The Micro and Macro of (Unconventional) Monetary Policy: the Role of the Banking Sector

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preliminary

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Abstract

Macroeconomic theory assigns a central role to the risk free savings rate, which in reality corresponds to the banks' deposit rate that is only indirectly controlled by the central bank. Backed by Euro-Area evidence, this paper shows that the pass-through of central bank reserves and interest-on-reserves policy on equilibrium rates may be state-dependent. I develop an industrial organization model of the banking sector where loans are financed by deposits and banks use reserves to hedge against liquidity risk from holding deposits. Swapping assets and reserves (i.e. central bank asset purchases) compresses the liquidity premium between lending and deposit rate, thereby stimulating lending. If the lending rate falls by less than the liquidity premium (if investment demand is elastic, e.g. in a recession), deposit rates may *increase*, thereby dampening consumption. Additionally, inflation may remain subdued as lower lending rates reduce firms' financing costs. Incorporated into a DSGE model, the estimated model suggests that the effects of the ECB policy amount to 0.25 percent of quarterly GDP, and the effects on inflation is negligible.

Keywords: Excess Reserves, Liquidity Facilities, Monetary Theory, Nonlinear Bayesian Estimation
JEL: E63, C63, E58, E32, C62

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1 Introduction

For more than a decade, central banks in major advanced economies have been resorting to unconventional tools such as large scale asset purchasing programs. Under these programs, a central bank buys assets from commercial banks in exchange for reserves which they then hold at their central bank account. With key central bank interest rates at the lower bound, the intention is to expand lending and provide liquidity, thereby further lowering interest rates in order to stimulate spending, investment and, thereby, inflation. Yet, the actual macroeconomic effects of asset purchases programs remain, to this day, controversial.

This paper asks a central question: what are the macroeconomic effects of unconventional monetary policy, especially asset purchasing programs, and what are the channels through which it operates? Macroeconomic theory assigns a central role to the risk free savings rate. In reality, this corresponds to the rate banks pay on private sector deposits, which the central bank controls only indirectly by means of reserves operations and by setting the interest rate on reserves. Based on an industrial organization model of the banking sector, I present a theory of the transmission of conventional and unconventional monetary policy through the banking sector. I show that this transmission can, depending on the state of the economy, be incomplete. Key to assessing the effectiveness of central bank policy measures is to understand why and when banks hold excess reserves, and how they pass-through to equilibrium rates.

Evidence suggests that a significant share of the central bank liquidity injections did not translate into additional loans, but led to the accumulation of massive excess reserves. The top panel of Figure 1 documents the evolution of banks' reserves in the Euro Area. Reserves effectively mirrored the minimal reserves requirement (MRR) until the onset of the Great Financial crisis in 2008Q4, implying that banks provision of deposits was constrained by the amount of reserves provided by the ECB. However, reserves built up over time as the ECB implemented their asset purchasing programs in 2012, 2014 and 2020 (vertical dashed lines), but were not matched by additional deposits. By 2022, reserves have reached a level 25 times the minimum requirement.

The bottom panel of Figure 1 shows the evolution of the bank deposit rate (the risk-free saving rate of the private sector), the BAA yield (a proxy for the bank lending rate), the EONIA (the overnight interbank market lending) and the ECB deposit facility rate (DFR), which is the rate which banks may use to make overnight deposits with the Eurosystem. Prior to 2009, all interest rates closely moved in tandem, consistent with standard theory (Bindseil, 2014). When the MRR is binding, reserves are scarce, and the amount of bank deposits is constrained by the minimal reserves requirement constraints. Hence, the deposit rate moves one-for-one with the market rate for reserves, the EONIA. Since banks require deposits to grant loans, the MRR is equivalently a constraint on lending activity.

This close comovement stopped after 2008 when the ECB reduced key central bank rates to levels close to zero. As the reduction of the EONIA rate required larger open market operations – that is, asset purchases – reserves became abundant and, in consequence, the EONIA rate quickly detached from the private sector deposit rate. Even more surprisingly, when the ECB launched their first asset purchases program in 2012, the deposit rate became significantly detached from the movements of lending rate and DFR and remained elevated even when other rates decline. Yet, the movement of the

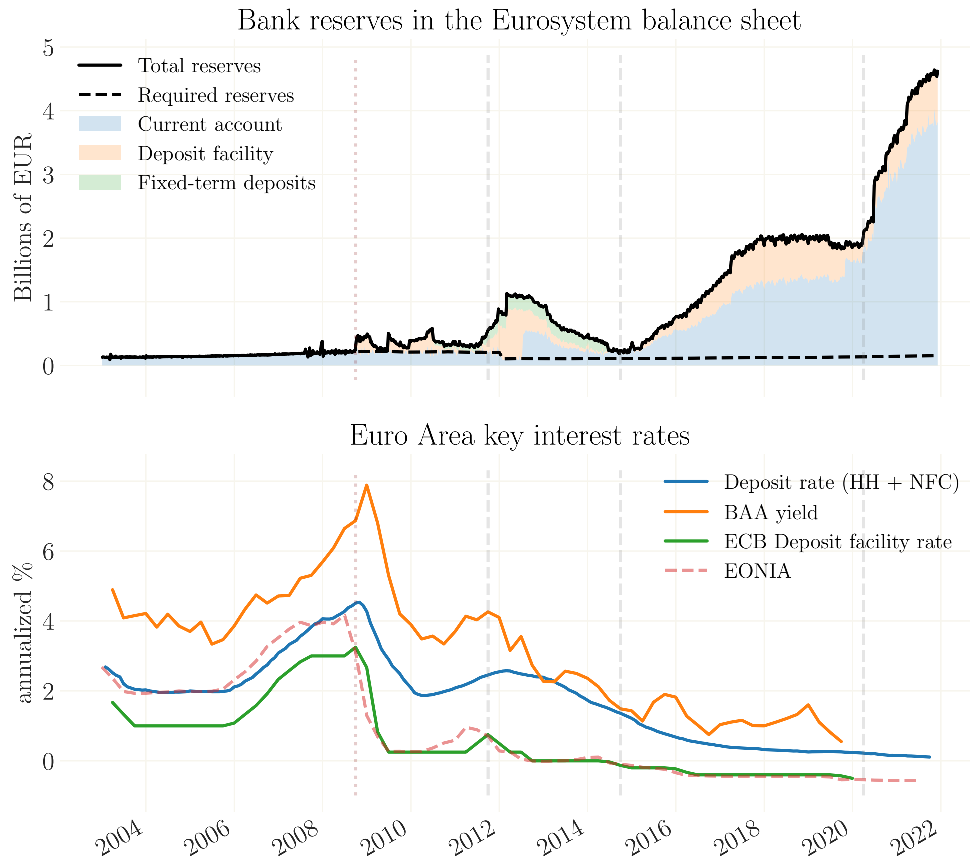

Figure 1: **Top panel:** reserves stored at the ECB, decomposed into the type of liability. Vertical dashed lines are the GFC and the announcements of LTROs, QE and the PEPP programme. **Bottom panel:** key interest rates (annualized) in the Euro Area. The DFR (deposit facility rate) is the rate payed on reserves in excess to required reserves. The deposit rate is a weighted average over different liquidity classes of bank deposits and the BAA yield is a measure for banks' lending rate. The EONIA rate stands for the interbank market rate Source: ECB SDW.

deposit rate might have unintended feedback effects at the aggregate level. Higher deposit rates encourages (relatively) more saving and distort the households' intertemporal consumption decisions, a key narrative of modern New Keynesian theory. Hence, in general equilibrium, the economy can end up with *less* spending and investment and lower inflation.

In this paper, I develop a tractable industrial organisation model of the banking sector to investigate the decisions of banks in response to central bank interventions at the micro level. I then incorporate the banking model into a New Keynesian medium-scale DSGE model to analyze the implications of the banking sector at the macro level. Based on this model, I develop a unifying theory of the transmission of monetary policy through the banking sector. The theory is unifying in so far, as it explains the pass-through of reserves and reserves rate policy, both in normal times *and* in times of unconventional monetary policy.

In my baseline model, banks face a liquidity problem similar to Poole (1968): lending activity is financed by deposits, and deposits may be withdrawn or wired to other banks. The corresponding transfers must be settled in central bank reserves. Banks hence wish to hold reserves to hedge the liquidity risk associated with lending. This liquidity risk is reflected in the spread between lending and deposit rate, the so-called interest rate margin.

Banks wish to hold some reserves to hedge against liquidity risk but their optimal level depends on the opportunity costs of reserve holdings. They choose to hold only the mandatory minimum reserve level (the MRR) when the opportunity costs in form of returns from granted loans are high. Yet, they are willing to hold reserves in excess of the MRR when the central bank offers to exchange reserves for assets at a competitive price or if returns on issued loans are low.

Monetary policy can work through asset purchases (also called "open market operations" when the MRR binds) or through setting the interest rate on reserves. Its effects, however, depend on whether the MRR is binding or slack. I show that, since mandatory reserves are generally relatively small (i.e. much less than 5% of total dopsits), the effectiveness of setting the interest rate on reserves on banks investment decisions is fairly limited if the MRR is binding. In contrast, setting the interest rate on reserves is very effective when the MRR is *slack*, i.e. when banks are holding excess reserves. In the latter case, changes in the reserves rate change deposit and lending rates almost one-for-one, before the zero lower bound constraint on deposit rates is reached.

Asset purchases (in exchange for reserves) are most effective when the MRR is binding. At the MRR, an expansion in the supply of reserves will be transformed one-for-one into loans and deposits. Central bank intervention can effectively determine the volume of lending, whereas the interest rate margin, i.e. the price for loans relative to deposits, will adjust in equilibrium. Away from the MRR, only part of the extra reserves will be transformed into loans. Their share depends on the equilibrium interest rate margin, where more reserves further lower the price for loans. Central bank intervention now effectively determines the interest rate margin.

How do lending and deposit rates move with a lower interest rate margin? The implications for the lending rate is fairly uncontroversial. Assuming a standard (downward-sloping) demand function for loans, more lending means a lower lending rate. However, the implications for the deposit rate are, a priori, ambiguous. By definition, the deposit rate is the difference between the lending rate and the interest margin. A fall in the interest margin hence *raises* the deposit rate, and their general equilibrium response then depends on the magnitude of the response of the lending rate. This, in turn, is determined by the elasticity of the demand for funds, that is, of aggregate investment. If the demand for funds is inelastic, a small increase in lending volume causes a larger fall in the lending rate which outweighs the positive effect of the interest margin, and the deposit rate falls. If however the demand for funds is elastic, i.e. if the lending rate does not react much to changes in the lending volume, the interest margin may shrink by more than the fall of the lending rate, actually causing the deposit rate to increase.

This can generates negative feedback effects at the macroeconomic level. Higher deposit rates, consistent with what has been observed in the early 2010s, can induce household to save more rather than spend, which reduces economic activity and inflation.

This novel finding illustrates the downside risk of central banks' asset purchases and has not been discussed so far in the literature. The second key finding is that, even if central banks' asset purchases are expansionary, their effects can diminish over time. This again depends on the elasticity of demand for loans. If the elasticity is very low, for example if there is lack of appetite for loans or saturated demand, the implications of asset purchases become negligible.

The predictions of my model suggest that the effectiveness of asset purchases are state-dependent. At the MRR, money supply (as measured by the volume of deposits) increases one-to-one with the volume of reserves that the central bank is willing to provide. However, the monetary multiplier collapses once banks hold excess reserves. The overall effect then depends on the price elasticity of investment: In booms, when the demand for loans tends to be less price sensitive, asset purchases may be highly effective in boosting lending. In recessions, when the economic outlook is depressed and firms are reluctant to invest, asset purchases may be less effective or even contractionary.

For tractability, I provide closed form solutions for major parts of the model. The MRR is an occasionally binding constraint which is binding or slack depending on the banks' marginal profits from reserves. The MRR is binding when marginal profits are negative and it is slack when marginal profits are zero. In the first step, I use this simple model of the banking sector to provide theoretical insights into the transmission of monetary policy at the banks' level. To asses the macroeconomic impact, I then implement the banking model into a New-Keynesian medium-scale DSGE model for the Euro Area. Finally, for a quantitative analysis, I estimate the model using nonlinear methods to account for the interest rate lower bounds. To identify the effects of the ECB's unconventional policy, I use data on deposit and lending rates, ECB balance sheet data, as well as standard macroeconomic variables (e.g. GDP and inflation). I provide estimates for the threshold of the reversal IOR rate. While the reversal rate has not been reached by the end of my sample in 2021, the results suggest that there was limited space before the threshold is hit.

Counterfactual analyses suggest that the measures of liquidity provision and quantitative easing had only a small effect on output – about one-quarter of quarterly GDP – and almost no effect on inflation. The latter stems from two opposite effects on inflation. On the one hand, asset purchases can help to reduce the household deposit rate and encourage more spending. All else equal, this has inflationary effects. However, on the other hand, asset purchases also reduce the lending rate for firms which turn reduces production costs. This has deflationary effects. Overall, the aggregate effect of inflation depends on the relative demand and supply effects. While asset purchases had limited effects, I document that the ECB's negative interest rate policy was quite successful in stimulating the economy with an effect of about one percent (quarterly) on GDP and a quarter percent on inflation. This is due to relatively stronger effects on consumption than on firm investment.

Related literature. This paper is most closely related to Bianchi and Bigio (2022), who also build on a banking model with liquidity frictions in the spirit of Poole (1968) and Frost (1971). While their banking model goes into greater detail and features an OTC interbank market, the main focus of their analysis lies on the empirical dynamics of the US interbank market, instead of studying the macroeconomic effects of (unconventional) monetary policy as in this paper. Afonso and Lagos (2015) also focusses on the US market for federal funds is treated in a similar fashion, but rather from the finance perspective. The models from those two papers are too complex to be included into a medium-scale DSGE model. Still, and arguably more parsimoniously, my model is able to reproduce the key mechanism discussed in both of the above papers. The work of Acharya and Rajan (2022) studies the effects the massive excess reserves holdings on the banking sector in the US from the perspective of the finance literature. They conclude that the effects of central bank liquidity provisions may go in either direction because as a response to such policy, banks may provide less for potential episodes of stress. Drechsler et al. (2017), much in line with my arguments, focus on the deposit side of the banking market and show that market concentration can have an important impact the pass-through of monetary policy.

One of the first papers that study the connection of the central bank balance sheet with macroeconomic dynamics is Cúrdia and Woodford (2011). Piazzesi et al. (2019) also incorporate a banking model into a NK framework to study the macroeconomic effects of a floor vs. a corridor system. Benigno and Nisticò (2020) study the interaction of the central bank with the fiscal authority. Diba and Loisel (2021) show that incorporating a reduced-form model of the banking sector in a New Keynesian (NK) framework allows to address some of the puzzles associated with the standard NK framework. My model confirms these findings, and its IO approach adds a profound microfoundation to the channels discussed in their paper.

A fast growing literature investigates the effects of negative interest policies. To only mention a few, Brunnermeier and Koby (2018) develops the concept of a *reversal rate* at which further decreases of the policy rate will have different (and often unwanted) macroeconomic effects. Heider et al. (2019) find in an empirical study that negative interest rates lead to less lending activity. In contrast, Demiralp et al. (2017); Altavilla et al. (2021), find no or rather positive effects of negative rates on lending and on firm activity. Eggertsson et al. (2019) also study the pass-through of negative interest rates to lending activity in a theoretical framework. They document limited pass-through of interest rate policy once the deposit lower bound is met.

Outline. The rest of this paper is structured as follows. Section 2 presents the IO model of the banking sector, and section 3 discusses the theoretical implications of this model. In section 4 the model in embedded into a medium-scale DSGE model and I present the setup for Bayesian estimation. Simulations and empirical results are presented in section 5. Section 6 concludes.

2 An IO Model of the Banking Sector

Banks lend funds to firms and the government in the form of (claims to) physical capital K_t and government bonds B_t. Loans are financed by deposits D_t, which are provided by households. Banks can exchange government bonds against interest-bearing reserves J_t at the central bank. The balance sheet of bank i then reads

$$Q_t^b B_{i,t} + Q_t K_{i,t} + J_{i,t} = D_{i,t}, \tag{1}$$

where Q_t^b is the price for government bonds and Q_t the price of one unit of capital. In the absence of additional frictions the Modigliani-Miller theorem holds, which allows to abstract from explicitly tracking banks' net-worth.[1]

Households use deposits as a medium of exchange for their expenditures which means that from the perspective of bank i, deposits may be subject to wire transfers to other banks. Assuming that assets $B_{i,t}$ and $K_{i,t}$ are illiquid during period t, banks use reserves to settle the associated cross-bank transfers. This implies that each bank is subject to liquidity risk: given a series of transfers, they may be temporarily unable to execute further transactions. Banks hence face a portfolio problem of holding assets (for their return) versus reserves (for their liquidity value).

Denote bank i's net outflow of deposits in period t through transfers by $\Delta D_{i,t}$. For each transfered unit of $\Delta D_{i,t}$ that exceeds the current stock of reserves $J_{i,t}$ the banker has to pay a cost γ. γ could for example be associated with the interbank lending spread, or the penalty rate for overshooting the discount window. Let χ be the unconditional probability for one unit of deposits to be transfered,[2] and note that the probability for any unit of deposits that is transfered from any bank to end up at bank i is given by the fraction $\frac{D_{i,t}}{D_t}$ of deposits that bank i already holds. Proposition 1 states that the distribution of $\Delta D_{i,t}$ approximately follows a normal distribution.

Proposition 1 (Liquidity risk)**.** *Given the probability χ that any unit of deposits get withdrawn, and the probability $\frac{D_{i,t}}{D_t}$ that any withdrawn unit (from any bank) is transfered to bank i, the probability for the event that $\Delta D_{i,t} = x$ for any $x \in \mathbb{R}$ is approximately normally distributed with*

$$\Delta D_{i,t} \sim \mathcal{N}\left(0, \frac{D_{i,t}D_{-i,t}}{D_t}(2\chi - \chi^2)\right). \tag{2}$$

Proof. See Appendix A.1. ■

Under the simplifying assumption that bankers are risk-neutral, this allows for an analytical expression for the expected costs of excess withdrawals, which is given in proposition 2. These are given by the expected value of the excess withdrawals conditional on excess withdrawals being positive, $E[\Delta D_{i,t} - J_{i,t} | \Delta D_{i,t} > J_{i,t}]$.

Proposition 2 (Liquidity costs)**.** *If bankers are risk-neutral, the expected volume of withdrawals in excess of reserves holdings is*

$$g(J_{i,t}, D_{i,t}) = E[\Delta D_{i,t} - J_{i,t} | \Delta D_{i,t} > J_{i,t}], \tag{3}$$

$$= h(D_{i,t}) f\left(J_{i,t} | 0, h(D_{i,t})\right) - J_{i,t}\left[1 - F\left(J_{i,t} | 0, h(D_{i,t})\right)\right], \tag{4}$$

with $h(D_{i,t}) = \frac{D_{i,t}D_{-i,t}}{D_t}(2\chi - \chi^2)$ and where $f\left(J_{i,t}|0, h(D_{i,t})\right)$ is the PDF of the normal distribution at $J_{i,t}$ with mean zero and variance $h(D_{i,t})$.

Proof. See Appendix A.2. ■

[1] See section 3 for a more detailed discussion.

[2] A somewhat more involved specification would link the *total number of transactions* in the economy to the volume of consumption expenditures. While this results in a rather complicated mathematical representation of the variance of in- and outflows, the model implications would remain similar.

Denote by $R_{i,t}$ the (gross) nominal rate bank i pays on households' deposits. Households can chose to hold cash instead of deposits, but, since deposits are perfectly safe and liquid for households, only have incentive to do so if $R_{i,t} < 1$. This gives rise to a zero lower bound on deposit rates (deposit lower bound, DLB). The banks' deposit services are heterogeneous (e.g. through diversification of services) and banks have some degree of market power (similar to Ulate (2021)). The aggregator takes the form

$$D_t = N^{1-\epsilon_D} \left(\sum_i^N D_{i,t}^{1/\epsilon_D} \right)^{\epsilon_D}, \tag{5}$$

where N is the number of banks. Then, bank i faces an inverse supply function of the form

$$\frac{R_{i,t}}{R_t} = N^{\frac{1-\epsilon_D}{\epsilon_D}} \left(\frac{D_{i,t}}{D_t} \right)^{\frac{1-\epsilon_D}{\epsilon_D}}, \tag{6}$$

$$R_{i,t} \geq 1, \tag{7}$$

where the latter is the DLB and it is assumed that $\epsilon_D \in (0,1]$. For a symmetric equilibrium it follows that

$$D_t = N D_{i,t}, \tag{8}$$

$$R_t = R_{i,t}. \tag{9}$$

Similarly, loan services to firms, $Q_t K_{i,t}$, are heterogeneous and bank i is facing the inverse demand function

$$E_t \left\{ \frac{R^k_{i,t+1}}{R^k_{t+1}} \right\} = N^{\frac{1-\epsilon}{\epsilon}} \left(\frac{K_{i,t}}{K^b_t} \right)^{\frac{1-\epsilon}{\epsilon}}. \tag{10}$$

with $\epsilon \geq 1$.

Following Woodford (2001), government bonds are modeled as perpetuities with decaying coupon payments. Let $\kappa \in [0,1]$ denote the decay parameter for coupon payments. The expected per-monetary-unit return on government bond holdings is then given by[3]

$$E_t R^b_{t+t} = E_t \left\{ \frac{1 + \kappa Q^b_{t+1}}{Q^b_t} \right\}. \tag{11}$$

The bond market clears with $\sum_i B_{i,t} = B^b_t$ (government bonds held by commerical banks) and $B_t = B^b_t + B^{cb}_t$, that is, commerical banks and the central bank together hold all bonds.

A regulatory authority enforces an (occasionally binding) minimal reserve requirement (MRR) of

$$\psi D_{i,t} \leq J_{i,t}, \tag{12}$$

and excess reserves are hence, if any, given by $J_{i,t} - \psi D_{i,t}$. The necessary conditions for an equilibrium of the banking sector are given by proposition 3.

[3]Variables with subscript t are those set in period t. All interest rates are given in nominal terms.

Proposition 3 (Equilibrium of the banking sector)**.** *Under the assumptions that*

1. *each bank i takes as given*
   (a) *the aggregate equilibrium variables $\{K_t, D_t, E_t R^k_{t+1}, E_t R^b_{t+1}, R_t\}$,*
   (b) *the nominal interest rate on reserves (IOR) rate R^j_t and the supply of reserves J_t,*
   (c) *the cumulated deposit choice of competitors $D_{-i,t}$;*
2. *the equilibrium is symmetric,*
3. *no entry and exit,*

and with $\nu = (N-1)(2\chi - \chi^2)$, $\hat{D}_t = D_t/\nu$ and $\hat{J}_t = J_t/\nu$, a competitive equilibrium in the banking sector is given by

$$R^k_{t+1} = \epsilon R^b_{t+1}, \tag{13}$$

$$Q^b_t B^b_t + Q_t K^b_t + J_t = D_t, \tag{14}$$

$$R_t/\epsilon_D = \max\left\{1/\epsilon_D,\ (1-\psi)R^b_{t+1} + \psi R^j_t + \gamma\left(\psi\left[1-\hat{F}\right] - 0.5\hat{f}\right)\right\}, \tag{15}$$

$$R^j_t - R^b_{t+1} + \gamma\left[1-\hat{F}\right] = \min\left\{0,\ R^j_t - R^b_{t+1} + \gamma\left[1-\hat{F}_\psi\right]\right\}, \tag{16}$$

where $\hat{f} = f\left(\hat{J}_t|0,\hat{D}_t\right)$, $\hat{F} = F\left(\hat{J}_t|0,\hat{D}_t\right)$, $\hat{f}_\psi = f\left(\psi\hat{D}|0,\hat{D}\right)$ and $\hat{F}_\psi = F\left(\psi\hat{D}|0,\hat{D}\right)$ are shorthand for the PDF and CDF of the normal distribution.

Proof. See Appendix A.3. ■

Equation (13) is the optimality condition w.r.t. physical capital and (14) is the aggregated bank balance sheet. (15) is the aggregated optimality condition for taking deposits, which equates the marginal profit of an additional unit of loans to the marginal costs of an additional unit of deposits, including the associated marginal liquidity risk. (16) is the optimality condition for holding reserves, linking the marginal profit of an additional unit of loans to the associated marginal increase in liquidity risk by reducing reserves holing. The equations (15) and (16) are central in this paper because they detemine the equilibrium deposit and lending rates. Note that both equations are expressed such that the minimal reserve requirement and the deposit lower bound act as an occasionally binding constraint.

The expression at the LHS of (16) corresponds to the marginal profit of reserve holdings, which is given by

$$MPJ\left(R^j_t, R^b_{t+1}, J_t, D_t\right) = R^j_t - R^b_{t+1} + \gamma\left[1 - F\left(\hat{J}_t|0,\hat{D}_t\right)\right]. \tag{17}$$

The MRR is binding whenever $MPJ_\psi = MPJ\left(\cdot,\cdot,\psi D_t, D_t\right) < 0$, i.e. when marginal profits of reserves *at the MRR* are negative. In this case equation (16) simply collapses to $J_t = \psi D_t$. If however $MPJ_\psi \geq 0$, banks have an incentive to hold excess reserves and an interior solution with $J_t > \psi D_t$ exists. Equation (16) then simply reads $MPJ_t = 0$. Thus, if banks are willing to hold excess reserves, these are determined by a conventional optimality condition which equates marginal profits to zero. Respectively, the DLB translates directly to a constraint on the optimality condition for deposits, which becomes inactive once the DLB binds.

Proposition 3 additionally reveals that the parameters N and χ can be summarized by the composite parameter for liquidity risk, ν, which scales J_t and D_t. Intuitively, a larger probability χ that deposits being withdrawn has a similar effect as a larger number of banks N, because a high N decreases the probability that deposits that are withdrawn from bank i will return to i. In the following $\hat{J} = \frac{J}{\nu}$ and $\hat{D} = \frac{D}{\nu}$ will be called *effective* reserves and deposits.

In this system banks are price-takers and equilibrium rates are equal to marginal costs in addition to monopolistic markups. The structure of the cost function gives rise to spreads between borrowing, lending, and the IOR rate. The next section analyses how these spreads and other monetary aggregates respond to monetary impulses in terms of variations in the IOR rate and the supply of central bank reserves.

3 Theoretical Insights: Transmission of (unconventional) Monetary Policy

The central bank can independently control the supply of reserves J_t and the interest rate on reserves (IOR) R_t^j of the economy. This section provides analytical insights into how the choice of the policy tools $\{J_t, R_t^j\}$ transmit through the banking sector,

a) to control the lending rate R_t^b, which determines the firms investment in capital and the cost of government debt,
b) to control the interest margin $s_t^m = R_t^b - R_t$, and
c) to control the household deposit rate R_t, which drives the households' consumption-savings decision and which results from the two above.

For the sake of simplicity I abstract from monopsonistic power in the deposit market throughout this section. To also simplify display let me define a composite asset $A_t = Q_t^b B_t + Q_t K_t$ and, since the banking problem is static, drop time subscripts.

Let asset holdings by commercial banks and the central bank be given by A^b and A^{cb} respectively. The aggregate bank balance now reads $A^b + J = D$ while the central bank balance sheet is given by $A^{cb} = J$. Assume that investment demand for assets is given by the demand function $A/\nu \equiv \hat{A} = d_A(R^b)$ with $\frac{\partial d_A}{\partial R^b} \leq 0$ and denote the demand elasticity (with respect to a one-percentage change in R^b) as $E_A = \frac{\partial d_A / A}{\partial R^b}$. From combining asset market clearing $A = A^b + A^{cb}$ with the two balance sheet equations, the equilibrium in the banking sector is given by

$$d_A\left(R^b\right) = \hat{D}, \tag{18}$$

$$R = \max\left\{1,\ (1-\psi)R^b + \psi R^j + \gamma\left(\psi\left[1-\hat{F}\right] - 0.5\hat{f}\right)\right\}, \tag{19}$$

$$R^j - R^b + \gamma\left[1-\hat{F}\right] = \min\left\{0,\ R^j - R^b + \gamma\left[1-\hat{F}_\psi\right]\right\}, \tag{20}$$

where (19) corresponds with the banks' optimality condition w.r.t. deposits, (15), and (20) corresponds with the optimality condition w.r.t. reserves (16). Given the policy variables $\hat{J}$ and R^j, these are three equations for the three unknowns R^b, R and $\hat{D}$. Note that in order to solve for an equilibrium it is not necessary to specify the households' supply of deposits.

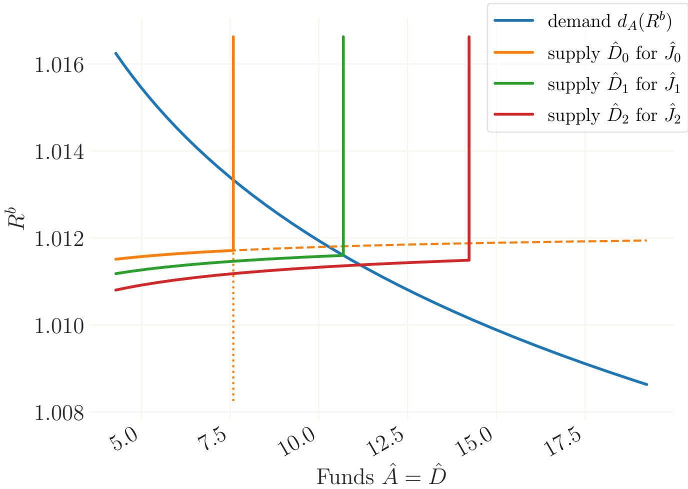

Figure 2: Banking equilibrium at the market for loans for a given demand function. Reserves increase exogenously from $\hat{J}_0 = 0.171$ (orange curve) to $\hat{J}_1 = 0.241$ (green) to $\hat{J}_2 = 0.321$ (red). This shifts the loan supply curve (eqns. (19) and (20)) outwards and downwards. In the equilibrium for $\hat{J}_0$ the MRR (dotted line where $\psi D = J$) is binding. The dashed line depicts the hypothetical supply of loans without MRR. In the equilibrium for $\hat{J}_1$ the MRR is just binding and for $\hat{J}_2$ the MRR is slack and banks voluntarily hold excess reserves. Parameters are $\psi = 2.26\%$, $R^j = 1$, and loan demand is assumed to follow $d_A(R^b) = 0.977{R^b}^{0.005}$.

Equations (18) to (20) provide an intuition for why the introduction of banks net worth would not fundamentally change the implications of the model: in a competitive equilibrium where banks accumulate net worth and maximize the expected stream of dividends, the optimality condition for net-worth accumulation would require that R^b equals the inverse of the discount factor times a liquidity premium. Under conventional assumptions the relative demand for net-worth is then increasing in R^b. Concurrently, a rise in R^b causes the lending volume A to fall since $E_A < 0$, thereby decreasing liabilities. If this would cause a (relative) increase in net-worth, then the decrease in deposits must be disproportionately high, thereby even amplifying the channel presented here.

Figure 2 illustrates the equilibrium in the loan market for different levels $J_0 < J_1 < J_2$ of reserves. Loan demand (blue line) is assumed to follow a standard, downwards sloping function. The MRR lends a hockey-stick-shape to the supply function, that is composed by two segments: the horizontal orange line (dashed once $\psi D > J$) represents the supply of loans in the absence of the MRR, i.e. all points where R^b equals the marginal costs of lending. This function is upwards sloping because, as loans require deposits, additional loans increase liquidity risk and thereby raise the associated marginal costs of lending. For the same reason (loans require deposits), the total volume of loans is constrained by the level of reserves supplied by the central bank, J, which leads to the vertical orange line (dotted wherever $MPJ > 0$) where $J = \psi D$. In the equilibrium for J_0 the MRR is binding. If the central bank expands its supply of reserves from J_0 to J_1, this shifts the vertical line outwards (more reserves enable more deposits if the MRR binds) and moves the horizontal line downwards because more reserves also mitigate liquidity risk.

The new equilibrium is exactly at the kink where the MRR starts binding. Finally, an additional increase in the supply of reserves $J_1 \rightarrow J_2$ will again shift both curves outwards/downwards and banks decide to hold some of the newly supplied reserves as excess reserves, i.e. the MRR is slack.

Below, I discuss the underlying mechanism and its implications on the effects of monetary policy measures in detail. I first derive general results for the case in which banks hold excess reserves, and then focus on the equilibrium where the MRR binds.

3.1 The economy with excess reserves

When neither the MRR nor the DLB binds, equations (19) and (20) from proposition 3 collapse to

$$R^b = R^j + \gamma \left(1 - \hat{F}\right), \tag{21}$$

$$R^b = R + 0.5\gamma \hat{f}, \tag{22}$$

additional to the equilibrium in the loan market given by (18) and, as in proposition 3, $\hat{f} = f\left(\hat{J}_t | 0, \hat{D}_t\right)$ and $\hat{F} = F\left(\hat{J}_t | 0, \hat{D}_t\right)$.

Recall that effective reserves and deposits $\hat{J}$ and $\hat{D}$ are simply reserves and deposits scaled by liquidity risk ν. Independent of the demand for loans, for a given IOR rate and a given ratio $\theta = D/J$, the lending rate and the deposit rate can be expressed in terms of ν. The findings are summarized in proposition 4.

Proposition 4 (Liquidity regimes)**.** *Ceteris paribus, if the DLB is slack, for a given IOR rate R^j and for a given ratio of deposits-reserves ratio D/J, the economy knows two liquidity regimes.*

1. *The deposit rate R is*
   (a) *increasing in liquidity risk ν if $\nu < (2J - J^2/D)$,*
   (b) *decreasing in liquidity risk ν if $\nu > (2J - J^2/D)$, and*
   (c) *goes from $\lim_{\nu \to 0} R(\nu) = R^j$ to $\lim_{\nu \to \infty} R(\nu) = -\infty$.*
2. *The lending rate R^b is continuously increasing in ν, bounded below by R^j for $\nu = 0$, and converges to 0.5γ when $\nu \to \infty$.*

Proof. See Appendix A. ■

Figure 3 illustrates the relationship between $\left\{R^b, R\right\}$ and ν graphically. When liquidity risk is low, expected liquidity costs are negligible and banks directly pass-through the IOR rate to market rates. The spread between the lending rate and the IOR rate can be interpreted as a liquidity premium: holding reserves implies to resign on lending returns in exchange for liquidity. The willingness to accept such premium increases as liquidity risk increases. When keeping D/J fixed and increasing liquidity risk, from a certain point the marginal benefit of holding reserves does not increase any further. This reflects that for a given, large liquidity risk a marginal unit of reserves is less effective in mitigating risk than when liquidity risk is low. A representation of the deposit rate relative to the IOR rate can be obtained by inserting (22) into (21),

$$R = R^j + \gamma \left(1 - \hat{F} - 0.5\hat{f}\right), \tag{23}$$

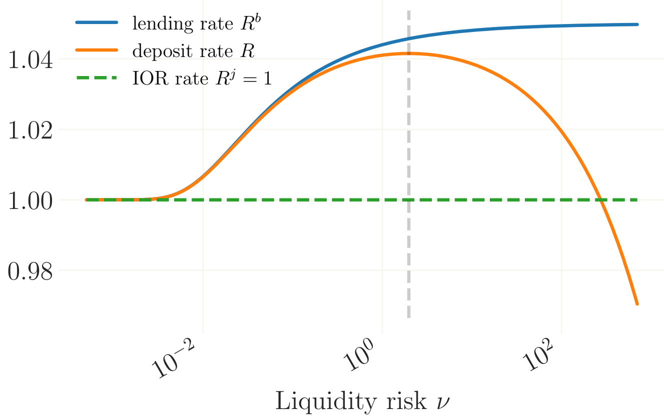

Figure 3: Lending rate R^b and deposit rate R as a function of liquidity risk ν, given the IOR rate $R^j = 1$. The reserve-deposits ratio $J/D = 2.26\%$ is fixed, reserves are set to unity and $\gamma = 0.1$. The vertical dashed line denotes maximum of R at $\nu = 2J - J^2/D$.

revealing that R it the net between R^b and the interest margin $s_m = \gamma 0.5\hat{f}$. Similar as the spread between R^b and R^j, s_m is continuously increasing in liquidity risk. Other than the former, s_m only increases gradually when liquidity risk increases, reflecting that for low liquidity risk a marginal unit of deposits does not alter expected liquidity costs as much as an additional unit of reserves. This causes the early co-movement of R^b and R as liquidity risk increases. While R^b converges, s_m continues to increase when liquidity risk rises because for large liquidity risk with $\nu > 2J - J^2/D$, an additional unit of deposits will raise expected liquidity costs more strongly.

Let us now turn to the effect of changes to the supply in reserves. Reserves can either be supplied via asset purchases (at normal times termed *open market operations*) or refinancing operations such as Long-term Refinancing operations (LTROs). In both cases the central bank exchanges reserves for government bonds or private assets. I will here refer to any such policy simply as *asset purchases*. The effect of any excess reserves policy can be decomposed into the direct (positive) effect of mitigating liquidity risk by providing additional reserves (the *liquidity effect*), and an indirect negative effect that I will call *the deposit effect*. In terms of figure 3, the liquidity effect can be illustrated as moving along the horizontal axis to the left. The deposit effect arises because any decrease in the lending rate R^b will lead to a surge in the demand for loans, which, as loans create deposits, will lead to an expansion of the volume of deposits. This in turn does lead to a relative increase in liquidity risk. To quantify the relative impact of both effects, it is useful to introduce the elasticity of deposits to reserves.

Proposition 5 (Elasticity of deposits to reserves)**.** *If MRR and DLB are slack, for* $0 \leq J \leq D$*,* $0 < D$ *and given* R^j*, the elasticity* E_{DJ} *of deposits with respect to reserves is*

$$E_{DJ} = \frac{1}{0.5 - \left(\gamma E_A \hat{J} \hat{f}\right)^{-1}} \in (0, 2). \tag{24}$$

Proof. See Appendix A.5. ■

Intuitively, proposition 5 states that the marginal effect of asset purchases on asset holdings (and thereby, deposits) cannot be negative since, given downwards sloping demand for loans, a fall in A would require an increase of R^b. However, the banks' optimality condition (21) only allows R^b to increase if either J declines or A expands. Similarly, $E_{DJ} > 2$ would imply that the effect of an increase in deposits would outweigh the liquidity effect which would increasing R^b.

Proposition 6 (Effectiveness of excess reserves)**.** *If MRR and DLB are slack, for* $0 \leq J \leq D$, $0 < D$ *and a given* R^j, *any policy that swaps assets against excess reserves*

1. *always reduces the lending rate* $R^b > R^j$ *with a pass-trough of*

$$\frac{\partial R^b}{\partial J/J} = \gamma \hat{J}(0.5E_{DJ} - 1)\hat{f}, \tag{25}$$

and decreasing marginal efficiency if $E_{DJ} \to 0$,

2. *always reduces the interest margin* s_m *with a pass-trough of*

$$\frac{\partial s_m}{\partial J/J} = 0.5\left[0.5\left(\frac{\hat{J}^2}{\hat{D}} - 1\right)E_{DJ} - \frac{\hat{J}^2}{\hat{D}}\right]\hat{f}, \tag{26}$$

3. *reduces the deposit rate* R *whenever*

$$2 - \frac{\hat{J}}{\hat{D}} > -\gamma E_A \hat{f}, \tag{27}$$

with a pass-trough of $\frac{\partial R}{\partial J/J} = \gamma \hat{J}(0.5\frac{\hat{J}}{\hat{D}} - 1)\hat{f}$ *if* $E_{DJ} \to 0$.

Proof. See Appendix A.6. ■

Proposition 6 summarizes the implications of our model for active reserves policy. For the lending rate (see part 1. of the proposition), liquidity effect and the deposit effect run in opposite directions. When the pass-through of asset purchases to the lending volume is maximal, i.e. in the limit of $E_{DJ} \to 2$, it is that $\lim_{E_{DJ}\to 2} \Delta R^b = 0$ and the effect of reserves policy on the lending rate is exactly zero. Part 2. of the proposition states that the additional supply of reserves always compresses the interest margin. While the direction of the deposit effect depends on the sign of $(J^2/D - 1)$, it never exceeds the liquidity effect.[4]

Part 3. of proposition 6 documents a key-finding: an increase in the supply of reserves can actually raise the deposit rate, which in general equilibrium will cause households'

[4]Mathematically, the ambivalence of the deposit effect on s_m can be seen via

$$\hat{f} = f(\hat{J}|0, \hat{D}) = \frac{1}{\sqrt{\hat{D}}}\varphi\left(\frac{\hat{J}}{\sqrt{\hat{D}}}\right). \tag{28}$$

If $\hat{J}$ is close to zero, the term $\frac{1}{\sqrt{\hat{D}}}$ has a strong discounting effect that dominates the expression as $\hat{D}$ increases.

consumption to fall. The intuition behind this result is that the response of the deposit rate is the net of the lending rate and the interest rate margin,

$$R = R^b - s_m. \tag{29}$$

As suggested by part 1. and 2. of the proposition, liquidity and deposit effect affects R^b and s_m differently. Take for example the limit case when $E_{DJ} \to 2$, i.e. deposits (and thereby, loans) react strongly to an increase in reserves. In this limit case, R^b will remain unchanged because liquidity and deposit effect exactly cancel out (see prop. 6.2.). However, as the $R^b - R$ spread is always decreasing in J (prop. 6.3.), the deposit rate must be increasing. More formally, the LHS term $\left(2 - \frac{J}{D}\right)$ in (27) is always positive since reserves cannot exceed deposits and, hence, $\frac{J}{D} < 1$. The RHS is also positive since $E_A \leq 0$. For the deposit rate to decrease, it is required that the demand for loans is rather inelastic (E_A small). A low level of reserves is not sufficient, because both J/D and $\hat{f}$ are increasing in $\hat{J}$. Crucially, this also suggests that for a rather elastic supply of loans (E_A larger) the net effect on the deposit rate becomes positive.

Proposition 7 (Effectiveness of IOR policy)**.** *Assume that MRR and DLB are slack,* $0 \leq J \leq D$, $0 < D$ *and take* J *as given.*

1. *Any IOR policy*
   (a) *has a pass-through to the lending rate* R_b *of*

$$\frac{\partial R^b}{\partial R^j} = \frac{1}{1 - 0.5\hat{J}E_A\hat{f}} \in (0, 1], \tag{30}$$

   (b) *has ambiguous effect on the interest margin* s_m *with*

$$\frac{\partial s_m}{\partial R^j} = \gamma 0.25 \left(\frac{\hat{J}^2}{\hat{D}} - 1 \right) E_{DR^j} \hat{f}. \tag{31}$$

   (c) *has a pass-through to the deposit rate of*

$$\frac{\partial R}{\partial R^j} = 1 + \gamma 0.25 \left[\hat{J} \left(\frac{\hat{J}}{\hat{D}} - 2 \right) - 1 \right] E_{DR^j} \hat{f} \geq 1, \tag{32}$$

   with $E_{DR^j} = \frac{\partial d_A/A}{\partial R^b} \frac{\partial R^b}{\partial R^j}$ *being the elasticity of deposits with respect to the IOR rate,*

2. *A stimulative (contractionary) IOR policy moves the economy back to (away from) the MRR.*

Proof. See Appendix A.7. ∎

Proposition 7 summarizes the effects of IOR rate policy for the economy when banks hold excess reserves. The pass-through of R^j on R^b is close to perfect and is slightly dampened by the deposit effect.[5] The interest margin spread increases in R^j if the

[5] The term $0.5\hat{J}E_A\hat{f}$ is small for reasonable assumptions on the interest margin and the demand elasticity of investment.

amount of excess reserves in the economy is large, but is decreasing otherwise. These two findings, taken together, show that IOR policy is highly effective in controlling the deposit rate, where as above, the pass-through is even amplified by the deposit effect.

A second, important result on the effects of IOR policy from proposition 7 is that any stimulative IOR policy will, through its positive effect on lending, ultimately lead the economy back to the MRR. This can be decomposed into two effects. The first, direct effect is that a low IOR rate increases the opportunity costs of holding reserves. This is dampened by the second, indirect effect, which comes from the fact that the lending rate decreases with the IOR rate, thereby causing a relative decrease in marginal costs of holding reserves.

3.2 The regime with a binding minimal reserve requirement

Let us now turn to the regime where the MRR is binding. In this case, the banking equilibrium from Equations (19) and (20) is given by

$$R = (1-\psi)R^b + \psi R^j + \gamma\left(\psi\left[1-\hat{F}_\psi\right] - 0.5\hat{f}_\psi\right), \tag{33}$$

$$J = \psi D, \tag{34}$$

and again $d_A(R^b) = \hat{D}$. Note that the deposit rate is a weighted average of R^b and R^j plus a liquidity spread and it always holds that $R < R^b$.[6] Proposition 8 establishes that, when deposits are directly linked to reserves, any exogenous increase in reserves will be reflected by an one-to-one increase in household deposits.

Proposition 8 (Elasticity of deposits to reserves with binding MRR)**.** *If the MRR is binding and DLB are slack, for $0 \leq J \leq D$, $0 < D$ and given R^j, the elasticity of deposits with respect to reserves E^{ψ}_{DJ} is given by*

$$E^{\psi}_{DJ} = 1. \tag{36}$$

Proof. At the MRR it holds that $J = \psi D$. The result follows directly from $d_A(R^b) = \hat{D}$, and $E^{\psi}_{DJ} = \frac{\partial d_A}{\partial J}\frac{J}{A}$. ■

Although seemingly trivial, this result has important implications on the pass-through of reserves policy to the rates in the banking equilibrium, which are summarized in proposition 9. Namely, the lending rate is solely determined by the equilibrium at the funds market, which in turn is directly tied to the supply of reserves via the MRR. The deposit rate is then closely linked to the lending rate, and the spread decreases in J as long as reserves are sufficiently small (or in terms of the proposition, until $\hat{J} = \psi^{-1}$).

Proposition 9 (Effectiveness of open market operations with binding MRR)**.** *If the MRR is binding and the DLB is slack, for $0 \leq J \leq D$, $0 < D$ and given J,*

---

[6] This can be seen by noting that (33) can be expressed as

$$R = R^b - 0.5\hat{f}_\psi + \psi MPJ_\psi, \tag{35}$$

where $MPJ_\psi < 0$ whenever the MRR binds.

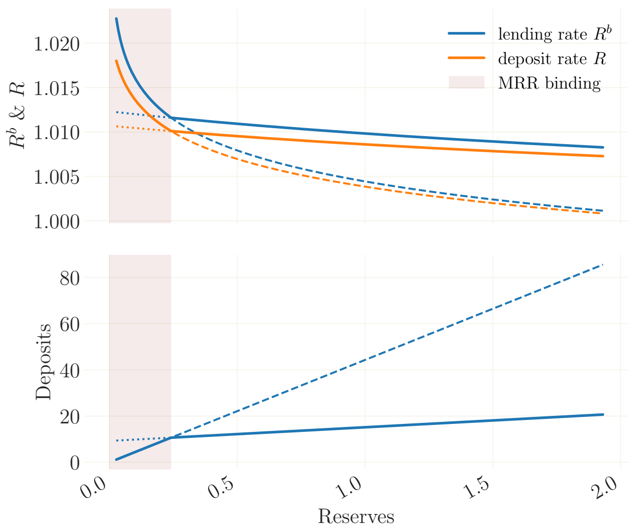

Figure 4: Responses of market rates in the banking equilibrium (equations (18) to (20)) when varying the quantity of supplied reserves. The red shaded area marks equilibria where the MRR is binding. Once the MRR is slack, the pass-through of the increase in reserves to equilibrium rates and the deposit volumen is incomplete. The dashed (dotted) lines depict the counterfactual equilibrium outcome if the MRR would continue to be binding (slack). R^j is fixed to 1, $\psi = 2.26\%$ and the inverse loan demand function is given by $\hat{A} = 0.977{R^b}^{0.005}$.

1. *for $E_A < 0$, any policy that actively increases the supply of reserves*
   (a) *always reduces the lending rate R^b with a pass-trough of*

$$\frac{\partial R^b}{\partial J/J} = E_A^{-1}, \tag{37}$$

   (b) *has a pass-through on the interest margin s_m of*

$$\frac{\partial s_m}{\partial J/J} = \psi E_A^{-1} + \gamma 0.25(\psi J - 1)\hat{f}. \tag{38}$$

   (c) *has a pass-trough to the deposit rate R_t of*

$$\frac{\partial R}{\partial J/J} = (1-\psi)E_A^{-1} + \gamma 0.25(1-\psi J)\hat{f} \tag{39}$$

2. *R^b, R and s_m are indetermined if $E_A = 0$.*

Proof. See Appendix A.8. ■

The equilibrium rates of the banking equilibrium for different levels of reserves are illustrated in figure 4. The solid lines in the red-shaded area to the left represent the equilibria when the MRR is binding. Deposit and lending rate decrease sharply when the

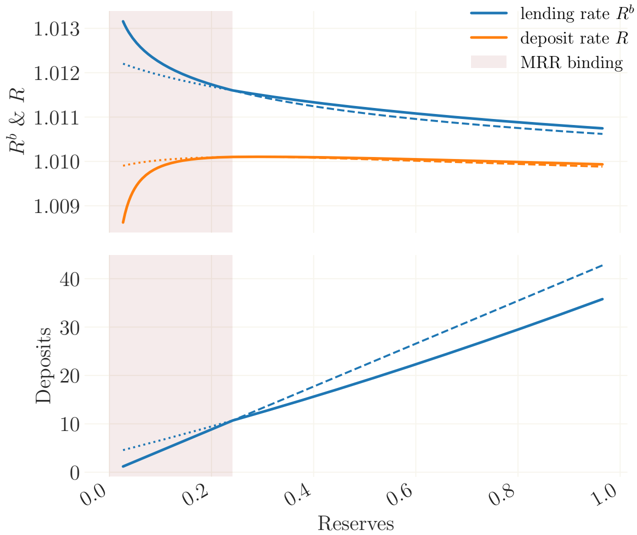

Figure 5: Responses of market rates in the banking equilibrium (equations (18) to (20)) when varying the quantity of supplied reserves but keeping the IOR rate fix. E_{DJ} is larger than in figure 4. The red shaded area marks equilibria where the MRR is binding. For smaller amounts of supplied reserves, the deposit rate increases when reserves increase. The dashed (dotted) lines depict the counterfactual equilibrium outcome if the MRR would continue to be binding (slack). R^j is fixed to 1, $\psi = 2.26\%$ and the inverse loan demand function is given by $\hat{A} = 0.987 {R^b}^{0.0007}$.

level of reserves rises, and the lending rate is solely determined by the demand function for loans. Once the MRR is slack, the pass-through of reserves policy to interest rates flattens out immediately, and the transmission to deposits is less than one-to-one. Note that, although only slowly, reserves policy with excess reserves successfully reduces the spread between deposit and lending rate. Figure 5 illustrates the same exercise for a more elastic investment demand function, leading to a more strongly attenuated deposit effect. Importantly, the deposit rate increases with the level of reserves shortly until the MRR becomes slack, and then slowly decreases with the amount of excess reserves supplied. Note that, while the interest margin decreases, the deposit rate remains almost constant when reserves increase.

Proposition 10 (Effectiveness of IOR policy with binding MRR)**.** *If the MRR is binding and DLB are slack, for* $0 \leq J \leq D$, $0 < D$ *and given* J*,*

1. *for* $E_A < 0$*, any IOR policy,*
   (a) *is fully ineffective in altering the lending rate* R_b*,*
   (b) *has a pass-through on the deposit rate of*

$$\frac{\partial R}{\partial R^j} = \psi, \tag{40}$$

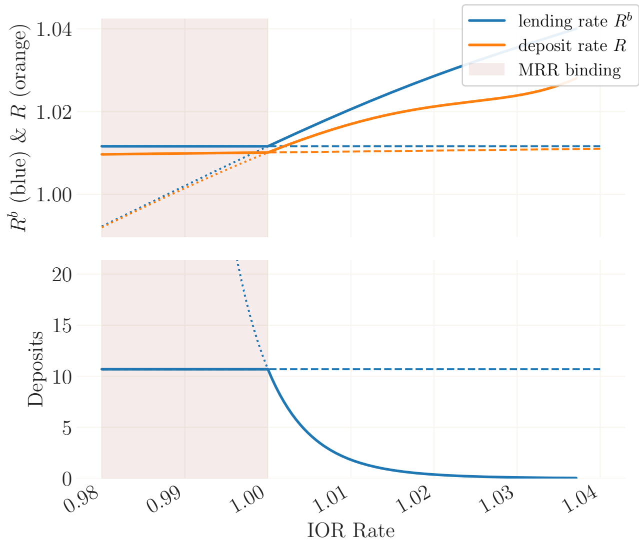

Figure 6: Partial equilibrium responses when varying the IOR rate and keeping $\hat{J}$ fixed. The inverse loan demand function is $\hat{A} = 0.977 R^{b^{0.005}}$, and $\psi = 2.26\%$. The dashed (dotted) lines depict the counterfactual equilibrium outcome if the MRR would be binding (slack).

Figure 7: Responses of market rates in the banking equilibrium (equations (18) to (20)) when varying the IOR rate but keeping the quantity of reserves fix. The red shaded area marks equilibria where the MRR is binding. When the MRR is binding, the pass-through of the IOR rate is negligible. The dashed (dotted) lines depict the counterfactual equilibrium outcome if the MRR would continue to be binding (slack). $\psi = 2.26\%$ and the inverse loan demand function is given by $\hat{A} = 0.977 R^{b^{0.005}}$.

(c) *has a pass-through to the interest margin of*

$$\frac{\partial s_m}{\partial R^j} = -\psi. \tag{41}$$

2. *R^b, R and s_m are indetermined if $E_A = 0$.*

Proof. See Appendix A.9. ■

Finally, proposition 10 documents a very limited transmission of IOR policy onto deposit and lending rates if the MRR is binding. In fact, because the lending rate is already determined by the supply of reserves, it is fully invariant to changes in the IOR rate. Since the deposit rate is a weighted average between lending and IOR rate with weights $1 - \psi$ and ψ, it is mainly determined by the lending rate and the pass-through of reserves policy is limited as well. For most countries the MRR is between 1% and 5%, and for the Euro Zone, the effective MRR was $\psi_{t<2012} \approx 3.6\%$ before 2012 and $\psi_{t>2012} \approx 1.3\%$ thereafter. This suggests that the pass-through of IOR policy to equilibrium rates is almost negligible.

Figure 7 represents the equilibria of the banking market for a given range of the IOR rate. The lines in the shaded red area to the left again represent equilibria where the

MRR is binding. For the reasons outlined above, the IOR has virtually no impact on equilibrium rates when the MRR is binding. Once the IOR rate exceeds a threshold value (here $R^j = 1$ by construction), the MRR becomes slack and banks wish to hold some of the supplied reserves as excess liquidity. This reflects in a decrease of deposit holdings, which are now endogenous, and the IOR rate becomes an efficient tool to steer equilibrium rates, as implied by proposition 7. In this regime with excess reserves, the IOR rate does only indirectly affect the spreads between the rates through the deposit effect, but, since spreads are mainly determined by the level of reserves, have a large impact on the level of deposit and lending rate.

3.3 The reversal IOR rate and the DLB

My model does not motivate why the IOR rate can not or should not be negative. It is easy to see that the optimality conditions of banks remain untouched as long as the deposit rate is positive. A prominent argument against negative IOR rates is that they are costly for banks. However, this argument may not hold necessarily because independently of whether the IOR rate is positive or negative, any $R^j < R^b$ implies that holding reserves is associated with opportunity costs. This holds equally for positive and negative IOR rates.

Correspondingly, the only constraint to this irrelevance result is the household's incentive constraint (7), which is the deposit rate lower bound (DLB). The DLB potentially affects the transmission of interest rate policy and reserves policy alike. The following propositions 11 and 12 suggest that the DLB affects the economy differently depending on whether the MRR is binding or not. Proposition 11 states that any IOR policy is ineffective at the MRR if the DLB is binding: the lending rate is fully determined by the supply of reserves while the deposit rate is constrained. This implies that there are no negative side effects of setting the IOR rate below the threshold, simply because there are no effects at all.

Proposition 11 (Effective lower bound)**.** *There exists a lower bound to the IOR rate below which any IOR policy is ineffective if*

$$r^j < \frac{\psi - 1}{\psi} r^b - \gamma \left(\left[1 - \hat{F}_\psi \right] - \frac{0.5}{\psi} \hat{f}_\psi \right) \tag{42}$$

and

$$r^j - r^b + \gamma \left[1 - \hat{F}_\psi \right] < 0, \tag{43}$$

i.e. when the MRR and the DLB are binding.

Proof. See Appendix A.10. ■

In contrast, when the MRR is not binding the lending rate directly depends on the IOR rate. This relationship (22) will hold independently of whether the DLB binds or not. This means that the DLB gives rise to a reversal rate at which the households' consumption decision is still indirectly affected by IOR policy changes via the general equilibrium effects of aggregate investment. Proposition 12 summarizes this result.

Proposition 12 (Reversal rate)**.** *Let a "reversal IOR rate" be a IOR rate such that marginal effects of IOR or reserves policy are nonzero but different from propositions 6 to 10. Then there exists a reversal IOR rate if*

$$r^j < \gamma(\hat{F} + 0.5\hat{f} - 1) \tag{44}$$

and

$$\psi d(R^b) < J, \tag{45}$$

i.e. when the MRR is slack and the DLB is binding.

Proof. See Appendix A.11. ■

The results from this subsection contradict some of the more recent findings concerning the risks associated with negative rates. However, some of these findings are based on models with rather ad-hoc assumption on dividend payments or the choice of net worth rather ad-hoc (e.g. Ulate, 2021; Sims and Wu, 2021). In contrast, my model assigns stimulative (and conventional) effects to any interest rate policy for which $R \geq 1$. Arguably, and as suggested by proposition 12, the reversal IOR rate is only relevant when there are excess reserves. Negative rates in combination with the DLB do indeed decrease profits in the above model, but it is unlikely that banks will immediately exit business or decrease their lending activity. In fact, the model suggests the opposite. Lastly, if for reasons exogenous to this model the central wants to avoid a binding DLB, this implies a careful trade-off between reserves and IOR-policy.

4 A medium Scale DSGE: model and estimation

In this section I first develop a fully-fledged medium-scale DSGE model of the Euro Area (EA) that incorporates the banking sector with liquidity frictions as proposed in section 2. I then specify the setup for nonlinear estimation in terms of data and methodology.

Apart from the banking sector, the backbone of the model by large follows the standard medium-scale setup of Smets and Wouters (2007). The setup of capital producers is adjusted, and I extend the toolbox of the central bank by balance sheet policies and negative interest rate policies. I here focus on the exhibition of these non-standard parts of the model and refer to the original papers for details on the baseline model. In addition, as suggested by Del Negro and Schorfheide (2013), let aggregate productivity be given by

$$Z_t = e^{\gamma t + \frac{1}{1-\alpha}\widetilde{z}_t}, \tag{46}$$

where γ is the steady-state growth rate of the economy and α is the output share of capital. $\widetilde{z}_t$ is the linearly detrended log productivity process that follows the autoregressive law of motion $\widetilde{z}_t = \rho_z \widetilde{z}_{t-1} + \sigma_z \epsilon_z$. For z_t, the growth rate of technology in deviations from γ, it holds that $z_t = \frac{1}{1-\alpha}(\rho_z - 1)\widetilde{z}_t + \frac{1}{1-\alpha}\sigma_z \epsilon_z$.

4.1 Firms

The setup of capital producers is adapted from Boehl et al. (forthcoming) such that the (expected) return on capital R^k_{t+1} can be expressed in terms of the monetary return on physical capital, $Q_t K_{t-1}$. The capital good producer's role in the model is to isolate the investment decision, that becomes dynamic through the introduction of convex investment adjustment costs.[7] At the end of each period, capital good producers buy used capital, restore it and produce new capital goods. Correspondingly, intermediate good producers sell the capital stock that they used for production to the capital producer, which repairs it, and purchase the capital stock that it is going to use in the next period from the capital producer. To finance the purchase of the new capital at the unit price Q_t, it issues a claim for each unit of capital it acquires to banks, which trade at the same price. As above, the interest rate the capital producer has to pay on the loans is R^k_{t+1}. I also assume that the firm incurs costs of capital utilization that are proportional to the amount of capital used, $\Psi(U_t)P_{m,t}K_{t-1}$.[8]

Capital evolves according to the law of motion

$$K_t = (1-\delta)K_{t-1} + e^{v_{i,t}}\left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right)I_t, \tag{47}$$

where δ is the depreciation rate and the function $S(\cdot)$, indicates a cost of adjusting the level of investment. In steady state it holds that $S = 0, S' = 0$, and $S'' > 0$. and $v_{i,t}$ follows an AR(1) process. The first order condition of capital producers reads

$$\begin{aligned} 1 \quad = \quad & Q_t e^{v_{i,t}}\left(1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right) \\ & + E_t\left\{\Lambda_{t,t+1}Q_{t+1}e^{v_{i,t+1}}S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right\}, \end{aligned}$$

and the choices for optimal labor input and optimal capital utilization yield the first order conditions

$$W_t = MC_t(1-\alpha)\frac{Y_t}{L_t}, \tag{48}$$

$$\Psi'(U_t)K_{t-1} = \alpha\frac{Y_t}{U_t} \Leftrightarrow \Psi'(U_t) = \alpha\frac{Y_t}{\overline{K}_t}, \tag{49}$$

where MC_t are marginal costs, U_t denotes the level of capital utilization, and $\overline{K}_t$ is the level of effective capital, and ex-post returns are given by

$$R^k_t/\pi_t = \frac{MC_t[\alpha\frac{Y_t}{\overline{K}_t} - \Psi(U_t)] + (1-\delta)Q_t}{Q_{t-1}}. \tag{50}$$

4.2 (Unconventional) Monetary policy

Setting up the monetary policy building block of a model of the post-1999 Euro Area is a nontrivial task. From an aggregate perspective, the MRR was effectively binding (for the majority of banks) until the end of 2008, when the ECB cut rates close to zero in

[7] Investment adjustment costs are a necessary feature to generate variation in the price of capital.

[8] This assumptions for the utilization costs are set to match the setting in Smets and Wouters (2007).

response to the Great Financial Crisis. Until that point the ECB was effectively using the IOR rate as well as open market operations (OMOs) to target a corridor system for the interbank lending rate, and by doing so it was very successful to pin down the interbank lending rate right in the middle of this corridor.[9] While banks started storing small amounts of liquidity in the deposit facility after 2008 – that is, as excess reserves –, they did not accumulate larger amounts of excess reserves before the ECB's 2010 *Security Market Programme*. Before 2009 neither the deposit facility nor the marginal lending facility were used in larger scale because banks could easily refinance in the interbank market. This means that when the MRR was binding, the MRO rate was the IOR rate in effect. However, when the MRR became slack in 2009, the deposit facility rate (DRF) became the relevant IOR rate since it is the rate banks receive on a marginal unit of reserves stored at the central bank.

The EONIA rate – as a measure of the interbank lending rate – was close to the MRO rate before 2009 but quickly converged to the DFR thereafter. Together with the fact that from 2009 to 2010 banks were holding small amounts of excess reserves even in the absence of unconventional monetary policy, this is indication that even before 2009 it was likely that $MPJ_t \approx 0$. This simplifies the setup of monetary policy considerably: under the assumption that $MPJ_t = 0$ at the MRR, J_t is uniquely pinned down given R_t^j (or vice versa), which fully determines the equilibrium of the banking sector.[10]

Hence, assume that the ECB sets the IOR rate to follow a conventional monetary policy rule of the form

$$\frac{R_t^s}{R^s} = \left(\frac{R_{t-1}^s}{R^s}\right)^{\rho} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^*}\right)^{\phi_y} \left(\Delta\left(\frac{Y_t}{Y_t^*}\right)\right)^{\phi_{dy}} e^{v_{r,t}}\right]^{1-\rho}, \tag{51}$$

where I refer to the unconstrained nominal policy rate R_t^s as the notional (or shadow) rate. Y_t^* denotes the potential output and $\Delta\left(\frac{Y_t}{Y_t^*}\right)$ denotes the growth in the output gap. Parameter ρ expresses an interest rate smoothing motive by the central bank over the notional rate and ϕ_π, ϕ_y, and ϕ_{dy} are feedback coefficients. $v_{r,t}$ is a conventional monetary policy shock that follows an AR(1) process.

As the nominal IOR rate, R_t^j, is under direct control of the central bank there are several options on how to model the policy rate and the effective lower bound. The model from section 2 gives no reason why the IOR rate should not go beyond zero. The model also does not imply that the ECB must respect the zero lower bound on the deposit rate (which must be respected by banks). However, the Federal Reserve Board remained reluctant to set the Federal Funds rate below zero, and the ECB kept the DFR above zero until 2014. This, together with anecdotal evidence, suggests that agents in the EA did not expect that the IOR rate could actually touch negative territory. Such

[9] The Euro System knows three policy rates: the deposit facility rate is the rate payed on excess liquidity parked at the central bank, the MRO (marginal refinancing operations) rate is payed on reserves subject to the MRR, and the marginal lending rate is the rate due when borrowing overnight reserves from the ECB.

[10] Note that if a central bank wishes to minimize the spread between borrowing and lending rates they seek to move MPJ_t close to zero. At the MRR it is that $MPJ_t < 0$. Plugging $R^b = R$ into (33) yields $\psi MPJ_t = 0.5\hat{f}_\psi$, which can never be achieved since $\hat{f}_\psi > 0$. However, increasing R^j raises MPJ_t and increasing J reduces $\hat{f}_\psi$.

perceived effective lower bound (PLB, perceived lower bound), although arguably only an intellectual constraint, can have a large impact on economic dynamics. Hence, assume that the ECB sets the IOR rate to follow the policy rule while maintaining that it does not go below zero:

$$R_t^j = \max\{1, R_t^s\} e^{v_{nir,t}}, \tag{52}$$

where we put the stochastic negative interest rate process $v_{nir,t}$ – which follows an AR(1) process – *outside* the max operator to allow for policy innovations that drive the IOR rate into negative territory, as observed in the Euro Area, while having agents to expect a classic zero lower bound ex-ante.

The central bank balance sheet is given by

$$J_t = Q_t^b B_t^{cb} + Q_t K_t^{cb},$$

where I assume that in normal times $K_t^{cb} = 0$. Imposing that in normal times the ECB always supplies enough reserves for banks to satisfy their desired liquidity needs, $J_t = \psi D_t$, the central bank's balance sheet can be written as

$$J_t = \psi D_t + X_t, \tag{53}$$

with X_t as the amount of excess reserves supplied. I assume that X_t follows an AR(2) process

$$x_t = \rho_{x,1} x_{t-1} + \rho_{x,2} x_{t-2} + \epsilon_{x,t}. \tag{54}$$

The advantage of an AR(2) process is that it can capture the hump-shaped response of the asset purchases, thereby also ensuring anticipation and stock effects at the moment the announcement was made. Note that by assumption $X_t = 0$ in steady state.

4.3 The linearized model

The full model is log-linearized around its growth path. By assuming that in steady state $MPJ = 0$ and $J = \psi D$ – that means, the model is linearized exactly at the kink of the banks decision function – a second occasionally binding constraint is avoided. The log-linear counterparts of the novel equations are given by

$$d_t + \frac{Y}{D}\hat{L}_t = \frac{Q^b B}{D}(q_t^b + b_t) + \frac{QK}{D}(q_t + k_t) \tag{55}$$

$$r_t^b = \frac{\kappa Q^b}{1 + \kappa Q^b} q_t^b - q_{t-1}^b, \tag{56}$$

$$E_t r_{t+1}^k = E_t r_{t+1}^b + \hat{\epsilon}_t, \tag{57}$$

$$E_t r_{t+1} - \frac{1}{\epsilon_D} r_t = -\frac{\gamma}{2} S_D \left(d_t - \hat{\nu}_t - \frac{\psi J}{\nu}\left(d_t + \hat{\nu}_t - 2j_t\right)\right) + \gamma S_D \hat{\gamma}_t, \tag{58}$$

$$E_t r_{t+1}^b - r_t^j = \gamma S_D \frac{J}{\nu}\left(d_t + \hat{\nu}_t - 2j_t\right) + \gamma S_L \hat{\gamma}_t, \tag{59}$$

$$j_t - d_t = x_t, \tag{60}$$

$$r_t^j = \max\{0, r_t^s\} + v_{nir,t}, \tag{61}$$

with $S_D = 0.5\sqrt{\frac{\nu}{D}}\varphi\left(\frac{J}{\sqrt{\nu D}}\right)$ the steady state value of $s_b(\cdot)$ and $S_L = 1-\Phi\left(\frac{J}{\sqrt{\nu D}}\right)$ being the steady state lending spread s_l (again, net of γ).[11] I assume that the steady-state deviations of the lending markup, $\hat{\epsilon}_t$, and of the liquidity cost parameter, $\hat{\gamma}_t$, both follow an AR(1) in logs. Note again that the term $v_{nir,t}$ stands outside of the max operator, thereby possibly driving the IOR rate into negative territory. The rest of the linearized model can be found in Appendix B.

4.4 Estimation

The fact that the data includes a long episode in which the PLB binds poses a host of technical challenges. These are related to the solution, likelihood inference and estimation of the model in the presence of an occasionally binding constraint. Boehl and Strobel (2020, henceforth BS) suggest a comprehensive collection of tools to tackle these challenges. To start with, they propose a solution method for occasionally binding constraints that performs roughly four magnitudes faster than alternative methods. For likelihood inference, BS suggest to use the Ensemble Kalman filter (Evensen, 1994), which can be understood as a hybrid of the particle filter and the Kalman filter. The Ensemble Kalman filter allows to efficiently approximate the distribution of states for large-scale nonlinear systems with only a few hundred particles (instead of several million as with the particle filter), which is computationally advantageous.[12] As proposed by BS, I use a nonlinear path-adjustment smoother to obtain the smoothed/historic shock innovations of the high-dimensional nonlinear model. To sample from the posterior distribution I use the differential evolution ensemble Monte Carlo Markov chain method (Ter Braak, 2006; ter Braak and Vrugt, 2008).[13] For further technical details see BS.

The model is estimated on quarterly data from 1999:II to 2019:IV using a total of eleven observables. As is standard in the estimation of medium-scale models, I include the real per capita growth rates of GDP, consumption, and investment, real wage growth, a measure of labor hours and the GDP deflator. I use the Libor as a proxy of the IOR rate because, as outlined in section 4.2, it closely followed the MRO rate when the MRR was binding, and then moved alongside the DFR rate after 2008. The Libor hence helps to homogenize the MRR and non-MRR parts of the sample. For the interest rate on bank deposits I use the household deposit rate supplied by the statistical data warehouse (SDW) of the ECB and I use the BAA yield as a measure of the lending rate. For unconventional monetary policy, I feed in the time series of reserves held at the ECB divided by required reserves as implied by deposits held by commercial banks and

[11] Additionally, linearized aggregated liquidity costs are given by $\gamma D S_D(d_t + \hat{\nu}_t + \hat{\gamma}_t) - \gamma J S_L(j_t + \hat{\gamma}_t)$.

[12] For all estimations and for the numerical analysis, we use an ensemble of 400 particles. This number is chosen to minimize sampling errors during likelihood inference. For the same reason we sample the initial distribution of states from quasi-random low-discrepancy series (e.g. Niederreiter, 1988). For our model, the evaluation of the likelihood for one parameter draw then takes less than 2 seconds on a single CPU. For a more detailed discussion of the properties of the Ensemble Kalman filter, also see Katzfuss et al. (2016).

[13] The fundamental idea is to have a large ensemble of Monte Carlo Markov chains that mutually exchange information. In practice, the posterior "chain" ensemble is initialized with 200 draws sampled from the prior distribution. I then let the sampler run 3500 iterations, of which the last 500 ensembles are kept. The posterior parameter distribution is thus represented by $500 \times 200 = 10000$ parameter draws. The full estimation is conducted on a machine with 40 Intel Xeon E5 CPUs and 32 GB RAM and takes about 3 hours.

subject to the MRR. In terms of the model's variables this hence reads as $X_t = \frac{J_t}{\psi D_t}$, which uniquely pins down $x_t \approx X_t - 1$. The advantage of this measure is that it is stationary and relatively insensitive to log-linear approximations. These time series are also obtained directly from the SDW.

Instead of using the IOR observable (the Libor) directly, it is further divided into $\text{IOR}^+ = \max\{\text{IOR}, 0\}$ and $\text{NIR} = \min\{\text{IOR}, 0\}$. This helps to clearly identify the negative impact of the PLB and to quantify the effects of the NIR policy. To facilitate the nonlinear filtering, I assume small measurement errors for all variables with a variance that is 0.01 times the variance of the respective time series. Since the IOR^+ and NIR rate and the amount of excess reserves are perfectly observable I divide the measurement error variance here again by 100. Except for the labor supply, the data is not demeaned as I assume the non-stationary model follows a balanced growth path, with a growth rate estimated in line with SW. In total, these eleven observables are matched with eleven economic shock processes.[14] The measurement equations and a detailed description of the data are delegated to Appendix C.

I fix several parameters prior to estimating the others. In line with SW, let the depreciation rate be $\delta = 0.025$, the steady state government share in GDP to $G/Y = 0.484$, and the curvature parameters of the Kimball aggregators for prices and wages to $\epsilon_p = \epsilon_w = 10$. The steady state wage markup is set to $\lambda_w = 1.1$. I set the decay factor for government bonds to 0.975, which implies an average maturity of 40 quarters. Lastly, we calibrate the empirical perceived lower bound of the nominal interest rate to 0.01% quarterly. ψ is fixed to 0.017, which is the relevant value of the MRR until 2012 as identified by the data.[15] I let $\epsilon_D = 0.99$, which is sufficient to guarantee the existence of a local maximum.

Finally, the choice of priors is summarized in table 1. I use standard priors from SW and BS wherever possible. The novel parameters are then ν, γ and the steady state values of the spreads between borrowing and lending rate and the IOR rate. To identify ν I redefine $\nu = \frac{\hat{\nu}}{1-\hat{\nu}}(2\psi - \psi 2)D$, which for $\hat{\nu} = 0.5$ sets ν to maximize the spread between IOR and deposit rate (see figure 3) while mapping $\hat{\nu} \in (0,1) \to \nu \in (0, \infty)$. I estimate $\hat{\nu}$ using a beta distribution with mean 0.5 and a standard deviation of 0.25 as prior, which is a very flat prior. The spread between the deposit rate and IOR rate and the lending rate and the IOR rate are also estimated (both using $\mathcal{N}(0.5, 0.2^2)$ as the prior distribution), which together can be used to pin down γ, S_D and S_L.

5 The quantitative effects of excess reserves

This section presents the results from the estimated model. I first briefly present the posterior distribution of parameters obtained from the estimation and then discuss the dynamics of an unconventional monetary policy (UMP) shock in the estimated model. I then use the estimated model to empirically quantify the effects of the post-2010 UMP measures undertaken by the ECB.

[14]The economic shocks are: TFP, government spending, marginal efficiency of investment, risk premium, (conventional) monetary policy, price markup, wage markup, loan markup, liquidity costs, liquidity provision, and negative interest rate policy.

[15]The value after 2009 is not relevant since banks were already holding large amounts of excess reserves.

	Prior			Posterior				
	distribution	mean	sd	mean	sd	mode	5% HPD	95% HPD
σ_c	normal	1.500	0.375	1.340	0.102	1.269	1.170	1.497
σ_l	normal	2.000	0.750	1.761	0.523	1.820	0.889	2.535
β_{tpr}	gamma	0.250	0.100	0.096	0.033	0.105	0.044	0.145
h	beta	0.700	0.100	0.672	0.062	0.749	0.579	0.782
S''	normal	4.000	1.500	3.790	1.036	2.801	2.001	5.397
ι_p	beta	0.500	0.150	0.262	0.096	0.229	0.093	0.402
ι_w	beta	0.500	0.150	0.320	0.093	0.329	0.167	0.471
α	normal	0.300	0.050	0.319	0.015	0.311	0.293	0.343
ζ_p	beta	0.500	0.100	0.860	0.026	0.855	0.817	0.901
ζ_w	beta	0.500	0.100	0.807	0.044	0.817	0.735	0.876
Φ_p	normal	1.250	0.125	1.688	0.068	1.667	1.576	1.796
ψ	beta	0.500	0.150	0.303	0.070	0.242	0.186	0.415
ϕ_π	normal	1.500	0.250	1.591	0.188	1.775	1.255	1.878
ϕ_y	normal	0.125	0.050	0.164	0.031	0.140	0.110	0.212
ϕ_{dy}	normal	0.125	0.050	0.204	0.029	0.212	0.154	0.249
ρ	beta	0.750	0.100	0.945	0.016	0.943	0.920	0.970
$\overline{\gamma}$	normal	0.440	0.050	0.329	0.021	0.335	0.294	0.364
$\overline{l}$	normal	0.000	2.000	2.622	0.778	2.834	1.405	3.919
$\overline{\pi}$	gamma	0.625	0.100	0.512	0.057	0.517	0.422	0.605
$\overline{spread}_D$	normal	0.500	0.200	0.200	0.044	0.185	0.123	0.263
$\overline{spread}_K$	normal	0.500	0.200	0.434	0.103	0.344	0.275	0.607
$\hat{\nu}$	beta	0.500	0.250	0.837	0.086	0.905	0.712	0.959

Table 1: Prior distribution and estimation results. The estimates for the parameters governing the exogenous shock processes can be found in Appendix D.

The posterior distribution is summarized in table 1. The estimates of the standard parameters that are in common with the baseline medium-scale model by large reflect the findings of Boehl and Strobel (2020) for the US economy over a similar sample. This includes in particular a low discount factor and estimates of the Calvo (1983) parameters, reflecting rather flat Phillips curves for prices and wages. The mean estimate of the parameter governing the liquidity risk of banks, $\hat{\nu}$, is pinned down at 0.837, which is significantly above its prior value. This value suggests that in terms of figure 3, the banking equilibrium is at the right side of the dashed line in the region where R_t is decreasing in ν. In this regime, the spread between borrowing and lending rate is mainly determined by the liquidity risk faced by banks. Together with the estimates of the two spreads this results in estimates of $\nu = 0.837$ and $\gamma = 0.005$ at the posterior mean.[16] These two estimates are central for the quantitative results of the model. The AR(2) process of unconventional monetary policy is characterized by $\rho_{x,1} = 1.631$ and $\rho_{x_2} = -0.665$, reflecting the hump-shaped response of asset purchases after announcement. The estimate of capital adjustment costs S'' below its posterior mean is rather uncommon, but entirely due to the assumption on G/Y.[17]

Figure 8 shows impulse response functions for unconventional reserves policy in the

[16]The fact that $\hat{\nu} \approx \nu$ is coincidental.

[17]It is commonly assumed that $G/Y = 0.2$, which is incorrect for the EA when measured average total

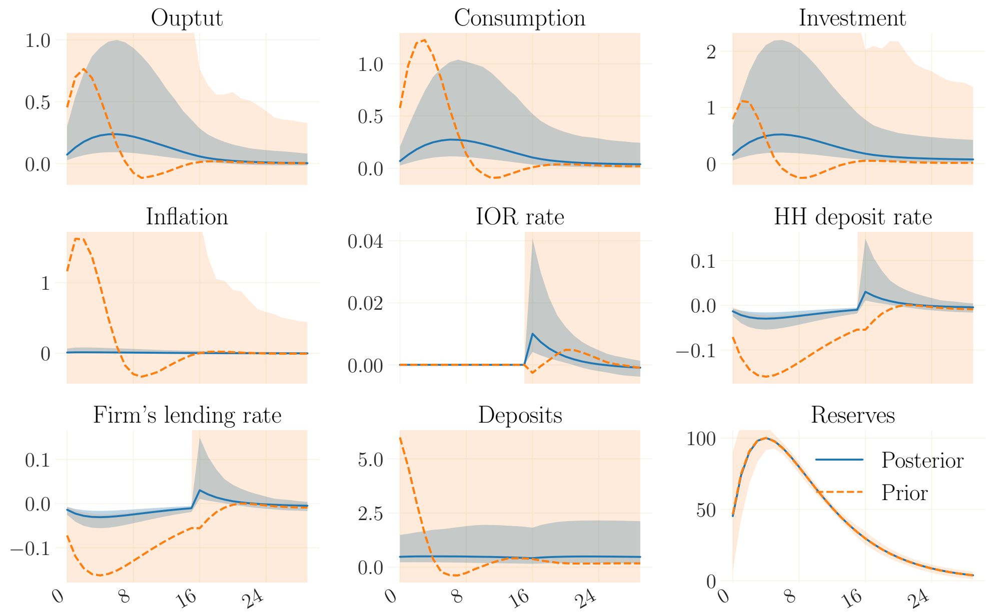

Figure 8: Impulse response functions for an unconventional monetary policy shock that doubles the amount of reserves. Simulations are based on 2500 parameter draws sampled from the posterior (blue) and the prior (orange). Shaded areas illustrate 95% credible sets. The ELB on the IOR rate is enforced for 16 quarters (4 years) to eliminate the effect of the policy rule reacting to the macroeconomic responses of the unconventional monetary policy measures. The AR(2) coefficients for all draws are set to the posterior mean to homogenize the simulations, and the shocksize is set such that the peak response of reserves is 100% of steady state reserves (i.e., reserves are doubled at the peak).

model. Specifically, the size of the shock is chosen to double the central banks' steady-state supply of reserves. The blue lines are sampled from the posterior parameter distribution, while the orange dashed lines represent the median over simulations sampled from the prior. Shaded areas represent 95% credible sets. In all simulations the lower bound on the IOR rate is enforced for 16 quarters to eliminate the feedback of conventional monetary policy to the UMP shock. For all draws the AR(2) parameters of the UMP shock process, x_t, are fixed to their posterior mean to homogenize the timing of the simulations.

The posterior simulations suggest rather conventional responses of macro variables to UMP: both the household deposit rate and the lending rate decrease moderately, where the impact on both rates is quantitatively similar. This triggers a quarter-percent increase in consumption via the IS curve and a mild increase in investment, both lifting up output. The inflation response is positive but very limited due to the flat Phillips curve. Although consumption increases, income rises such that households are also able to increase savings and hence the volume of deposits also increases. However, this increase

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government expenditures over GDP. The specification used here is preferred by the data in terms of a higher data density, but has no major implications on the effects of UMP.

in deposit holdings by no means reflects the surge in reserves.

In contrast, the simulations sampled from the prior suggest far stronger median responses – in particular of inflation – and a much larger dispersion. In fact, the responses of output and inflation can in theory also turn negative. This can for example occur when the response of the lending rate is much stronger than that of the borrowing rate. Lower lending rates will cause a decrease in marginal costs, which may trigger a fall in the price level and hence deflation. Deflation in turn, in combination with an only weak response of the household deposit rate to the UMP measures, can cause the households' real interest rate to turn positive, which eventually may depress consumption. A negative consumption response may again reinforce the negative effect on inflation and cause output and investment to fall as well. Boehl et al. (forthcoming) term such deflationary effects *the cost channel of QE* and give account that this channel may have been important for the effects of QE in the US economy. However, such deflationary effects are absent in the Euro Area in the simulations sampled from the posterior. Note that this also documents that none of the effects reported here are actually hardwired into the model. Other than the responses of macroeconomic aggregates, the prior responses of borrowing and lending rates are always negative as suggested by the findings from section 3.

Figure 9 finally shows counterfactual simulations. The methodology is similar to Boehl et al. (forthcoming): I take draws from the posterior distribution. For each draw, I use a nonlinear Bayesian filter to obtain a sequence of shocks that drives the economic dynamics (according to the filter). I then mute the shock that drives the UMP measures and use the rest of the shocks to again simulate a set of time series. The plots then show the net difference between the simulations with all shocks and without the UMP shock. I repeat the same exercise for the shock $v_{nir,t}$ that drives the IOR rate into negative territory. The crucial difference to the impulse response functions in figure 8 is that this exercise takes into account the actual endogenous expected durations of the ELB, which are important when quantifying the impact of the UMP measures.

Overall, the unconventional monetary policy measures lead to a swift and proportional decline in deposit and lending rates. Both rates are affected similarly. The median impact of unconventional reserves policy on output turns out to be quite small with a median response of about 0.25% in 2013:I and again from 2018 to 2019. Notably, the 2018/2019 response was not (much) larger than the earlier spike in 2013, which is due to the fact that in the later period agents are expecting the PLB to be binding for short period (expected PLB durations as implied by the estimation can be found in Appendix E). As already implied by the impulse response functions, the increase in GDP comes from a one-quarter percent median response in consumption, which is quite large in relation to the 0.5% impact on investment. This, in combination with the relatively flat Phillips curve, leads to a muted response of inflation that is quantitatively negligible.

This estimate of the effectiveness of unconventional reserves policy is independent of the specification of the PLM. In fact, an estimated *linear* model will conclude with very similar quantitative findings. However, a linear model would suggest that the dynamics before 2015 would to a large extend be driven by contractionary (conventional) monetary policy shocks, which biases the overall shock decomposition and could have lead to misleading results. Additionally, using in a purely linear model the effects of reserves policy would always scale proportionally, and hence cause the output effects in 2013 and 2018–2019 to differ more strongly.

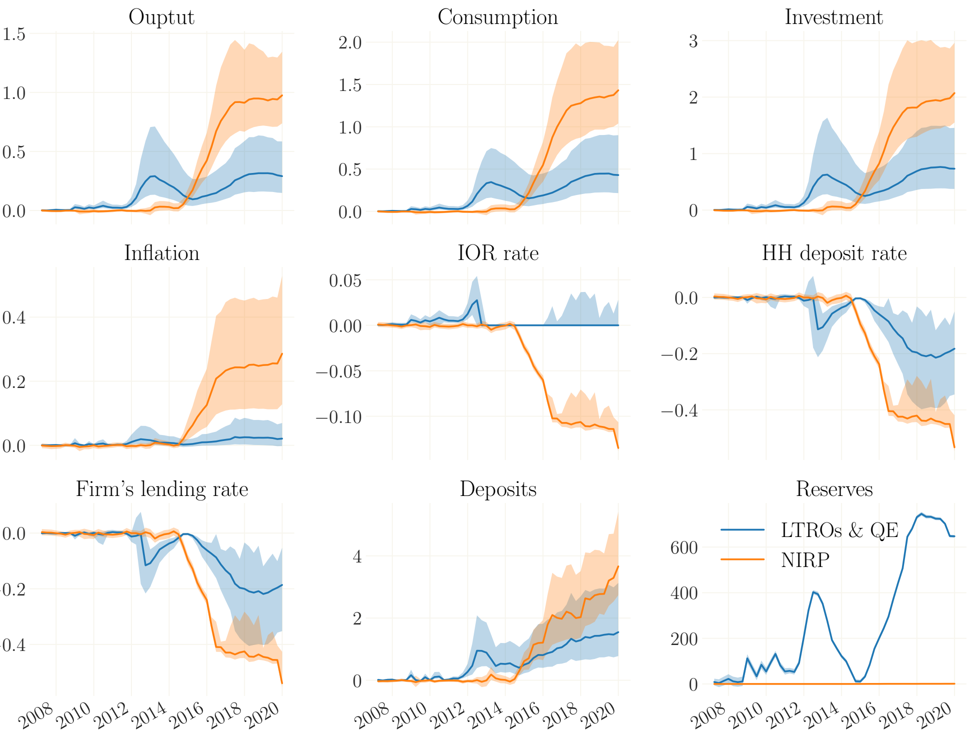

Figure 9: Counterfactual simulations for the effects of reserves policy (blue) and negative interest rate policy (orange). The figure is constructed from 1000 simulated series with and without the shocks driving both policies. All measures in percentage rates. Interest rates and inflation are annualized, the rest is expressed in quartely terms. The nonlinear effects of the ZLB binding in expectations are implicitely included.

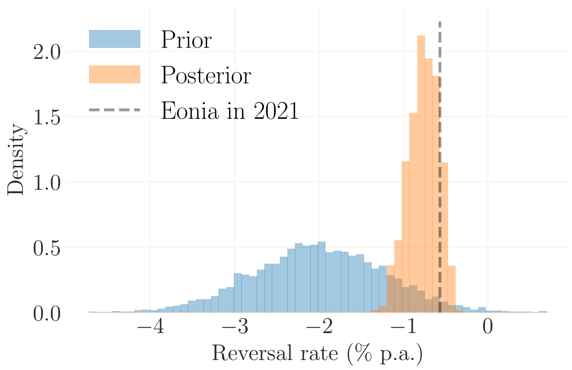

Figure 10: Prior (blue) and posterior (orange) distribution of the annualized steady-state reversal rate.

In contrast, the estimation identifies the gradual decline of the IOR rate into negative territory to be an efficient tool to stimulate both, output and inflation. The peak median response of output lies at about 1% of quarterly GDP, which mainly reflects a sharp increase in consumption and less the rather moderate rise in investment. This strong demand-sided effect of the NIRP sparks a stronger response in inflation of about 0.25% annually. Figure 10 provides estimates of the steady-state reversal IOR rate, whih is given by (c.f. proposition 12)

$$r^r_{ss} = \gamma(\hat{F}_{ss} + 0.5\hat{f}_{ss} - 1). \tag{62}$$

Note that this neither takes into account the effect of the ECB's measures of liquidity provision, that would *increase* the reversal IOR rate, nor does it take into account any shocks that may increase the liquidity demand by banks, which would lower the actual reversal IOR rata. The prior mean estimate of the reversal rate lies at about -0.76% while the actual Eonia rate was already as low as -0.57% in 2021, which is at the 0.85%-percentile of the posterior distribution. This suggests that the further beneficial effects of the NIRP come with the risk of hitting the DLB rather soon.

6 Conclusion

This paper develops a fully-fledged medium scale DSGE model with a banking sector that supplies inside money. Lending activity creates deposits, and banks use reserves to hedge liquidity frictions associated with deposits. This gives rise to spreads between the borrowing and lending rate, and the interest-on-reserves rate. When the minimal reserve requirement is binding, i.e. when the marginal profit from holding reserves is negative, the central bank can effectively steer borrowing and lending rates through open market operations, but setting the interest on reserves is rather ineffective. Inside money, as measured in terms of the economy wide deposit volume, then increases one-to-one with the volume of reserves provided. Once the minimal reserve requirement is slack and banks are willing to hold excess reserves, a quantitative easing or liquidity provision policy that supplies additional reserves has only limited effect on borrowing and lending rates. Inside money then becomes fully endogenous and depending on banks discretion, and the interest-on-reserves rate is a powerful policy tool.

I show that the general equilibrium effects coming from loan demand – that is, investment demand – can further dampen the effects of LTROs and quantitative easing. If loans create deposits, a decrease in the loan rate triggers more loans and hence more deposits. These deposits however may cause a relative increase in the liquidity risk faced by banks, that may have important quantitative implications. I estimate the model using nonlinear Bayesian methods on data of the 1999-2019 Euro Area while feeding in household deposit rates and various measures of the central bank balance sheet policies such as excess reserves. Counterfactual analysis suggests that the unconventional monetary policy measures undertaken by the ECB had only limited effect on output (about one-quarter percent of quarterly GDP) and almost no measurable impact on inflation.

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Appendix A Proofs of the propositions

Appendix A.1 Proof of proposition 1 (liquidity risk)

To simplify notation, denote bank js expected net inflow of deposits with $Z = \Delta D_{i,t}$, which consists of inflows A from other banks to j and outflows Y from j to other banks. Note that a negative Z implies a net outflow of deposits with $Y > A$. A and Y both follow a binomial distribution with

$$Pr\,(A = x) = f_B(x|D_{-i,t}, \chi \frac{D_{i,t}}{D_t}), \tag{A.1}$$

$$Pr\,(Y = x) = f_B(x|D_{i,t}, \chi \frac{D_{-i,t}}{D_t}), \tag{A.2}$$

that is for A, the number of the units of deposits *not* hold by j that we denote by $D_{-i,t}$, and for each of these units the probability to end up at bank j is the probability χ to get transfered away from its current bank times the probability to be transfered to j, which is given by $\frac{D_{i,t}}{D_t}$. Bank j's deposits after transfers are hence given by the random variable

$$Z = A - Y. \tag{A.3}$$

We cannot directly sum over two Binomial distributions with different probabilities. However, for large sample sizes a Binomial distribution with PDF $f_B(k|n, p)$ can be well approximated by a normal distribution with PDF $f_N\,(k|np, np(1-p))$. Since both $D_{i,t}$ and $D_{-i,t}$ are large (in a stochastic context), we can rewrite

$$A \sim \mathcal{N}\left(\chi \frac{D_{-i,t}D_{i,t}}{D_t}, \chi \frac{D_{-i,t}D_{i,t}}{D_t}\left(1 - \chi \frac{D_{i,t}}{D_t}\right)\right), \tag{A.4}$$

$$Y \sim \mathcal{N}\left(\chi \frac{D_{-i,t}D_{i,t}}{D_t}, \chi \frac{D_{-i,t}D_{i,t}}{D_t}\left(1 - \chi \frac{D_{-i,t}}{D_t}\right)\right). \tag{A.5}$$

Since now Z is the difference of two normal distributed variables it follows that

$$Z \sim \mathcal{N}\left(0, \frac{D_{i,t}D_{-i,t}}{D_t}(2\chi - \chi^2)\right). \tag{A.6}$$

Note that this results assumes that the *number* of units of deposits, and the real value of deposits is exactly equal. It is however easy to show that the result would hold up to some scaling factor of the variance if I would instead assume that the number of units of deposits is proportial to the value of deposits. For simplicity I hence assume that this scaling factor is included in the parameter for the transfer probability χ.

Appendix A.2 Proof of proposition 2 (liquidity costs)

Using f as the PDF of $Z = \Delta D_{i,t}$ from proposition 1 and the definition of $h(D_{i,t})$ yields

$$g(J_{i,t}, D_{i,t}) = \int_J^\infty (z - J_{i,t}) f(z) dz \tag{A.7}$$

$$= \int_J^\infty z f(z) dz - J_{i,t} \int_J^\infty f(z) dz, \tag{A.8}$$

$$= -h(D_{i,t}) \int_J^\infty f'(z) dz - J_{i,t}(1 - F(J_{i,t})), \tag{A.9}$$

$$= h(D_{i,t}) f(J_{i,t}) - J_{i,t}(1 - F(J_{i,t})). \tag{A.10}$$

Appendix A.3 Proof of proposition 3 (equilibrium of the banking sector)

Bank i's profit maximization problem is

$$\max_{K_{i,t}, B_{i,t}, J_{i,t}, D_{i,t}, R^k_{i,t+1}, R_i} E_t \left\{ R^b_{t+1} Q^b_t B_{i,t} + R^k_{i,t+1} Q_t K_{i,t} \right\} + R^j_t J_{i,t} - R_t D_{i,t} - \gamma g(J_{i,t}, D_{i,t}) \tag{A.11}$$

s.t.

$$Q^b_t B_{i,t} + Q_t K_{i,t} + J_{i,t} = D_{i,t}, \tag{A.12}$$

$$g(J_{i,t}, D_{i,t}) = h(D_{i,t}) f(J_{i,t}, h(D_{i,t})) - J_{i,t}(1 - F(J_{i,t}, h(D_{i,t})) \tag{A.13}$$

$$h(D_{i,t}) = \frac{D_{i,t} D_{-i,t}}{D_{i,t} + D_{-i,t}} (2\chi - \chi^2) \tag{A.14}$$

$$E_t \left\{ \frac{R^k_{i,t+1}}{R^k_{t+1}} \right\} = N^{\frac{1-\epsilon}{\epsilon}} \left(\frac{K_{i,t}}{K_t} \right)^{\frac{1-\epsilon}{\epsilon}}, \tag{A.15}$$

$$\frac{R_{i,t}}{R_t} = N^{\frac{\epsilon_D - 1}{\epsilon_D}} \left(\frac{D_{i,t}}{D_t} \right)^{\frac{\epsilon_D - 1}{\epsilon_D}}, \tag{A.16}$$

$$1 \leq R_{i,t}, \tag{A.17}$$

$$\psi D_{i,t} \leq J_{i,t}. \tag{A.18}$$

For the derivatives with respect to $g(\cdot)$ we can exploit that for the normal distribution it holds that $f'_Z(z) = -\frac{z}{h(D_{i,t})} f_Z(z)$, which simplifies the algebra considerably:

$$
\begin{aligned}
\frac{\partial g}{\partial J_{i,t}}(J_{i,t}, D_{i,t}) &= -\gamma(1 - F_Z(J_{i,t})), && \text{(A.19)}\\
\frac{\partial^2 g}{\partial J_{i,t}^2}(J_{i,t}, D_{i,t}) &= \gamma f_Z(J_{i,t}), && \text{(A.20)}\\
\frac{\partial g}{\partial D_{i,t}}(J_{i,t}, D_{i,t}) &= 0.5\gamma h' f_Z(J_{i,t}), && \text{(A.21)}\\
\frac{\partial^2 g}{\partial D_{i,t}^2}(J_{i,t}, D_{i,t}) &= 0.5\gamma \left(h'' + 0.5 \left(\frac{J^2}{h} - 1 \right) \frac{h'^2}{h} \right) f_Z(J_{i,t}), && \text{(A.22)}\\
&= 0.5\gamma h'' f_Z(J_{i,t}) + 0.25 h'^2 f''_Z(J_{i,t}), && \text{(A.23)}\\
\frac{\partial^2 g}{\partial J_{i,t} D_{i,t}}(J_{i,t}, D_{i,t}) &= -\gamma 0.5 J \frac{h'}{h} f_Z(J_{i,t}), && \text{(A.24)}\\
& && \text{(A.25)}
\end{aligned}
$$

where

$$
\begin{aligned}
h(D_{i,t}) =& \frac{D_{i,t} D_{-i,t}}{D_{i,t} + D_{-i,t}} (2\chi - \chi^2), && \text{(A.26)}\\
h'(D_{i,t}) =& \left(\frac{D_{-i,t}}{D_{i,t} + D_{-i,t}} \right)^2 (2\chi - \chi^2), && \text{(A.27)}\\
h''(D_{i,t}) =& -2 \frac{D_{-i,t}^2}{(D_{i,t} + D_{-i,t})^3} (2\chi - \chi^2), && \text{(A.28)}\\
=& -\frac{2}{D_{i,t} + D_{-i,t}} h'(D_{i,t}). && \text{(A.29)}
\end{aligned}
$$

Under $\epsilon_D \to 0$ and omitting expectations operators for this proof, the FOCs are

$$
\begin{aligned}
J_{i,t} : -R^b_{t+1} + R^j_t + \gamma \left[1 - F(J_{i,t}, h(D_{i,t})) \right] + \hat{\mu}_{J,t} &= 0, && \text{(A.30)}\\
D_{i,t} : -R^b_{t+1} + R_t + \gamma \frac{1}{2} \left(\frac{D_{-i,t}}{D_t} \right)^2 (2\chi - \chi^2) f(J_{i,t}, h(D_{i,t})) + \hat{\mu}_{J,t} + \mu_{D,t} &= 0, && \text{(A.31)}\\
R^k_{i,t+1} \& K_{i,t} : R^k_{i,t+1} - \epsilon R^b_{t+1} &= 0, && \text{(A.32)}
\end{aligned}
$$

together with (A.12) to (A.14), (A.17), (A.18) and the (modified) Kuhn-Tucker conditions

$$\hat{\mu}_{J,t} \geq 0 \tag{A.33}$$

$$\mu_{D,t} \geq 0, \tag{A.34}$$

$$\hat{\mu}_{J,t}(\psi D_{i,t} - J_{i,t}) = 0, \tag{A.35}$$

$$\mu_{D,t}(1 - R_{i,t}) = 0. \tag{A.36}$$

Under the assumption of a symmetric equilibrium and no entry and exit ($N = \frac{D_t}{D_{i,t}}$) all N banks make identical choices and with $B^b_t = \sum B_{i,t}$ their best-responses can be aggregated to

$$Q^b_t B^b_t + Q_t K^b_t + J_t = D_t, \tag{A.37}$$

$$E_t R^b_{t+1} = \frac{1 + \kappa Q^b_{t+1}}{Q^b_t}, \tag{A.38}$$

$$R^b_{t+1} = R^j_t + \gamma \left[1 - \Phi\left(\frac{J_t}{\sqrt{\nu D_t}}\right)\right] + \mu_{J,t}, \tag{A.39}$$

$$R^b_{t+1} = R_t + 0.5\gamma \frac{N-1}{N}\sqrt{\frac{\nu}{D_t}}\varphi\left(\frac{J_t}{\sqrt{\nu D_t}}\right) + \psi\hat{\mu}_{J,t} + \mu_{D,t}, \tag{A.40}$$

$$R^k_{t+1} = \epsilon R^b_{t+1}. \tag{A.41}$$

where $\nu = (N-1)(2\chi - \chi^2)$ is a measure of *liquidity (tail) risk* and with $\Phi(\cdot)$ as the standard normal CDF and $\varphi(\cdot)$ as the standard normal PDF. The result from the proposition follows after assuming $\frac{N-1}{N} \approx 1$.

We must however ensure that this is a (local) maximum of the profit function. This is unproblematic for the constrained case. For the unconstrained case the second partial derivative test requires that the hessian of the profit function is positive,

$$\det H_\Pi(D, J) > 0, \tag{A.42}$$

and that $\Pi_{JJ} < 0$. The latter is easy to see since $g_{JJ} < 0$ always. The condition on the determinant leads to

$$\frac{1 - \epsilon_D}{\epsilon_D^2} R > 0.25\gamma h' f(J_{i,t}), \tag{A.43}$$

which implies that some monopsonism with $\epsilon_D < 1$ is necessary for a valid equilibrium (the RHS will always be positive). Plugging in the optimality condition for deposits results in

$$\frac{2 - \epsilon_D}{\epsilon_D^2} > \frac{R^b}{R}, \tag{A.44}$$

or

$$\epsilon_D < \frac{\sqrt{1 + 8R^b/R} - 1}{2R^b/R}. \tag{A.45}$$

Appendix A.4 Proof of proposition 4 (liquidity regimes)

Let $\theta = J/D$. For the first part, the deposit spread $s_d = \frac{R-R^j}{\gamma}$ in terms of ν is given by

$$s_d(\nu) = 1 - \Phi\left(\theta\sqrt{\frac{D}{\nu}}\right) - 0.5\sqrt{\frac{\nu}{D}}\varphi\left(\theta\sqrt{\frac{D}{\nu}}\right), \tag{A.46}$$

with first derivative

$$\frac{\partial s_d(\nu)}{\partial \nu} = \left(0.5\theta\sqrt{D}\nu^{-1.5} - 0.25/\sqrt{\nu D} - 0.25\theta^2\sqrt{D}\nu^{-1.5}\right)\varphi\left(\theta\sqrt{\hat{D}}\right), \tag{A.47}$$

which has a unique root at $\nu = (2\theta - \theta^2)D = 2J - J^2/D$.

The second part can be seen by defining $s_l = \frac{R^b - R^j}{\gamma}$ and acknowledging that $\partial s_l/\partial \nu > 0$ and then

$$\lim_{\nu\to\infty} s_l(\nu) = \lim_{\nu\to\infty} 1 - \Phi\left(\sqrt{\frac{J}{\nu D}}\right) \iff \lim_{x\to 0} 1 - \Phi(x) = 0.5. \tag{A.48}$$

Appendix A.5 Proof of proposition 5 (elasticity of deposits to reserves)

Express (21) in terms of the standard normal distribution with PDF φ and CDF Φ and insert (18). The lending rate is then given by

$$R^b(\hat{J}) = R^j + \gamma\left(1 - \Phi\left(\hat{J}/\sqrt{d_A(R^b(\hat{J}))}\right)\right). \tag{A.49}$$

Let $d'_A = \frac{\partial d_A}{\partial R^b}$ and define

$$h(J) = d_A(R^b(J))^{-1/2}, \tag{A.50}$$

with first derivative $h' = -0.5A^{-1.5}R^{b\prime}d'_A$. Inserting into (A.49) and taking the derivative w.r.t. J yields

$$\begin{aligned} R^{b\prime} &= -\gamma(h + Jh')\phi(\cdot), && \text{(A.51)}\\ &= -\gamma h\phi(\cdot) + \gamma J0.5A^{-1.5}R^{b\prime}d'_A\phi(\cdot), && \text{(A.52)}\\ &= -\gamma\frac{h\phi(\cdot)}{1 - 0.5\gamma JA^{-1.5}d'_A\phi(\cdot)}, && \text{(A.53)}\\ &= -\frac{1}{(\gamma\hat{f})^{-1} - 0.5Jd'_A/A}, && \text{(A.54)} \end{aligned}$$

which is negative for all $d'_A < 0$. Since the elasticity of A w.r.t. J is $E_{DJ} = \frac{\partial d_A/A}{\partial J/J} = \frac{\partial d_A}{\partial R^b}\frac{\partial R^b}{\partial J}\frac{J}{A} = d'_A R^{b\prime}J/A$, the first result from the proposition follows.

(A.52) can also be rewritten as

$$\begin{aligned} R^{b\prime} &= -\gamma h\phi(\cdot) + \gamma J0.5A^{-1.5}R^{b\prime}d'_A\phi(\cdot), && \text{(A.55)}\\ &= \gamma(0.5E_{DJ} - 1)\hat{f}. && \text{(A.56)} \end{aligned}$$

Since $R^{b'} < 0$ and $d'_A < 0$, it follows that

$$\{E_{DJ} \geq 0 \wedge \gamma(0.5E_{DJ} - 1)\phi(\cdot) < 0\} \quad \Longrightarrow \quad E_{DJ} \in [0, 2). \tag{A.57}$$

Appendix A.6 Proof of proposition 6 (effectiveness of asset purchases)

i) The first part follows directly from the proof of proposition 5 in Appendix A.5. For the result on the marginal efficiency, the second derivative is given by

$$R^{b''} = \left[(J^2/A - 1)h' + Jh^3)\phi(\cdot) - 0.5\left(d''_A R^{b'} J/A + d'_A/A - {d'_A}^2 J/A^2 R^{b'}\right)\right] {R^{b'}}^2. \tag{A.58}$$

For large J/D we have that $E_A \to 0$, from which it follows that $d'_A = 0$ and $h' = 0$ and the equation collapses to

$$\frac{\partial^2 R^b}{\partial J^2} = \frac{J}{D}\hat{f}, \tag{A.59}$$

which is always positive.

ii) The interest margin is given by

$$s_m\left(\hat{D}, \hat{J}\right) = \frac{1}{2\sqrt{\hat{D}}}\varphi\left(\hat{J}/\sqrt{\hat{D}}\right), \tag{A.60}$$

with first derivative

$${s_m}' = 0.5\left(h' - \frac{J}{D}(h + Jh')\right)\varphi(Jh) \tag{A.61}$$

where from using $Jh' = -0.5E_{DJ}$ it is that

$${s_m}' = 0.5\left(\frac{J}{D}(0.5E_{DJ} - 1) - 0.5E_{DJ}/J\right)\hat{f}, \tag{A.62}$$

after acknowledging that $E_{DJ} - 2 < 0$ it follows that $s'_m < 0$ iff

$$J\frac{J}{D} > \frac{E_{DJ}}{E_{DJ} - 2}. \tag{A.63}$$

iii) In terms of the standard normal distribution with PDF φ and CDF Φ, the deposit rate is given by

$$R(J) = R^j + \gamma\left(1 - \Phi(Jh) - 0.5h\varphi(Jh)\right), \tag{A.64}$$

with derivative

$$R' = \gamma \left(-(h + Jh') - 0.5h' + 0.5\frac{J}{D}(h + Jh') \right) \varphi(\cdot), \tag{A.65}$$

$$= \gamma \left(-(1 - 0.5E_{DJ}) + 0.5E_{DJ}/J + 0.5\frac{J}{D}(1 - 0.5E_{DJ}) \right) \hat{f}, \tag{A.66}$$

$$\frac{\partial R}{\partial J/J} = \gamma \left(J(0.5J/D - 1)(1 - 0.5E_{DJ}) - 0.5E_{DJ} \right) \hat{f}. \tag{A.67}$$

Plugging in $E_{DJ} = \frac{1}{0.5 - \left(\gamma E_A J \hat{f}\right)^{-1}}$ yields the expression from the proposition.

Appendix A.7 Proof of proposition 7 (effectiveness of IOR policy)

1. (a) Again, the lending rate is then given by

$$R^b(\hat{J}) = R^j + \gamma \left(1 - \Phi\left(Jh\right)\right), \tag{A.68}$$

with

$$h(R^j) = d_A(R^b(R^j))^{-1/2}, \tag{A.69}$$

and first derivative $h' = -0.5hR^{b'}d'_A/A$. Again define the elasticity as $E_{DR^j} = \frac{\partial d_A/A}{\partial R^b}\frac{\partial R^b}{\partial R^j} = R^{b'}d'_A/A$. The derivative of the lending rate is then given by

$$R^{b'} = 1 - \gamma Jh'\varphi(Jh) \tag{A.70}$$

$$= \frac{1}{1 - 0.5\gamma J E_A \hat{f}}. \tag{A.71}$$

which is always positive.

(b) The interest margin is given by

$$s_m = 0.5h\varphi\left(Jh\right), \tag{A.72}$$

with first derivative

$$s_m{}' = 0.5\left(h' - \frac{J}{D}h\right)\varphi(Jh). \tag{A.73}$$

(c) The deposit rate is

$$R(J) = R^j + \gamma \left(1 - \Phi(Jh) - 0.5h\varphi(Jh)\right), \tag{A.74}$$

with derivative

$$R' = 1 + 0.5\gamma \left(0.5\frac{J^2}{D} - 0.5 - J\right) E_{DR^j}\hat{f}. \tag{A.75}$$

The conjecture $R' \geq 1$ follows from the fact that $J\frac{J}{D} - J < 0$ since $0 \leq J \leq D$ while $E_{DR^j} < 0$ since $d'_A < 0$.

2. The MRR is binding whenever $MPJ_\psi < 0$. Conversely, when the MRR is slack

$$R^j - R^b + \gamma\left[1 - \hat{F}_\psi\right] > 0. \tag{A.76}$$

The economy will hence move towards the MRR for stimulative IOR policy (when $\Delta R^j < 0$) if $\frac{\partial MPJ_\psi}{\partial R^j} > 0$. The term $\gamma\left[1 - \hat{F}_\psi\right]$ is independent of R^j (D is constrained by J at the MRR) and we know from above that

$$R^{b\prime} = \frac{1}{1 - 0.5\gamma J E_A \hat{f}}. \tag{A.77}$$

Hence,

$$\frac{\partial MPJ_\psi}{\partial R^j} = 1 - \frac{1}{1 - 0.5\gamma J E_A \hat{f}}, \tag{A.78}$$

$$= \frac{-\gamma J E_A \hat{f}}{1 - 0.5\gamma J E_A \hat{f}}, \tag{A.79}$$

$$> 0, \tag{A.80}$$

since $E_A < 0$.

Appendix A.8 Proof of proposition 9 (effectiveness of open market operations with binding MRR)

i) As above, it is that

$$E^\psi_{DJ} = \frac{\partial d_A/A}{\partial J/J} = \frac{\partial d_A}{\partial R^b}\frac{\partial R^b}{\partial J}\frac{J}{A} = d'_A R^{b\prime} J/A. \tag{A.81}$$

From $E^\psi_{DJ} = 1$ we can rearrange to

$$\frac{\partial R^b}{\partial J} J = \left(\frac{\partial d_A}{\partial R^b}\right)^{-1} A = E_A^{-1}. \tag{A.82}$$

ii) The proof follows from iii) via

$$s^{b\prime} = R^{b\prime} - R'. \tag{A.83}$$

iii) In terms of the standard normal distribution with PDF φ and CDF Φ, the deposit rate when the MRR is binding is given by

$$R = (1-\psi)R^b + \psi R^j + \gamma\left(\psi\left[1 - \Phi(Jh)\right] - 0.5h\varphi(Jh)\right). \tag{A.84}$$

Taking the derivative w.r.t. J:

$$R' =(1-\psi)R^{b'} + \gamma\left(0.5\frac{J}{D}(h+Jh') - 0.5h' - \psi(h+Jh')\right)\varphi(Jh), \tag{A.85}$$

$$=(1-\psi)R^{b'} - \gamma\left(0.5\psi(h+Jh') + 0.5h'\right)\varphi(Jh). \tag{A.86}$$

Since we know that $Jh' = -.5E_{DJ}h = -.5h$:

$$R' =(1-\psi)R^{b'} - \gamma\left(0.25\psi - 0.25/J\right)\hat{f}. \tag{A.87}$$

Appendix A.9 Proof of proposition 10 (effectiveness of IOR policy with binding MRR)

i. The proof follows from the MRR, $\psi D = J$, and the funds market equilibrium, $d_A(R^b) = D$. Since D is fixed because J is given

$$\frac{\partial d_A}{\partial R^j} = \frac{\partial d_A}{\partial R^b}\frac{\partial R^b}{\partial R^j} = 0. \tag{A.88}$$

If $\frac{\partial d_A}{\partial R^b} < 0$ it must be that $\frac{\partial R^b}{\partial R^j} = 0$. If however $\frac{\partial d_A}{\partial R^b} = 0$, $\frac{\partial R^b}{\partial R^j}$ is indetermined.

ii. The proof follows again from

$$s^{b'} = R^{b'} - R' \tag{A.89}$$

with R' given from below.

iii. As above, the deposit rate is

$$R = (1-\psi)R^b + \psi R^j + \gamma\left(\psi\left[1-\Phi(Jh)\right] - 0.5h\varphi(Jh)\right). \tag{A.90}$$

We know from i. that the derivative of the first term is zero. Since $\frac{\partial d_A}{\partial R^j} = 0$ it follows that $h' = 0$ and the derivative of the last term is also zero.

Appendix A.10 Proof of proposition 11 (effective lower bound)

If the MRR is binding, R^b is determined by $d(R^b) = D = \psi J$ and thereby fully independent of R^j. This means that only R is determined by the IOR rate, which at the DLB is fixed at 1.

Appendix A.11 Proof of proposition 12 (reversal rate)

For the slack MRR, the equations follow from (19) and (23). I.e., if the MRR is slack, then for the above $R = 1 + r = 1$ and (22) is inactive. However, R^b is still determined by (21), implying that the borrowing-lendign spread is directly affected by the DLB.

The second condition $\psi d(R^b) < J$ is important because otherwise the MRR binds and we are in the case of proposition 11, where there is no reversal.

Appendix B The linearized model

TBD

Appendix C Data

The observables of GDP, consumption, investment, wages, labor and inflation for the standard Smets and Wouters (2003, 2007) part of my model are also standard and are obtained from the ECB. Due to artificial dynamics in the civilian noninstitutional population series that arise from irregular updating (Edge et al., 2013), I use a 4-quarter trailing moving average to calculate per capita variables. Data on the loan rate and the household deposit rate is obtained directly from the ECB (similar data can be found in the ECB's statistical data warehouse, SDW).

The time series for liquidity provision policy X=JoMRR is using SDW data and is constructed by

- JOMRR: TOTRES/MINRES
- TOTRES: ILM.W.U2.C.L020100.U2.EUR + ILM.W.U2.C.L020200.U2.EUR + ILM.W.U2.C.L020300.U2.EUR
- MINRES: $\psi_{\text{pre-2012}}$*DEPOSITS
- $\psi_{\text{pre-2012}}$ is the mean of (RESERVES/DEPOSITS) until 2011:IV
- RESERVES: BSI.M.U2.N.R.LRE.X.1.A1.3000.Z01.E + BSI.M.U2.N.R.LRR.X.1.A1.3000.Z01.E
- DEPOSITS: BSI.M.U2.N.R.L2A.H.1.A1.3000.Z01.E + BSI.M.U2.N.R.L2B.L.1.A1.3000.Z01.E

Note that as of Jan 2022, the ECB's time series for excess reserves (BSI.M.U2.N.R.LRE.X.1.A1.3000.Z01.E) is missleading because the deposit facility and fixed term deposits are not counted as (excess) reserves, which they are by definition.

Appendix D More parameter estimates

	Prior			Posterior				
	distribution	mean	sd	mean	sd	mode	5% HPD	95% HPD
ρ_r	beta	0.500	0.200	0.340	0.093	0.381	0.187	0.487
ρ_{nir}	beta	0.500	0.200	0.983	0.006	0.984	0.974	0.993
ρ_g	beta	0.500	0.200	0.940	0.016	0.938	0.914	0.967
ρ_z	beta	0.500	0.200	0.992	0.006	0.995	0.986	0.998
ρ_u	beta	0.500	0.200	0.931	0.050	0.894	0.857	0.988
ρ_p	beta	0.500	0.200	0.596	0.149	0.683	0.353	0.818
ρ_w	beta	0.500	0.200	0.828	0.054	0.906	0.748	0.914
ρ_i	beta	0.500	0.200	0.752	0.044	0.709	0.684	0.826
ρ_ϵ	beta	0.500	0.200	0.787	0.046	0.846	0.716	0.860
ρ_γ	beta	0.500	0.200	0.963	0.012	0.970	0.943	0.982
$\text{root}_{x,1}$	beta	0.500	0.200	0.822	0.092	0.767	0.683	0.959
$\text{root}_{x,2}$	beta	0.500	0.200	0.809	0.103	0.862	0.651	0.955
μ_p	beta	0.500	0.200	0.570	0.154	0.628	0.282	0.780
μ_w	beta	0.500	0.200	0.688	0.098	0.807	0.541	0.839
ρ_{gz}	normal	0.500	0.250	1.160	0.132	1.075	0.967	1.382
σ_r	inv.gamma	0.100	0.250	0.063	0.007	0.058	0.054	0.075
σ_{nir}	inv.gamma	0.100	0.250	0.006	0.000	0.007	0.006	0.007
σ_g	inv.gamma	0.100	0.250	0.226	0.019	0.215	0.196	0.256
σ_z	inv.gamma	0.100	0.250	0.199	0.018	0.189	0.172	0.230
σ_i	inv.gamma	0.100	0.250	0.381	0.051	0.399	0.298	0.465
σ_p	inv.gamma	0.100	0.250	0.143	0.015	0.129	0.119	0.167
σ_w	inv.gamma	0.100	0.250	0.204	0.025	0.193	0.165	0.244
σ_u	inv.gamma	0.100	0.250	0.105	0.077	0.177	0.022	0.222
σ_ϵ	inv.gamma	0.100	0.250	0.002	0.000	0.002	0.002	0.002
σ_γ	inv.gamma	0.100	0.250	0.221	0.049	0.246	0.141	0.297
σ_x	inv.gamma	0.100	0.250	0.326	0.024	0.344	0.289	0.365

Table D.2: Estimation results for parameters govering the exogenous shock processes.

Appendix E Estimated expected PLB durations

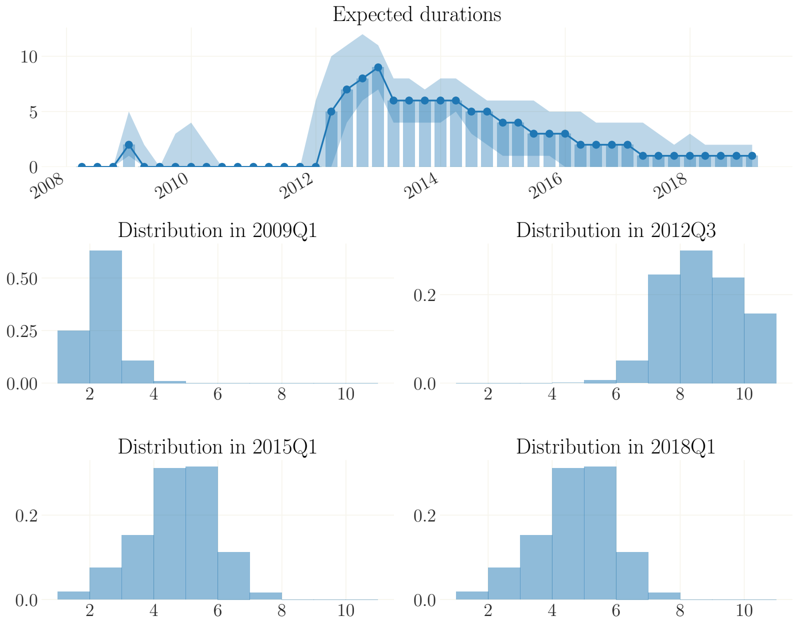

Figure E.11: Estimated expected ELB durations based on the benchmark estimation. Bars in the top panel mark the mean estimate. The shaded area represents 90% credible sets. The lower panels show histograms of the distribution of ELB durations. The last bar to the right marks the probability of a duration of 10 or more quarters.