a sensible range of parameter values the Blanchard-Kahn-Conditions are satisfied. The closest bound for which eigenvalues cross the unit circle is if $\phi_s < -0.305$. Exchanging shocks $\mathbf{v}_t$ by perceived shocks $\tilde{\mathbf{v}}_t$, in expectations it has to hold that

$$\tilde{a}_{t+1} = \rho_a \tilde{a}_t^\pi \tag{B.2}$$

$$\tilde{d}_{t+1} = \rho_d \tilde{d}_t^y. \tag{B.3}$$

Using this form, the PLM can be written by using the system of equations (17) – (21) and by bringing all expectations to the LHS:

$$\beta E_t\pi_{t+1} = \pi_t + \kappa x_t - \tilde{a}_t \tag{B.4}$$

$$E_t\pi_{t+1} + E_t y_{t+1} = r_{t+1} + y_t - \tilde{d}_t \tag{B.5}$$

$$E_t\pi_{t+1} = x_t + \psi y_t + r_{t+1} - \nu s_t - \frac{1+\eta}{1+\tilde{nu}} a_t \tag{B.6}$$

$$0 = -r_{t+1} + \phi_\pi \pi_t + \phi_s s_t \tag{B.7}$$

$$E_t\pi_{t+1} + \beta E_t s_{t+1} = s_t + r_{t+1} \tag{B.8}$$

$$\tilde{a}_{t+1} = \rho_a \tilde{a}_t \tag{B.9}$$

$$\tilde{d}_{t+1} = \rho_d \tilde{d}_t \tag{B.10}$$

Using (B.6) and (B.7) to substitute for $r_{t+1}$ and $x_t$ and rewriting as a matrix yields the System (25). Express this system as

$$\mathbf{\tilde{P}} E_t \mathbf{\tilde{x}_{t+1}} = \mathbf{M}\mathbf{\tilde{x}_t}.$$

$\mathbf{\tilde{N}} = \mathbf{\tilde{P}^{-1}}\mathbf{\tilde{M}}$ is the $5 \times 5$ matrix which summarizes the dynamics of the perceived law of motion of rational agents. I use eigenvector/eigenvalue decomposition to obtain $\mathbf{\Gamma\Lambda\Gamma^{-1}} = \mathbf{\tilde{N}}$, where $\mathbf{\Lambda}$ is the diagonal matrix $\mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_5)$ of the eigenvalues of $\mathbf{\tilde{N}}$ ordered by size (smallest in modulus first) and $\mathbf{\Gamma}$ the associated eigenvectors, with columns ordered in the same fashion. The expectation system can then be rewritten as

$$\mathbf{\Gamma^{-1}} E_t \mathbf{x_{t+1}} = \mathbf{\Lambda\Gamma^{-1}}\mathbf{x_t}.$$

Denote the sub-matrix of $\mathbf{\Lambda}$ that only contains unstable eigenvalues as $\mathbf{\Lambda_u}$, and the associated eigenvectors as $\mathbf{\Gamma_u}^{\mathbf{-1}}$. In order to be consistent with the transversallity condition it must hold that $\mathbf{\Gamma_u^{-1}} E_t \mathbf{x_{t+1}} = \mathbf{0}$. Using this fact, solve for $E_t \mathbf{x_{t+1}}$ by

$$E_t \mathbf{x_{t+1}} = \mathbf{\Gamma^{-1}_{u,1:3}}\mathbf{\Gamma_{u,4:5}} E_t \mathbf{\tilde{v}_{t+1}} = \mathbf{\Gamma^{-1}_{u,1:3}}\mathbf{\Gamma_{u,4:5}}\boldsymbol{\rho}\mathbf{\tilde{v}_t}.$$

Note that the requirement that $\mathbf{\Gamma_{u,1:3}}$ is invertible implies the Kuhn-Tucker condition, imposing that $\mathbf{\Gamma_{u,1:3}}$ is a square matrix with full rank. This means that the number of forward looking variables has to equal the number $n_u$ of unstable eigenvalues $\lambda_i > 1$ of $\mathbf{\tilde{N}^{-1}}$. Let me define $\mathbf{\bar{\Omega}} = \mathbf{\Gamma^{-1}_{u,1:3}}\mathbf{\Gamma_{u,4:5}}$. The solution from the main body is then $\mathbf{\bar{\Omega}_{1:2,1:2}}$.[25]

[25] This implies that rational agents do not take asset prices into account when forming expectations. However, a more general approach including the adjustment for measurement errors of projecting three endogenous variables on two shock terms (stochastic indeterminacy) approximately lead to the same $\Omega$. Assuming that agents use OLS to regress $\mathbf{x_t}$ on $\mathbf{\tilde{v}_t}$, $\mathbf{\tilde{\Omega}} = \left(\mathbf{\bar{\Omega}^T}\mathbf{\bar{\Omega}}\right)\mathbf{\bar{\Omega}^T} \in \mathbb{R}^{3\times 2}$ and $\mathbf{\tilde{\Omega}_{1:2,1:2}} \approx \mathbf{\Omega}$.

## Appendix C The full model (for online publication only)

### *Appendix C.1 Households*

The household block is defined by the households' optimization problem in (1) and (2) and, for household $i$, results in the optimality conditions

$$e^{d_{i,t}} C_{i,t}^{-1} = E_t \left\{ \beta R_{t+1} \frac{P_t}{P_{t+1}} C_{i,t+1}^{-1} \right\}, \tag{C.1}$$

$$H_{i,t}^{\eta} = \frac{W_t}{C_{i,t}}. \tag{C.2}$$

From $\epsilon_{i,t} = \epsilon_t^d$ it follows that $\int_0^1 d_{i,t} = d_t$ and all households will make identical decisions. Since then $C_{i,t} = C_{j,t}$ for all $i, t \in [0, 1]$, aggregation is trivial and gives rise to Equations (4) and (5) from the main body.

However, following Assumption 1, the information set of each individual household $i$ contains $\tilde{d}_{i,t}$ but not $d_t$. This implies that the aggregate variables in Equations (4) and (5) are not known to the individual agents, and cannot be used to draw conclusions on possibly boundedly rational behavior of other agents.

The law of motion of the true aggregated, unobserved preference shock is given by (3):

$$d_t = \rho_d d_{t-1} + \epsilon_t^d. \tag{C.3}$$

### *Appendix C.2 Firms*

The wholesale firms' problem is stated in detail in Appendix A and yields Equations (10) and (11) (corresponding to (A.12) and (A.15)). Together with the aggregated production function (6) these are:

$$\frac{S_{j,t}}{N_{j,t}} = z_1 \left( \frac{N_{j,t}}{W_t H_{j,t}} \right), \tag{C.4}$$

$$\frac{A_t}{X_t W_t} = z_2 \left( \frac{N_{j,t}}{W_t H_{j,t}} \right) R_{t+1} \frac{P_t}{P_{t+1}}, \tag{C.5}$$

$$Y_t = A_t H_t. \tag{C.6}$$

The aggregated technology shock $A_t$ is given by (7):

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_t^a, \tag{C.7}$$

which is only observable to retailer. This is, as with the preference shock of the households, necessary to avoid that agents are able to learn about the existence of boundedly rational agents.

To complete the setup of the firm sector, an additional equation is the solution to the problem of retail firms, which results in the Phillips curve in terms of the markup $X_t$. The nonlinear Phillips curve under Calvo pricing has no closed form solution.

*Appendix C.3 Financial intermediaries*

The problem of the financial intermediary, specified in Sections 2.3 and 4.2, give rise to the no-arbitrage condition 13, which includes the boundedly rational expectations operator $\hat{E}_t$:

$$S_t = \hat{E}_t \left\{ \frac{\Theta_{t+1} + S_{t+1}}{R_{t+1} \frac{P_t}{P_{t+1}}} \right\}. \tag{C.8}$$

Aggregate assets supply of assets $J_t$ is assumed to be fixed at unity at all $t$.

Notably, this is the only part of the model where agents are purely non-rational in a strict sense. The participants of all other agents are modeled as "conditionally model consistent rational", as defined in Section 2.6.

*Appendix C.4 Government sector*

The central bank follows a nonlinear version of (14):

$$\frac{R_t}{R} = \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^*}\right)^{\phi_y} \left(\frac{S_t}{S}\right)^{\phi_s} \tag{C.9}$$

*Appendix C.5 Market clearing*

Market clearing requires

$$Y_t = C_t. \tag{C.10}$$

*Appendix C.6 Equilibrium*

A competitive equilibrium consists of sequences of the 11 variables

$$\left\{C_t, d_t, R_{t+1}, \frac{P_{t+1}}{P_t}, W_t, H_t, S_t, N_t, A_t, X_t, Y_t\right\}_{t=0}^{\infty} \tag{C.11}$$

that satisfy equations (C.1) to (C.10) additional to the New-Keynesian Phillips curve (a total of 11 equations).

*Appendix C.7 Linearized system of equations*

The linearized system of equations is given by (17) to (21) and the equations (C.3) and (C.7), which are repeated here for convenience:

$$\pi_t = \beta E_t \pi_{t+1} - \kappa x_t, \tag{C.12}$$

$$y_t = E_t y_{t+1} - (r_{t+1} - E_t \pi_{t+1}) + d_t, \tag{C.13}$$

$$x_t = \nu s_t - \psi y_t - (r_{t+1} - E_t \pi_{t+1}) + \frac{1+\eta}{1+\bar{\nu}} a_t, \tag{C.14}$$

$$r_{t+1} = \phi_\pi \pi_t + \phi_y \hat{y}_t + \phi_s s_t, \tag{C.15}$$

$$s_t = \beta \hat{E}_t s_{t+1} - (r_{t+1} - E_t \pi_{t+1}), \tag{C.16}$$

$$d_t = \rho_d d_{t-1} + \epsilon_t^d, \tag{C.17}$$

$$a_t = \rho_a d_{t-1} + \epsilon_t^a. \tag{C.18}$$

Equation (C.12) is the log-linearized Phillips curve. (C.13) can be derived by plugging (C.10) into (C.1) (this eliminates $C_t$) and subsequent log-linearization. (C.14) comes from log-linearizing (C.4) and (C.5) and then combining both equations to eliminate the log-linearized $N_t$. (C.15) is again the linearized monetary policy rule

(C.9). Finally, linearizing (C.8) results in

$$s_t + r_{t+1} - E_t\pi_{t+1} = (1-\beta)\hat{E}_t\left(\frac{\Theta_{t+1}-\Theta}{\Theta}\right) + \beta\hat{E}_t s_{t+1}, \tag{C.19}$$

and then (C.16) comes from acknowledging that $1-\beta$ is very small for $\beta$ close to 1.

Note that this is a system of seven equations and seven unknowns due to the fact that we eliminated $C_t$ and $N_t$ and do not explicitly have to keep track of $W_t$ and $H_t$.

## Appendix D Additional figures and bifurcation diagrams (for online publication only)

Figures D.11 and D.12 show the bifurcation diagram for $\gamma_s$. An increase in trend extrapolation has a similar effect as an increase in bias. The amplitude of cycles increases with $\gamma_s$, for parameter values larger than 1.2, the trajectory again approaches a homoclinic orbit. For higher values the system exhibits explosive dynamics. An increase in the behavioral parameters $\gamma_b$ and $\gamma_s$ hence implies two effects. The *quantitative* aspect is, that the amplitude increases with increases in the parameters. The *qualitative* aspect is that the type of dynamics can also change. Figure D.10 shows the time series of trader types and profits for each of the times for the limit cycle/financial cycle.

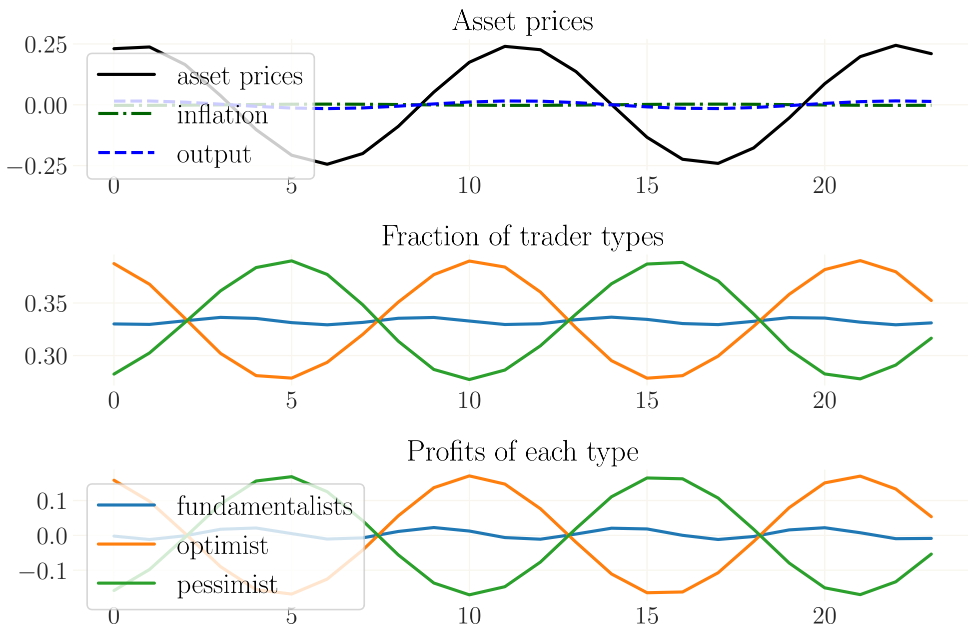


Figure D.10: *Top panel:* time series of asset prices, inflation and output at the estimated parameters (s. Table 2) in the absence of additional shocks. *Middle panel:* switching dynamics of the different trader types. *Bottom panel:* the cyclical component is reflected in changes in profitability of each trader type.

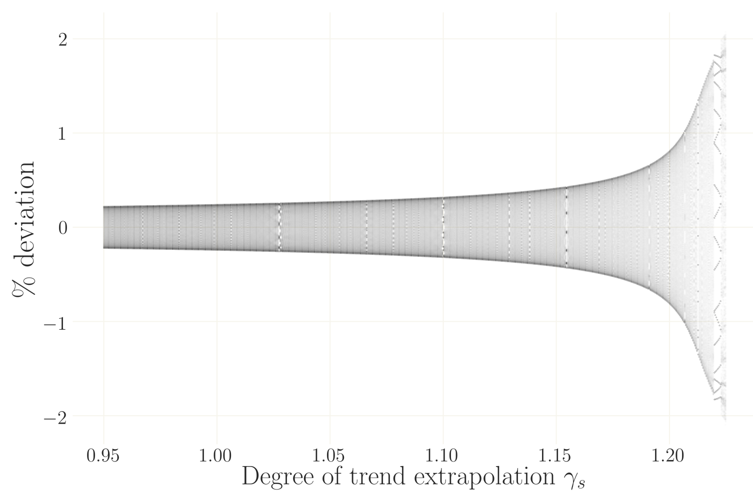


Figure D.11: Bifurcation diagram of inflation and output w.r.t. $\gamma_s$. All other parameters as in Table 2.

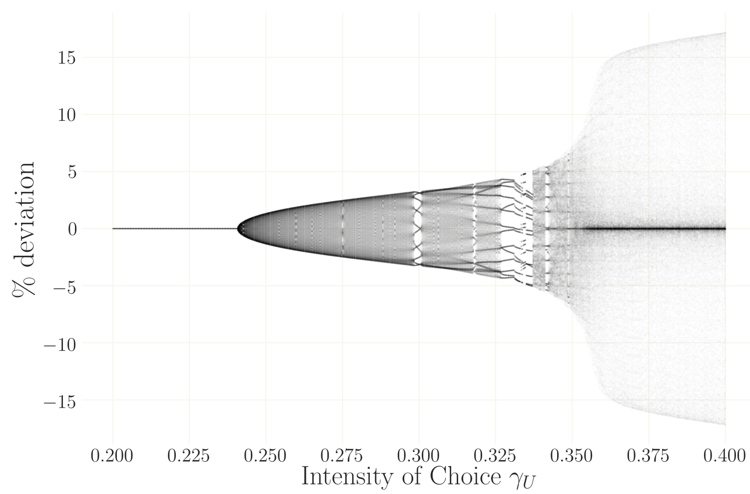


Figure D.12: Bifurcation diagram of inflation and output w.r.t. $\gamma_U$. All other parameters as in Table 2.