The Hockey Stick Phillips Curve and the Effective Lower Bound

Gregor Boehl[a,*], Philipp Lieberknecht[b]

[a] *University of Bonn*
[b] *Deutsche Bundesbank*

September 25, 2024

Abstract

We show that the interplay between a binding effective lower bound (ELB) on nominal interest rates and the costs of external financing weakens the disinflationary effect of financial shocks. In normal times, the real costs of production factors dominate in firms' marginal costs and are therefore key for inflation dynamics. In contrast, financing costs normally play a subordinate role as higher credit spreads are balanced-out by lower nominal rates. At the ELB, however, higher spreads following financial shocks can offset the effect of lower production factor costs on firms' price setting. The relationship between inflation and output hence features a hockey stick shape: the Phillips curve is flat at the ELB, but conventionally upward-sloping during normal times. This mechanism also weakens the power of forward guidance at the ELB, since such policy reduces spreads and financing costs.

Keywords: Phillips Curve, Financial Frictions, Effective Lower Bound, Disinflation, Forward Guidance

JEL: C62, C63, E31, E32, E44, E52, E58, E63

[⋆] The views expressed in this paper are those of the authors and do not necessarily coincide with the views of the Deutsche Bundesbank or the Eurosystem. We are grateful to Joachim Keller, Keith Kuester, Taisuke Nakata, Johannes Wieland, and participants of several conferences and seminars for helpful discussions and comments. Parts of the research leading to the results in this paper has received financial support from the Alfred P. Sloan Foundation under the grant agreement G-2016-7176 for the MMCI at the IMFS Frankfurt. Gregor Boehl gratefully acknowledges financial support by the DFG under CRC-TR 224 (projects C01 and C05) and under project number 441540692.

[*] Corresponding author. Email: `gboehl@uni-bonn.de`, address: Institute for Macroeconomics and Econometrics, University of Bonn, Adenauerallee 24-42, 53113 Bonn, Germany

Email addresses: `gboehl@uni-bonn.de`, `philipp.lieberknecht@bundesbank.de`

1 Introduction

What is the relationship between inflation and economic activity? Given the fundamental role of these two concepts, it is quite troubling that this question is still puzzling the economic profession. In particular, the Global Financial Crisis of 2007/2008 and the associated financial turmoil led to the *missing disinflation puzzle*: despite substantially negative output gaps, inflation fell only modestly and thus seemed disconnected from economic activity.[1] This observation raised considerable interest in analyzing the seemingly flat Phillips curve (Ball and Mazumder, 2011; Coibion and Gorodnichenko, 2015; Harding et al., 2022). While the explanations put forward are numerous and manifold, we found one key contributing factor yet to be missing: the effective lower bound (ELB) on nominal interest rates, which was reached by several central banks around the globe at the same time as the observed inflation puzzles.

In this paper, we show that the interplay of the ELB and financial frictions may reshape the relationship between inflation and output if financial shocks are prevailing. Recent research documents that supply-side financial frictions can be crucial for firms' price setting behavior and, thereby, for inflation dynamics (e.g. Gilchrist et al., 2017). We argue that firms' marginal costs are dominated by the procyclical costs of production factors in normal times. In the presence of financial frictions, however, marginal costs also contain the costs of external financing. These costs consist of the risk-free interest rate and a countercyclical credit spread reflecting financial frictions. We show that higher credit spreads can substantially offset lower production factor costs if the nominal rate is constrained by the ELB. In this case, the costs of external financing considerably weaken the supply-side link between output and prices. As a result, financial shocks at the ELB induce only moderate disinflation, and may in extreme cases even be inflationary.

Taking the ELB into account, the resulting *observational Phillips curve*[2] is thus shaped like a hockey stick. For normal times with positive or mildly negative output gaps, it exhibits a conventional positive slope in output gap - inflation space. In contrast, the slope is considerably flat for significantly negative output gaps when the ELB is binding. This non-linear, kinked Phillips Curve provides an explanation for the puzzles of missing disinflation consistent with the observed timing of events. As seen in Figure 1, corporate spreads peaked at almost 6% in the

---

[1] For example, the US output gap was -5.3% in Q2 2009, accompanied by a core inflation rate (excluding food and energy) of 1.8%, only slightly below the central bank target of 2%.

[2] We use this term to refer to the reduced-form relationship between realized (equilibrium) values for inflation and output gap, i.e. the *observed* or *empirical* Phillips curve. As discussed below, this is not equivalent to the New Keynesian Phillips curve describing firms' price setting behavior.

last quarter of 2008, while the Federal funds rate reached almost 0% in the first quarter of 2009. Both elevated corporate spreads and the binding lower bound persisted until the end of 2015, coincidental with the observed weakening of the relationship between inflation and output.

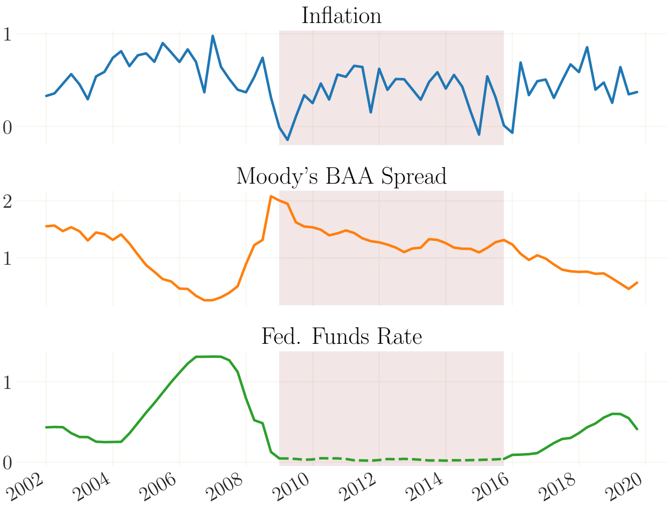


Figure 1: Time series of inflation, BAA spread and US interest rates from 2002 to 2020. Quarterly data in percentage points. The shaded area depicts the episode during which the ELB was binding.

We show these results in a tractable New Keynesian DSGE model featuring financial frictions. In the model, workers need to be paid before production (as in Ravenna and Walsh, 2006), generating external financing needs for the entrepreneurs operating the firms. Due to a costly state verification problem à la Townsend (1979) and Bernanke et al. (1999), the costs of external financing comprise a risk premium in the form of a countercyclical credit spread which depends on entrepreneur leverage. This key model feature is well-established in the empirical literature (see e.g. Gilchrist and Zakrajšek, 2012; Anderson and Cesa-Bianchi, forthcoming). While entrepreneurs choose the credit volume, their available net worth depends positively on the state of the aggregate economy, in the spirit of empirical evidence (Greenwood and Shleifer, 2014; Adam et al., 2017). The overall model setup implies that financial frictions influence firms' marginal costs and price setting, in line with a large literature showing that supply-side financial frictions can help explaining inflation dynamics during the Great Recession and its aftermath (see e.g. Christiano et al., 2015; Gilchrist et al., 2017, and further below). Concerning financial shocks, we focus on the effects of risk premium shocks in the spirit of Smets and Wouters (2007). These are known to have a large explanatory power for the joint dynamics of consumption, investment and inflation following the 2007/2008 recession (Gust et al., 2017; Kulish et al., 2017; Boehl, 2022a; Boehl and Strobel, 2024a; Boehl et al., 2024).

Our first contribution is to provide analytic solutions for macroeconomic dynamics after financial shocks, both for normal times and for a binding ELB. We show that a longer expected ELB duration can be associated with weaker disinflationary effects of financial shocks. This case occurs for sufficiently large financial shocks if the elasticity of the credit spread with respect to entrepreneur leverage is sufficiently high. Accordingly, this effect depends crucially on the presence and degree of financial frictions and is hence absent in the standard New Keynesian model. The analytic solutions furthermore highlight that even an overall increase of inflation following contractionary financial shocks is possible, and may in particular occur if the ELB is expected to bind for an extended period of time. In this case, financial shocks move output and inflation in opposite directions, thus appearing as supply-type disturbances, in sharp contrast to their usual appearance as demand-type shocks away from the ELB.

As our second contribution, we discuss the associated implications for monetary policy. Shocks to the monetary policy rule generate macroeconomic dynamics that are very similar to financial shocks. As a consequence, forward guidance shocks with relatively low persistence can even be disinflationary: the effect of keeping the expected costs of external financing lower in the future may dominate the long-run effect of increasing the price level by stimulating consumption. Hence, this also provides an explanation for the forward guidance puzzle (Carlstrom et al., 2015; Del Negro et al., 2015a; Kiley, 2016) and suggests that any forward guidance measures must be undertaken with vigor. Furthermore, the central bank might find itself in a knife-edge scenario where the appropriate window for systematic policy responses guaranteeing a determinate equilibrium is rather small.

Our results are well-supported by empirical evidence showing that financial shocks can be associated with weak disinflation or even be inflationary if supply-side effects dominate demand effects. Various contributions find empirical evidence in favor of such a (financial) cost channel via higher borrowing costs (Barth III and Ramey, 2001; Chowdhury et al., 2006; Tillmann, 2008; Abbate et al., 2021). Similarly, using firm-level data, Gaiotti and Secchi (2006) find this cost channel to be proportional to working capital, while Gilchrist et al. (2017) show that financially constrained firms increased their prices amid the stark economic downturn in the Great Recession. Our paper provides a theoretical foundation for these papers and highlights that both the degree of financial frictions and a binding ELB are particularly relevant.

The issue of missing (dis-)inflation in recent years was first brought up by Ball and Mazumder (2011) and subsequently confirmed for many advanced economies by Friedrich (2016). A wealth of explanations was put forward, encompassing anchored expectations (Ball and Mazumder, 2018; Coibion and Gorodnichenko, 2015), various measures of economic slack (Gordon, 2013; Watson, 2014), supply shocks and wage rigidities (Daly and Hobijn, 2014; Harding et al., 2022), optimal monetary policy, potentially in combination with financial frictions (Lieberknecht, 2019; Sims and Wu, 2019; McLeay and Tenreyro, 2020) or global factors (Bobeica and Jarociński, 2019; Forbes, 2019). We provide a complementary explanation for inflation dynamics that also matches the particular timing of the observed missing (dis-)inflation: the ELB affects the cyclicality of marginal costs via the costs of external financing in the presence of supply-side financial frictions, thereby leading to an observational disconnect between inflation and output.

A related strand of the literature investigates these inflation dynamics through the lens of New Keynesian DSGE models, notably Christiano et al. (2015), Del Negro et al. (2015b) and Gilchrist et al. (2017). In line with our paper, they show that adding supply-side financial frictions to DSGE models helps to explain the missing disinflation puzzle. Closely related to our work, Gilchrist et al. (2017) argue that financial distortions affecting firms' markups and their price setting behavior are key. While our paper shares this argument, we provide additional insights that a binding ELB strongly amplifies the effects of financial frictions such that credit spreads may even dominate inflation dynamics. This is in line with Bianchi and Melosi (2017) and Boehl (2022a), who find that accounting for the ELB substantially improves the empirical fit of estimated DSGE models. We also shed light on the necessary conditions for inflationary effects of financial shocks – a feature that is present in several financial friction models (Christiano et al., 2010; Meh and Moran, 2010; Gerali et al., 2010). Additionally, we also contribute to the literature on how to conduct monetary policy when the ELB is present (Adam and Billi, 2006; Budianto et al., 2023; Budianto, 2023) by documenting that forward guidance may have disinflationary effects.

Lastly, our paper is related to the literature on Neo-Fisherianism, which argues that the causality between the policy rate and inflation is positive even in the short run (Gabaix, 2020; Cochrane, 2011, 2016, 2017). We show that such effects may arise at the ELB, and – theoretically – even in normal times if the degree of financial frictions is strong enough. This is in contrast to García-Schmidt and Woodford (2019), who argue that Neo-Fisherian effects arise only after credible changes in long-run monetary policy targets.

We continue in Section 2 by outlining the model and discussing the components of marginal costs in this framework. In Section 3, we derive closed-form solutions for macroeconomic dynamics following financial shocks. Section 4 complements by showing numerical solutions and analyzing the resulting observational Phillips curve. In Section 5, we investigate the implications for monetary policy at the ELB. Section 6 concludes.

2 Model

Our analysis is based on a tractable New Keynesian DSGE model featuring financial frictions, similar to Boehl (2022b) and Lieberknecht (2019). Wholesale firms are operated by entrepreneurs who maximize their returns. Production is subject to a working capital channel as in Ravenna and Walsh (2006): Workers have to be paid before production, which creates a need for external financing. To finance the wage bill and enable production, entrepreneurs use their available net worth and borrow from financial intermediaries. The supply of entrepreneurial equity depends on the aggregate state of the economy. The lending contract is subject to financial frictions in the form of a costly state verification problem in the spirit of Townsend (1979) and Bernanke et al. (1999). The combination of the working capital channel and costly state verification implies that firms' marginal costs contain a credit spread over the real interest rate that depends on entrepreneur leverage. The homogeneous good produced by wholesale firms is sold to a monopolistic retailer. The resulting final good is bought by homogeneous households for consumption. The labor market is perfectly competitive. A central bank follows an interest rate rule subject to an effective lower bound.

2.1 Households

Households are homogeneous and maximize the expected present value of lifetime utility by choosing consumption of a composite good C_t and hours supplied to the labor market H_t. They can deposit monetary savings at financial intermediaries (also synonymously called banks in the following), for which they receive a return of R_tU_t in the next period, consisting of the gross nominal risk-free interest rate R_t adjusted by an exogenous *financial shock* U_t. The financial shock constitutes a premium on the risk-free interest rate reflecting the state of the financial system or, equivalently, a wedge between the central bank interest rate and the return on household assets as in Smets and Wouters (2007).

Aside from the financial shock, the households' optimization problem is completely standard and yields the usual inter-temporal Euler equation and an intra-temporal labor supply equation

$$C_t^{-\sigma} = \beta E_t \left[\frac{R_t}{\Pi_{t+1}} U_t C_{t+1}^{-\sigma} \right], \tag{1}$$

$$H_t^{\eta} = W_t C_t^{-\sigma}, \tag{2}$$

where $\Pi_t = P_t / P_{t-1}$ is gross inflation and W_t is the real wage. The parameters σ, η and β are the inverse elasticity of intertemporal substitution, the inverse Frisch elasticity of labor supply and the discount rate, respectively.

2.2 Wholesale firms and retailers

The wholesale sector consists of a continuum of firms. Each firm j is operated by a risk-neutral entrepreneur and produces a homogeneous good $Y_{j,t}$ using a production function that is linear in labor (the only production factor), subject to a firm-specific idiosyncratic productivity shock $\omega_{j,t}$:

$$Y_{j,t} = \omega_{j,t} H_{j,t} \tag{3}$$

Workers have to be paid before production takes place, while returns are realized at the end of the period. This working capital channel follows Ravenna and Walsh (2006) and is supported by empirical work (see the references in the introduction). In our setup, this assumption provides a role for external finance. To finance production, the entrepreneurs use bank loans and net worth (or synonymously equity). While entrepreneurs choose the loan volume, they take their available net worth as given. The loan volume is thus the difference between working capital $W_t H_{j,t}$ (with the amount of labor chosen by the entrepreneur) and net worth $N_{j,t}$.[3]

For simplicity, we assume that entrepreneurs only live for one period and are at birth (at the beginning of the period) endowed with a certain amount of net worth. This startup capital is proportional to aggregate output, i.e. net worth is procyclical. We represent the evolution of entrepreneurs' net worth by

$$N_t = \Psi \left(Y_t \right), \tag{4}$$

with $\psi \equiv \frac{Y}{N} \Psi'(Y) > 0$.

---

[3] The notation N_t for entrepreneur net worth follows Bernanke et al. (1999), not to be confused with labor supply (H_t in our notation).

One interpretation of this functional representation is that entrepreneurial net worth is traded on the financial market, where agents do not form rational expectations but rely on backward-looking forecasting heuristics. Such a configuration would be in line with a large literature documenting that financial market dynamics cannot be reconciled with purely rational expectations (see Boehl, 2022b, for a summary of this literature). Our modelling approach thus follows the spirit by Boehl (2022b), who considers a behavioral model of financial traders switching endogeneously between simple forecasting heuristics.[4] The procyclicality of net worth is also consistent with the data as well as the standard financial accelerator models à la Bernanke et al. (1999), in which entrepreneurs accumulate retained earnings. Importantly, this assumption implies that net worth is not a state variable, which is required to obtain tractable closed-form solutions, as shown in Section 3.

The lending contract is subject to a costly state verification problem: The idiosyncratic productivity shock is private information of the entrepreneur; banks incur monitoring costs if they wish to observe the firm's produced output. In line with Carlstrom and Fuerst (1997) and Bernanke et al. (1999), we assume that the contract is written in real terms, and that bargaining power accrues to the entrepreneur. Entrepreneurs maximize the expected discounted sum of profits over equity – subject to a participation constraint for the bank, i.e. ensuring that the bank is willing to grant the loan – by choosing labor, dividends and a threshold value of productivity. The realization of the idiosyncratic shock relative to the threshold value decides whether the entrepreneur is able to repay the loan at the end of the period (if the realized value falls above threshold) or has to default (if it falls below). The threshold, in turn, determines the interest rate on the loan $R^L_{j,t}$. The loan rate contains an endogenous risk premium over the bank's opportunity cost of lending, i.e. the real deposit rate (which includes the financial shock).[5] This risk premium takes the form of a credit spread that depends positively on the individual firm's leverage, i.e. the ratio of the loan volume to net worth, $LEV_{j,t} = \frac{W_t H_{j,t}}{N_{j,t}}$. Intuitively, when the leverage ratio rises, less collateral is provided relative to the loan volume such that the

---

[4] In a financial market setup, representing equity as the discounted stream of profits would give

$$N_t = E_t \left\{ \frac{N_{t+1} + Profit_{t+1}}{R_t/\Pi_{t+1}} \right\} \tag{5}$$

where profits equal output Y_t minus costs. The steady state relationship between net worth and output would then be $N = \frac{1}{R/\Pi - 1}\frac{1}{\sigma}Y$, where σ is the households' elasticity of substitution.

[5] The real deposit rate being the relevant reference rate can be seen as a form of indexation to the aggregate state of the economy. Carlstrom et al. (2014) and Carlstrom et al. (2016) show that the privately optimal contact – i.e. the one that lenders and borrowers would choose – is state contingent. Intuitively, in our setup, the loan contract is repaid at the end of the period and is thus only available for consumption in the next period. Consequently, the repayment is priced with the next period's consumption basket.

loan repayment becomes more risky from the viewpoint of the lender. Accordingly, the bank demands a higher risk compensation and the external finance premium over the real interest rate increases.

In equilibrium, all entrepreneurs make identical decisions, such that the aggregate leverage ratio and the aggregate loan rate are given by

$$LEV_t = \frac{W_t H_t}{N_t}, \tag{6}$$

$$R_t^L = z\left(\frac{W_t H_t}{N_t}\right)\frac{R_t}{E_t[\Pi_{t+1}]}U_t, \tag{7}$$

where $\nu \equiv z'(\cdot) > 0$. A detailed derivation of the entrepreneurs' optimization problem and the external finance premium in Equation (7) is provided in Appendix A.

Since the wholesale sector is assumed to be perfectly competitive, wholesale firms are price takers. In the aggregate, no-arbitrage requires the rate of return on working capital to equal the rate on external funding, since otherwise, wholesalers would have an incentive to adjust the loan volume. It follows that firms' marginal costs are given by

$$MC_t = W_t R_t^L = W_t\, z\left(\frac{W_t H_t}{N_t}\right)\frac{R_t}{E_t[\Pi_{t+1}]}U_t, \tag{8}$$

and hence contain both labor costs and external financing costs; we explore these components of marginal costs in close detail in Section 2.4.

After wholesale goods have been produced, retailers buy the homogeneous good on the wholesale market. Subsequent to differentiation, they sell it in the monopolistically competitive good market. Firms' price setting decisions are subject to nominal rigidities à la Calvo (1983), i.e. they can only adjust their prices each period with a probability of $(1-\zeta)$. The aggregate production function is thus given by

$$Y_t = \frac{H_t}{v_t^p}, \tag{9}$$

where v_t^p is a measure of price dispersion.

2.3 The central bank

The central bank follows a standard monetary policy rule in the spirit of Taylor (1993) for the notional gross nominal interest rate R_t^n in response to inflation and output[6]

$$\frac{R_t^n}{R^n} = \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} \exp(v_t), \tag{10}$$

where v_t is a monetary policy shock following an AR(1) process. The policy rate R_t is subject to an ELB constraint and cannot fall below a threshold level $\bar{R}$:

$$R_t = \max\left\{\bar{R}, R_t^n\right\} \tag{11}$$

At the ELB, negative realizations of the shock v_t can be interpreted as *forward guidance.* Even if the policy rate is constrained by the ELB, the monetary policy shock still affects the (expected path of the) notional nominal interest rate (the "shadow rate"[7]), which in turn affects the expectations of private agents about future interest rates. We elaborate on this interpretation of the shock and investigate the associated implications for macroeconomic dynamics as well as monetary policy at the ELB in Section 5.

2.4 Understanding the components of marginal costs

In our model framework, financial frictions originate in the firm sector and therefore primarily affect the supply side of the economy. The role of financial frictions for marginal costs and inflation dynamics is thus best understood by investigating the New Keynesian Phillips curve. Linearizing around an efficient steady state[8], and denoting log-deviations from steady state via small-case letters, the New Keynesian Phillips curve may be represented in the familiar textbook form (see e.g. Galí, 2015)

$$\pi_t = \kappa\, mc_t + \beta E_t[\pi_{t+1}], \tag{12}$$

---

[6] For financial shocks, the steady state deviation of output and the output gap are equal. We show in Section 5 that financial shocks and monetary policy shocks yield identical macroeconomic dynamics. The canonical result that nominal rigidities are necessary for real effects of monetary policy shocks thus also applies to our financial shocks.

[7] In the literature, several concepts of shadow rates exist. In this paper, we use the definition of a notional policy rate, i.e. the policy rate that the policymaker would choose if the ELB would not pose a constraint. A different concept is a summary measure of the overall monetary policy stance at the ELB, constructed to capturing the effects of unconventional policy instruments (e.g. Jones et al., 2024).

[8] Steady state subsidies from the government (financed by lump-sum taxes) can correct for the two inefficiencies arising from monopolistic competition and the presence of financial frictions (see Lieberknecht, 2019).

with slope $\kappa \equiv \frac{(1-\zeta\beta)(1-\zeta)}{\zeta}$. Financial frictions thus do not alter the price setting behavior of firms per se; prices are tied to marginal costs and expectations of future inflation. However, financial frictions determine and affect the components of marginal costs, which are given by

$$mc_t = w_t + (r_t - E_t[\pi_{t+1}]) + s_t, \tag{13}$$

where s_t denotes the linearized credit spread $s_t = \nu\, lev_t + u_t$. This highlights that marginal costs consist of three components: a) the real wage (called *factor costs* in the following) as in the standard New Keynesian model[9], b) the ex-ante real interest rate (the *cost channel*, similar to Ravenna and Walsh, 2006) and c) the credit spread (also known as the *external finance premium*). The latter two components jointly constitute the costs of external finance.

In the following, we take a closer look at the cyclicality of these components, i.e. their co-movement with output. The real wage is unambiguously procyclical in our setup: as output increases, firms need to offer a higher real wage in order to attract more labor to expand production.[10] The cyclicality of the cost channel depends on the source of aggregate fluctuations. A stable equilibrium requires that the real interest rate rises in response to an increase in inflation to dampen demand. The cost channel is hence procyclical for demand-side shocks (inflation and output move in the same direction), and countercyclical for supply-side shocks (inflation and output move in opposite directions).

Lastly, credit spreads are countercyclical and depend positively on firm leverage in our model setup. Both of these features are in line with empirical evidence (see e.g. Gilchrist and Zakrajšek, 2012; Anderson and Cesa-Bianchi, forthcoming) and standard DSGE models with financial frictions (see e.g. Bernanke et al., 1999; Del Negro et al., 2015b). This implies that firm leverage is countercyclical as well. Using the household's intra-temporal optimality condition, firm leverage can be written as

$$lev_t = -(\psi - 1 - \sigma - \eta) y_t, \tag{14}$$

where ψ denotes the elasticity of equity with respect to output. The necessary and sufficient condition for firm leverage to be countercyclical is thus that the term in brackets is larger than

---

[9] The real wage is equivalent to the labor share of income in our setup. The latter is given by $w_t + h_t - y_t$ in linearized form; inserting the aggregate production function $y_t = h_t$ cancels out the last two terms.

[10] This follows directly from the intra-temporal labor supply equation (2). In contrast, real wages can be countercyclical in models with wage stickiness, different household preferences or a more detailed modelling of the labor market.

zero, i.e. that the procyclicality of net worth outweighs the procyclicality of the wage bill:

Assumption 1. *The elasticity of net worth with respect to output satisfies*

$$\psi > 1 + \sigma + \eta > 0. \tag{15}$$

The second inequality follows from the conventional parameter properties $\sigma > 0$ and $\eta > 0$. Empirical data suggests that this assumption is comfortably satisfied (see Section 4.1 for the calibration).

The three components of marginal costs are thus characterized by opposing cyclicality over the business cycle: for shocks originating from the demand side of the economy, the real wage and the real interest rate are procyclical, whereas the credit spread is countercyclical. Since firms' price setting decisions are tightly connected to current and expected marginal costs, the relative dynamics of these components over the business cycle are thus crucial for inflation dynamics. In particular, the supply-side financial frictions imply that the inflation effects of financial shocks are non-trivial and depend on the degree of financial frictions.[11]

2.5 The linearized representation of the model

The whole linearized model can be represented in three equations only (see Appendix B for more details)

$$\pi_t = \kappa\gamma\, y_t + (\beta - \kappa)E_t[\pi_{t+1}] + \kappa r_t + \kappa u_t, \tag{16}$$

$$y_t = -\sigma^{-1}\left(r_t - E_t[\pi_{t+1}] + u_t\right) + E_t[y_{t+1}], \tag{17}$$

$$r_t = \max\left\{\phi_\pi \pi_t + \phi_y y_t + v_t, \bar{r}\right\}, \tag{18}$$

where $\gamma \equiv \sigma + \eta - \nu(\psi - 1 - \sigma - \eta)$. The exogenous processes for the financial shock u_t and the monetary policy shock v_t are

$$u_t = \rho\, u_{t-1} + \epsilon_t, \tag{19}$$

$$v_t = \rho_r\, v_{t-1} + \epsilon_{r,t}. \tag{20}$$

---

[11] This stands in contrast to alternative modelling approaches for financial frictions which do not imply a direct link between marginal costs and financial frictions such that financial shocks are inherently both contractionary and disinflationary. Among others, this includes the original financial accelerator model by Bernanke et al. (1999).

Equation (16) again represents the New Keynesian Phillips curve, where the first term captures factor costs and the endogenous credit spread, while the third and fourth term reflect the cost channel and the purely exogenous markup effect that arises from financial shocks, respectively. Equation (17) is the Euler equation, and Equation (18) is the monetary policy rule setting the (notional) interest rate as a function of inflation and output. In normal times, the ELB constraint does not bind, such that (17) and (18) are identical to the textbook New Keynesian model. Financial frictions thus manifest solely in the New Keynesian Phillips curve, highlighting once again that the combination of the working capital channel and the costly state verification problem creates a supply-side financial friction that directly affects inflation dynamics via firms' price setting decisions. This representation also showcases that the financial shock – as a wedge between central bank interest rate and the deposit rate – acts different from pure demand shocks (e.g. preference shocks) that would appear solely in the Euler equation. In the following two sections, we investigate this financial shock in close detail, and turn to the monetary policy shock thereafter. In Appendix E, we show and discuss the dynamic impulse responses to conventional preference and cost-push shocks.

3 Analytical results and financial shocks at the effective lower bound

In this section, we analyze how a binding ELB affects the transmission of financial shocks in the model economy. To this end, we derive closed-form general equilibrium solutions for normal times and for a binding ELB. Contrasting these two cases highlights that macroeconomic dynamics at the ELB may be fundamentally different.

3.1 The propagation of financial shocks in normal times

We first analyze the macroeconomic effects of financial shocks in normal times, i.e. when the ELB is not binding. Assuming that the determinacy conditions hold[12], we solve the model via the method of undetermined coefficients and guess that the equilibrium responses of the endogenous variables are linear functions of the exogenous financial shock.

Proposition 1. *The impact responses of inflation and output to a financial shock in normal*

---

[12] See Section 5.2 and Footnote 13 for a closer analysis of the corresponding determinacy requirements for the coefficients in the monetary policy rule.

times are given by

$$\pi_t = \lambda_0^\pi u_t, \tag{21}$$

$$y_t = \lambda_0^y u_t, \tag{22}$$

where

$$\lambda_0^\pi = -\frac{\kappa\gamma - \kappa\sigma(1-\rho)}{(1-\beta\rho)(\sigma(1-\rho)+\phi_y) + \kappa\gamma(\phi_\pi - \rho) - \kappa\sigma(1-\rho)(\phi_\pi - \rho)}, \tag{23}$$

$$\lambda_0^y = -\frac{1 + (\phi_\pi - \rho)\lambda_0^\pi}{\sigma(1-\rho)+\phi_y}. \tag{24}$$

Proof. See Appendix. ■

Lemma 1. *The impact responses of inflation and output to a financial shock in normal times are negative, i.e.*

$$\lambda_0^\pi < 0, \tag{25}$$

$$\lambda_0^y < 0, \tag{26}$$

if the elasticity of the credit spread to entrepreneur leverage satisfies

$$\nu < \frac{\eta + \rho\sigma}{\psi - 1 - \sigma - \eta}. \tag{27}$$

Proof. See Appendix. ■

Proposition 1 and Lemma 1 show that financial shocks are usually demand-type shocks. A financial shock increases the wedge between the interest rate controlled by the central bank and the return on deposits for households, thereby reducing current consumption. Thus, an adverse financial shock decreases overall output. Via the New Keynesian Phillips curve, inflation decreases as well, since factor costs dominate over external financing costs.

The analytic solutions from Proposition 1 clearly display the different channels through which the financial shock operates. In the inflation policy function λ_0^π, the first term in the numerator is the slope of the Phillips curve with respect to output, whereas the second term captures the exogenous markup effect of the financial shock. Following an adverse financial shock, factor costs decrease, because labor demand falls given the decline in demand (the first part of $\kappa\gamma$). Ceteris paribus, this reduces inflation. At the same time, the financial shock increases marginal costs via the external finance premium, as financial frictions in the firm sector intensify (the second part of $\kappa\gamma$ and the markup effect). This increase in the credit spread partially counteracts the decline in factor costs, weakening the overall disinflationary effect.

The cost channel is represented by the last term in the denominator of λ_0^π. This term features a negative sign and is thus – ceteris paribus – disinflationary. Generally, if the central bank reacts stronger (weaker) to fluctuations in inflation and output (which is captured by ϕ_π and ϕ_y), the denominator is larger (smaller), such that the overall response of inflation is smaller (larger). However, lower nominal interest rates in reaction to an overall decline in inflation also decrease marginal costs directly. This amplifies the disinflationary response. The cost channel thus weakens the overall stabilizing property of the central bank's interest rate policy.

Following financial shocks, the various components of marginal costs thus move in different directions. Whereas factor costs and the cost channel induce a disinflationary response, the credit spread channel weakens it. As seen in Lemma 1, the overall inflation response in normal times is negative, as long as the elasticity of the credit spread to entrepreneur leverage is not excessively large.[13] Existing empirical results that are informative for the calibration of ν suggest that condition (27) in Lemma 1 is satisfied (see Section 4.1 on the calibration). We therefore postulate the following Assumption:

Assumption 2. *The elasticity of the credit spread to entrepreneur leverage satisfies Condition* (27) *from Lemma 1.*

In this case, the cost channel and credit spreads approximately balance out. As a consequence, the procyclical factor costs dominate the price setting of firms. This implies that λ_0^π is negative in normal times, such that the financial shock is a classic demand shock and the Phillips curve is upward sloping, i.e. a positive relationship between inflation and output.

Nevertheless, as summarized in Lemma 2 below, the analytic solutions also reveal that an overall increase of inflation following adverse financial shocks is in principle possible. This result relies crucially on the presence of financial frictions linking marginal costs to credit spreads: in the absence of financial frictions, the policy functions in Proposition 1 are unambiguously

---

[13] Lemma 1 guarantees that the denominator in λ_0^π is positive, which is required for determinacy (as shown in the Appendix). Intuitively, the model is only determinate if a stronger central bank reaction to deviations from steady state translates into lower deviations in general equilibrium. The combination of a positive numerator from Lemma 1 and determinacy thus yields $\lambda_0^\pi < 0$ (note the minus in front of the fraction).

negative. The converse situation – inflation and output move in opposite directions – may occur if the credit spread channel dominates both factor costs and the cost channel because the elasticity of the credit spread to leverage is (excessively) large:

Lemma 2. *The impact response of inflation to a financial shock in normal times is positive if the elasticity of the credit spread to entrepreneur leverage satisfies*

$$\nu > \frac{\eta + \rho\sigma}{\psi - 1 - \sigma - \eta}. \tag{28}$$

Proof. See Appendix. ■

Due to Assumption 2, Lemma 2 does not play a role in the subsequent analysis of financial shocks, but we return to it in our analysis of monetary policy Section 5.

3.2 The propagation of financial shocks at the effective lower bound

We now turn to the case of a binding ELB. To this end, we assume that a financial shock (or any other shock) endogenously brought the economy to the ELB and makes private agents expect the ELB to bind for a certain number of periods (often called the *ELB spell duration*, see e.g. Holden, 2019).[14] For the following analysis, we take this ELB spell duration as given and do *not* adjust agents' expectations on the spell duration. This scenario hence focuses on *marginal* effects of *additional* financial shocks at the ELB that are sufficiently small not to alter the expected ELB length. While this perspective abstracts from the mapping between shocks and the expected duration of the ELB, it allows for a straightforward analytical comparison to the case of normal times. The associated analytical policy functions shed light on the propagation of financial shocks at the ELB, and thereby provide intuition for interpreting the impulse responses, which we obtain using numerical methods and discuss in Section 4).

We guess that the equilibrium responses of endogenous variables are linear functions of the exogenous financial shock and the ELB value for the nominal interest rate $\bar{r}$. Again using the method of undetermined coefficients shows that the equilibrium responses of inflation and output can be characterized by recursive policy functions which are conditionally linear given the expected ELB spell duration:[15]

---

[14] The empirical DSGE literature (see e.g. Jones et al., 2022) finds that actual ELB durations can differ considerably from model-implied ELB durations and market participants' expectations (based on surveys).

[15] The use of recursive representations for solving models with occasionally binding constraints like the ELB is quite standard, see e.g. Kulish et al. (2017) and Kulish and Pagan (2017).

Proposition 2. *Suppose that the ELB on nominal interest rates is expected to bind for* $T > 0$ *periods. The impact responses of inflation and output to a financial shock that does not alter the expected ELB duration are then given by*

$$\pi_t = \lambda_T^{\pi}\, u_t + \mu_T^{\pi}\, \bar{r}, \tag{29}$$

$$y_t = \lambda_T^{y}\, u_t + \mu_T^{y}\, \bar{r}, \tag{30}$$

where

$$\lambda_T^{\pi} = \kappa\left(1 - \gamma\sigma^{-1}\right) + \rho\left(\beta - \kappa + \kappa\gamma\sigma^{-1}\right)\, \lambda_{T-1}^{\pi} + \rho\kappa\gamma\, \lambda_{T-1}^{y}, \tag{31}$$

$$\mu_T^{\pi} = \kappa(1 - \gamma\sigma^{-1}) + \left(\beta - \kappa + \kappa\gamma\sigma^{-1}\right) \mu_{T-1}^{\pi} + \kappa\gamma\mu_{T-1}^{y} \tag{32}$$

$$\lambda_T^{y} = -\sigma^{-1} + \rho\sigma^{-1}\, \lambda_{T-1}^{\pi} + \rho\, \lambda_{T-1}^{y}, \tag{33}$$

$$\mu_T^{y} = \sigma^{-1} + \sigma^{-1}\mu_{T-1}^{\pi} + \mu_{T-1}^{y} \tag{34}$$

where $\{\lambda_0^{\pi}, \lambda_0^{y}\}$ *as in Proposition 1 and* $\mu_0^{\pi} = \mu_0^{y} = 0$.

Proof. See Appendix. ■

To interpret Proposition 2, consider the inflation response under an expected ELB duration of one quarter (λ_1^{π}) and recall that the financial shock is assumed not to prolong the given ELB duration. Under Assumption 2, the impact responses of inflation and output are negative in normal times ($\lambda_0^{\pi} < 0$, $\lambda_0^{y} < 0$). In λ_1^{π}, the second and third term are thus negative. For persistent shocks, these terms induce a monotonically decreasing recursion. This shows the ELB's amplification property: the impact response of inflation increases (ceteris paribus) in the expected ELB spell duration. The amplification reflects the inability of the central bank at the ELB to counteract further contractionary shocks via additional (conventional) monetary stimulus. At the same time, the resulting upward pressure on real interest rates depresses consumption, and accordingly output such that the financial shock is contractionary.

However, Proposition 2 shows that there is an opposing effect on the overall inflation response at the ELB. The first term in Equation (31) can be positive, such that there is a potential for a policy function for inflation that is *concave* and thus partially increasing in the given expected ELB spell duration. In other words, it is possible that the disinflationary effect following adverse financial shocks is *lower* if the ELB is expected to bind for a longer period of time. Based on

Equations (31) and (32), this requires two necessary conditions, which we postulate in the following Lemma.

Lemma 3. *A concave inflation policy function in the expected ELB spell duration requires that the elasticity of the credit spread with respect to entrepreneur leverage satisfies*

$$\nu > \frac{\eta}{\psi - 1 - \sigma - \eta} \tag{35}$$

and that the size of the financial shock satisfies

$$u_t > -\bar{r} = \beta^{-\sigma} - 1. \tag{36}$$

Proof. See Appendix. ■

The first part of Lemma 3 shows that the overall response of inflation following financial shocks at the ELB depends crucially on the elasticity of the credit spread with respect to entrepreneur leverage. Intuitively, a concave policy function for inflation requires that the credit spread dominates both the factor cost and the cost channel. If financial frictions are sufficiently pronounced such that ν is large, credit spreads may dominate the price setting of firms at the ELB, thereby increasing inflation even though output contracts. This result also directly implies that the corresponding effect is absent in the standard NK model (in which $\nu = 0$).[16] We show in Section 4.1 that calibrating ν in our model setup based on available empirical evidence suggests that Equation (35) is comfortably satisifed. We hence capture this scenario via the following assumption:

Assumption 3. *The elasticity of the credit spread with respect to entrepreneur leverage satisfies Condition* (35) *from Lemma 3.*

The second necessary condition in Lemma 3 refers to the magnitude of the financial shock: a concave policy function in the expected ELB spell duration requires sufficiently large financial shocks. This can be seen by noting that the first terms in Equation (31) and Equation (32) are identical. As a consequence, the sum $u_t + \bar{r}$ needs to be positive as well. Intuitively, this

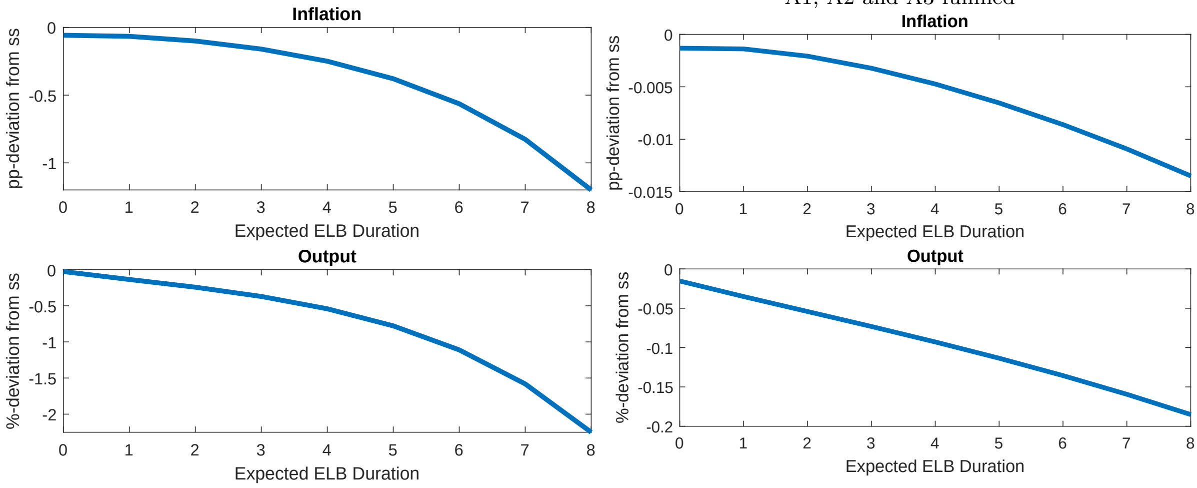
---

[16] Given financial frictions, a concave policy function is also possible for preference shocks, see Appendix E. This requires a larger elasticity of the credit spread to compensate for the missing purely exogenous markup effect. It holds that $\lambda_T^{\pi,\mathrm{ps}} = \lambda_T^{\pi,\mathrm{fs}} - \kappa$, where "ps" stands for preference shock, and "fs" for financial shock.

highlights once more that the financial shock needs to be large enough such that the disinflationary effect via the cost channel is outweighed. However, the condition itself is relatively weak for a standard calibration of the associated household preference parameters and does not contradict the assumption in this section that the financial shock is small enough not to alter the expected ELB spell duration (as shown in the numerical analysis in Section 4.2). We capture this necessary condition in the following Assumption:

Assumption 4. *The size of the financial shock is large enough to satisfy Condition* (36) *from Lemma 3, but sufficiently small not to alter the expected ELB spell duration.*

Figure 2 displays the impact responses of inflation and output to a financial shock for a given expected ELB duration T. We plot the policy functions λ_T^π and λ_T^y under three alternative illustrative calibrations. In the first case, Assumptions 1, 2 and 4 hold: the spread is weakly countercyclical, financial shocks have conventional demand-type character in normal times and the shock is relatively large. In the second case, the calibration satisfies Assumption 3 such that financial frictions are relatively severe, but the financial shock is rather small. Case 3 shows the scenario of a large financial shock for severe financial frictions. In the first two cases, the policy functions for inflation and output are strictly decreasing in the expected ELB spell duration: inflation and output move in the same direction, and a longer expected ELB duration implies a stronger macroeconomic effect of additional financial shocks. In the third case, however, the policy function for inflation is concave in the ELB spell duration, peaking at an expected ELB duration of six quarters in positive territory. In other words, if the ELB is expected to bind for a longer period of time, the overall inflation response may even turn positive. In this case, financial shocks appear as supply-type disturbances, i.e. moving output and inflation in opposite directions. This illustrates that inflation dynamics following financial shocks may be fundamentally different at the ELB compared to normal times. In particular, the slope of the Phillips curve (the ratio of the inflation policy function to the output policy function) may decrease in the expected ELB spell duration and even turn negative in certain cases. We hence explore the Phillips curve in more detail in the next section using numerical methods.



(a) Case 1: financial frictions not pronounced, A1, A2 and A4 fulfilled

(b) Case 2: small financial shock, A1, A2 and A3 fulfilled

(c) Case 3: large financial shock and severe financial frictions, A1-A4 fulfilled

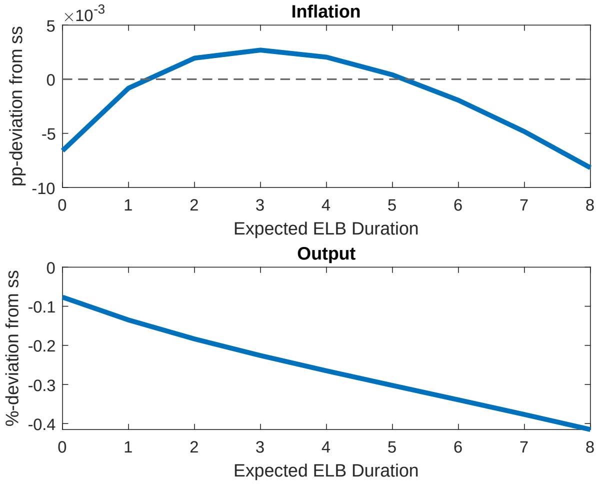


Figure 2: Policy functions for the impact responses to financial shocks for a given expected ELB duration.

4 Numerical results and the hockey stick Phillips curve

In this section, we complement our closed-form solutions by a numerical analysis of the full general equilibrium rational expectations solution. We employ numerical methods to treat the expected ELB spell duration as endogenous. This allows us to present impulse responses to financial shocks and trace out the corresponding observational Phillips curve.

4.1 Calibration and solution method

Throughout this section, we fix the model's structural parameters to standard values taken from Woodford (2003) and recent empirical estimates (up until 2019) from Boehl (2022a, BS20 henceforth). We set $\beta = 0.99$, representing the standard view of a quarterly model. We set $\sigma = 1$, which is a common assumption in line with a balanced growth path and also backed by

BS20. Following the same line of reasoning, we set $\eta = 0.5$. The fraction of non-adjusting price setters ζ is set to the commonly found textbook value of 0.66. This is deliberately lower than the larger estimates from Smets and Wouters (2007) and BS20, as we want to avoid assuming a flat New Keynesian Phillips curve ex-ante.[17]

Concerning the monetary policy parameters, we set ϕ_π to 1.5 (a commonly used standard value), and ϕ_y to 0.2. In line with the estimates of BS20, the latter value is large relative to the standard prior mean of 0.125. As the authors argue, this reflects the strong reaction of the Fed to output during the ELB episode from 2009–2015, during which inflation was close to its target value, while the level of output remained persistently depressed. We set $\rho = 0.9$ as suggested by estimates for the persistence of such shocks over the post-2009 US and Euro Area data (BS20, Boehl et al., 2024). This reflects a lasting, quite persistent financial shock, which resembles the scenario of the Great Recession and its aftermath.

For the parameters referring to the financial frictions, we proceed as follows. We calibrate $\psi = 8$ based on the relative volatility of real stock prices and real output in historical US data.[18] This value implies that Assumption 1 is comfortably satisfied such that firm leverage is countercyclical. For the elasticity of the credit spread with respect to firm leverage ν, we consider a range of values and conduct comparative exercises with regard to this parameter. In particular, existing empirical evidence by Anderson and Cesa-Bianchi (forthcoming) on the effect of monetary policy shocks on the credit spread based on detailed firm-level data for 1999-2017 suggests a value of $\nu = 0.16$ (taking our remaining calibration into consideration).[19] This value is between the two thresholds given by Assumption 2 ($\nu = 0.091$) and Assumption 3 ($\nu = 0.255$), which would imply that financial shocks are disinflationary in normal times but may feature a concave inflation policy function at the ELB. In the analyses further below and in Section 5, we consider slightly higher values ranging from $\nu = 0.2$ up to $\nu = 0.25$ (right below the threshold of Assumption 3). This is motivated by the empirical results by Akinci and Queralto (2022) showing that the relationship of credit spreads to real activity is highly asymmetric, i.e. that their correlation is much stronger when credit spreads are elevated (as was

---

[17] A higher value directly amplifies our key results, as it yields an even flatter observational Phillips curve at the ELB, see Appendix F.

[18] Equation 4 implies $sd(n_t)/sd(y_t) = \psi$. In empirical data, the standard deviation of stock prices (deflated by the GDP deflator) to the standard deviation of real output is between 7 and 9 when considering quarterly HP-filtered ($\lambda = 1600$) US data from 1959-2019 (depending on the starting point). We choose 8 as the middle of this range.

[19] Anderson and Cesa-Bianchi (forthcoming) find that a monetary policy shock of 25 bps increases credit spreads by 27 bps. Based on the closed-form solutions from Proposition 1, a value of $\nu = 0.16$ generates a similar amplification, i.e. the response of the credit spread is approx. 10% higher than the initial shock.

the case in the Global Financial Crisis and its aftermath, see Figure 1).

The analytical solutions shown in the previous section hold for the impact period when the shock occurs, under the assumption that the expected duration of the ELB k is given. However, in general and in the absence of special policy measures such as forward guidance, k is an equilibrium outcome to be determined endogenously at each point in time, given the contemporaneous exogenous disturbances that causes the ELB constraint to bind. To solve the model at the ELB, we use the numerical solution method proposed by Boehl (2022a). The main advantage of this method is a considerable boost in computational speed. A brief description of the solution method is outlined in Appendix D.

4.2 Impulse responses to financial shocks

Figure 3 displays impulse responses following contractionary financial shocks of differing size in the left panel. A 1%-shock (yellow line) is not strong enough to cause the ELB to be binding. As a result, the macroeconomic dynamics look like typical demand-side shocks. Output and inflation (as well as marginal costs) fall in response to the shock. Note that in this case, the impact responses of inflation and output correspond to the analytical policy functions shown in Proposition 1.

As the financial shock becomes larger, the ELB starts to bind and the spell duration increases. Correspondingly, the initial response of inflation shifts upwards such that the shock is less disinflationary, in line with the analytical insight of a concave inflation policy function in the expected ELB spell duration from Proposition 2.[20] Notably, this occurs for $u_t = 1.5\%$, which decreases output by approx. 5%, similar to the reaction of US output during the Global Financial Crisis. For a larger value of u_t and a particularly long expected ELB duration, the initial response of inflation even becomes positive, while output still falls. These impulse responses also provide a numerical confirmation that Assumption 4 is a rather weak constraint on the financial shock, and confirm the validity of the policy functions derived in the previous section. The lower bound on the shock size to generate a concave policy function under our calibration is $u_t > 0.99^{-1} - 1 \approx 1\%$, while it takes a considerably larger shock to increase the expected ELB spell duration. For example, Figure 3 shows that increasing the ELB duration from three to five quarters requires a financial shock $u_t = 2\%$ instead of $u_t = 1.5\%$.

---

[20] Note that the lines are simply shifted outwards in case of a larger initial shock, since the responses of endogenous variables are a simple linear map of u_t and u_t decreases each period by $(1 - \rho)$. Appendix E shows that, similar to the financial shock, conventional preference and cost-push shocks are also associated with considerably weaker disinflationary effects at the ELB.

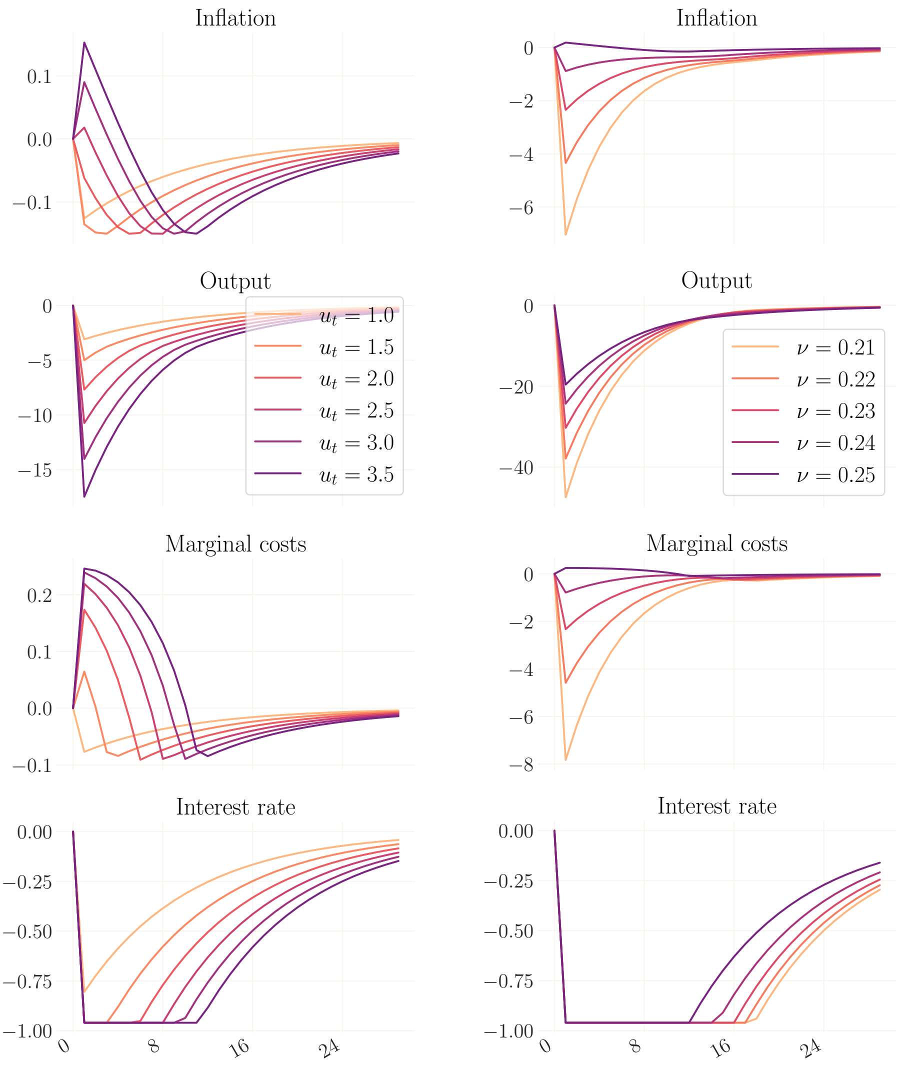


Figure 3: Impulse response functions. *Left:* to different financial shocks for $\nu = 0.25$. *Right:* to a 3% financial shock for different values of ν causing the ELB to be binding.

In the right panel of Figure 3, we consider a large financial shock pushing the economy to the ELB for different values of ν. As highlighted by the graphs, marginal costs decrease less if financial frictions are stronger, which also translates to a smaller fall of inflation. For $\nu = 0.25$, inflation actually increases, whereas the same calibration yields regular dynamics in the absence of the ELB (left side of Figure 3). These impulse responses corresponds to the standard case outlined in the previous section: the elasticity of the credit spread with respect to entrepreneur leverage is large enough to generate a concave inflation policy function, but not excessively large such that a positive inflation response emerges in normal times.[21]

---

[21] As Proposition 2 suggests, the persistence of financial shocks ρ is another central parameter for inflation dynamics, in particular at the ELB. A lower value of ρ yields a more concave inflation policy function (c.f. Equation 31). A lower ρ also implies a stronger discounting and hence a less dominant effect of the anticipated course of the financial shock. We illustrate this in Figure F.8 in the Appendix. We discuss the role of persistence in more detail in Section 5.

4.3 The observational hockey stick Phillips curve

Figure 4 plots the impulse responses to financial shocks projected into $\{y_t, \pi_t\}$-space. We interpret this as the *observational Phillips curve*, i.e. the realized values of inflation and output (gap) that would be observed in general equilibrium.[22] This is in contrast to the theoretical New Keynesian Phillips curve – as shown in Equations (12) and (16) – which represents firms' price setting under the assumption of nominal price rigidities. The most remarkable observation in Figure 4 is the striking hockey stick shape of the observational Phillips curve. For positive values of output, the observed slope of the Phillips curve is positive, in line with standard theory. However, for substantially negative values of output (caused by large financial shocks) the observational Phillips curve flattens out at the ELB.

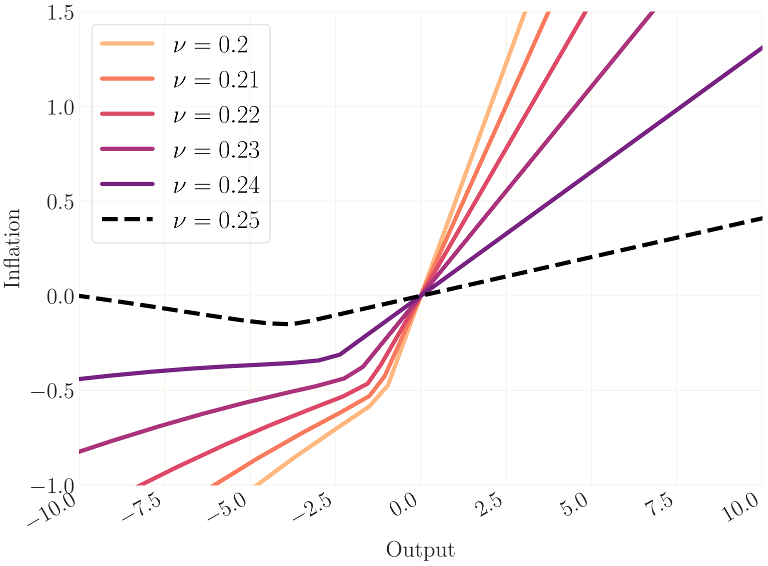


Figure 4: Observed Phillips Curve for an economy facing financial shocks. For each value of ν, we simulate the model for u_t on the interval $[-4, 4]$ and plot the respective combination of π_t and y_t.

As the elasticity of the credit spread to leverage, ν, increases, the Phillips Curve becomes flatter for both regimes: the hockey stick rotates in the origin and the ratio of the two slopes decreases. For $\nu = 0.24$, the observed slope in the region of -3% output is almost zero, while having a conventional slope in the origin. For a value of $\nu = 0.25$, we observe that the credit spread effect at the ELB is strong enough that inflation actually increases as output contracts more strongly, while the Phillips curve is still upwards sloping in normal times. In interpreting these results, it is worthwhile to recall that our calibration avoids pre-assuming a flat New

---

[22] In the model framework, the output response following financial shocks is identical to the output gap response, see Footnote 6. As such, the figure can equivalently be interpret as showing the output gap - inflation space.

Keynesian Phillips curve (as our Calvo parameter is considerably lower than empirical estimates, see Section 4.1.) A higher degree of price stickiness would have the same effect as a higher degree of financial frictions: the Phillips curve would be flatter both in normal times and at the ELB.

In other words, an economic observer aiming to infer the slope of the Phillips curve in times of a binding ELB and financial frictions would inherently conclude that the Phillips curve is "dead". This observation emerges even though the New Keynesian Phillips curve is well and alive: the relationship between firms' prices and marginal costs, governed by the Calvo parameter, is intact. However, the credit spread channel dominates firms' price setting at the ELB and thus blurs the supply-side link between output and prices. The flat observational Phillips curve at the ELB implies that a proper identification of the relationship between inflation and output is challenging. This holds in particular if financial shocks are prevailing because of their pronounced inflationary impact via the credit spread and firms' marginal costs.

5 Monetary policy at the effective lower bound

From the viewpoint of central banks, the difficulties of interpreting the observational Phillips curve translate into delicate decisions about the appropriate design of monetary policy at the ELB. To make matters worse, the effects of monetary policy itself are also affected by financial frictions and the ELB. We analyze this aspect by considering both monetary policy shocks – in particular forward guidance shocks at the ELB – and the systematic behavior of central banks as governed by the monetary policy rule.

5.1 Monetary policy shocks and forward guidance

The first crucial insight regarding monetary policy shocks v_t is that they generate *identical* macroeconomic dynamics as financial shocks in normal times. To understand this result, recall that the linearized policy rate rule is given by

$$r_t = \max\{\phi_\pi \pi_t + \phi_y y_t + v_t, \bar{r}\} \tag{37}$$

where v_t is an AR(1) monetary policy shock following

$$v_t = \rho_r \, v_{t-1} + \epsilon_{r,t}. \tag{38}$$

Abstracting from the max operator and inserting the policy rate rule into the linearized New Keynesian Phillips curve and the Euler equation yields

$$\pi_t = \kappa\gamma\, y_t + (\beta - \kappa)E_t[\pi_{t+1}] + \kappa(\phi_\pi \pi_t + \phi_y y_{)} + \kappa(u_t + v_t), \tag{39}$$

$$y_t = -\sigma^{-1}\left(\phi_\pi \pi_t + \phi_y y_t - E_t[\pi_{t+1}] + u_t + v_t\right) + E_t[y_{t+1}]. \tag{40}$$

This shows that monetary policy shocks appear *in the same places* as financial shocks. Therefore, in this framework and away from the ELB, monetary policy shocks and financial shocks are observationally equivalent in terms of inflation and output; they are only distinguishable via the responses of the interest rate (which increases for monetary policy shocks but decreases for financial shocks) or the credit spread (which increases by more after financial shocks).

As a consequence, all results from the previous sections concerning financial shocks in normal times are valid for monetary policy shocks as well. Notably, this includes the closed-form solutions. This implies that the model-based impulse responses to monetary shocks qualitatively match the vast empirical evidence (see e.g. Anderson and Cesa-Bianchi, forthcoming; Gertler and Karadi, 2015) and standard financial accelerator models (see e.g. Bernanke et al., 1999; Del Negro et al., 2015b): contractionary monetary policy shocks decrease inflation and output, while credit spreads (and firm leverage) behave countercyclically and increase. It also follows immediately that the central bank can, in principle, offset financial shocks perfectly in normal times. Moreover, the applicability of our closed-form solutions indicates the possibility of *Neo-Fisherian* effects of monetary policy shocks in normal times: as captured in Lemma 2, extremely pronounced financial frictions may induce an increase of inflation after a restrictive monetary policy shock that raises interest rates.

At the ELB, monetary policy can still stimulate output and inflation via *forward guidance*, i.e. by increasing the expected ELB duration and thus shifting longer-run interest rates. Notably, given Equation (37), the monetary policy shock v_t can be interpreted as forward guidance policy at the ELB. The intuition behind this is that the policy rule prescribes a notional rate – or ”shadow rate”, see Equation (10) – that the policymaker would choose if the ELB did not exist. Whenever the policy rule suggests a value below the ELB, the actual interest rate is set equal to $\bar{r}$, but this policy choice is still informed by the rule. At the ELB, v_t can thus be understood as a promise by the central bank to deviate from this shadow rate in the future, typically by keeping the policy rate for an extended period below the level that would normally be prescribed by the policy rate rule. In turn, v_t affects the expectations of households and firms regarding the future interest rate path. If a negative value of v_t shifts the interest rate below $\bar{r}$ for a given point in the future, this implies that the ELB will hold for this period. A sufficiently large and persistent shock to v_t can thus cause the ELB to hold for several periods in expectations, and, respectively, that the expected ELB spell duration must increase.[23]

Given that monetary policy shocks constitute forward guidance shocks at the ELB, the insight that both shocks – the financial and the monetary policy shock – appear in the same places features major implications for forward guidance at the ELB, which is the second key contribution of this paper. In particular, the macroeconomic effects of forward guidance at the ELB are identical to those of financial shocks. Given the same persistence, they are not even distinguishable via the policy rate since it remains constant in both cases. Unfortunately for monetary policy, our previous results thus imply that forward guidance at the ELB might not be particularly effective and may even be associated with unintended effects on inflation. Notably, this includes the possibility that forward guidance at the ELB may be *disflationary.*

Intuitively, forward guidance shocks induce three different and partially opposing effects on inflation. First, expected future interest rates are lower, which transmits to the economy via the standard (demand-side) Euler channel. Second, lower expected interest rates decrease expected marginal costs via the external finance premium. Third, agents expect that the inversion of the policy function will remain active for more periods. The first effect leads to an unambiguous increase in output. The second effect clearly depresses inflation. The third effect prolongs the reversal of the inflation response that is induced by the ELB via the credit channel. As forward guidance raises output, this could also trigger a drop in inflation. Which of these effects dominates depends crucially on the forward guidance persistence and the degree of financial frictions.

Lemma 4. *At the ELB, forward guidance shocks v_t may be associated with Neo-Fisherian effects such that expansionary forward guidance is disinflationary iff*

$$\rho_r < \rho. \tag{41}$$

---

[23] An alternative way of modelling forward guidance is to directly consider an increase in the ELB spell, as e.g. in Kulish et al. (2017). Also note that, alternatively to persistent monetary policy shocks, we could assume a policy rule with interest rate smoothing. The effect of monetary policy shocks on the expected path of future interest rates would remain unchanged, whereas the new influence of lagged inflation and lagged output would induce some minor quantitative differences.

This condition is necessary, but not sufficient. To see this, assume a combination (ρ, ν) for which a given shock u_t is disinflationary. As the mechanics behind forward guidance and financial shocks are equal, we learn from Equation (31) in Proposition 2 that a smaller ρ (or here: ρ_r) can reduce the weight on the (negative terminal) second and third term. A decrease in ρ thus has a similar effect as an increase in ν. We show this effect in Figure F.8 in the Appendix.

As an illustration, Figure 5 shows impulse responses following forward guidance shocks at the ELB given different values for ν. In the left panel, the stronger internal propagation of the forward guidance shock caused by a stronger degree of financial frictions leads to a longer ELB period than in the right panel. This means that, given the same financial shock, during the extended ELB period the interest rate is much lower than in the absence of forward guidance, which causes marginal costs to fall. Since expected lower marginal cost are anticipated by firms (via the Phillips curve), the fall in inflation is *larger* than without forward guidance. This effect is absent in the right panel of Figure 5 because the shock does not prologue the ELB period significantly.

While it is safe to assume a high persistence of the financial shock, the persistence of the forward guidance shock is to some extent a policy parameter that can in principal be chosen by the central bank. However, it also depends on how successful the central bank is in its communication strategies and the perceived credibility from the perspective of private agents. Central banks are hence not fully in control of the persistence ex ante. Moreover, unanticipated and rapid changes in the macroeconomic environment may force central banks to deviate substantially ex post from their forward guidance (e.g. as experienced in the euro area and in Australia amid the rapid economic rebound and inflation surge after the Covid19-pandemic). As illustrated in Figure 5, a monetary policy shock with low persistence can trigger negative inflation responses because the short-run effect of decreasing the costs of external finance via credit spreads dominates the longer-term effect that operates through the household Euler Equation. As such, non-credible or non-persistent forward guidance may be associated with undesirable macroeconomic dynamics for monetary policy. This result constitutes a cautionary tale for central banks; it highlights potential difficulties and the crucial role of credibility in the application of forward guidance.

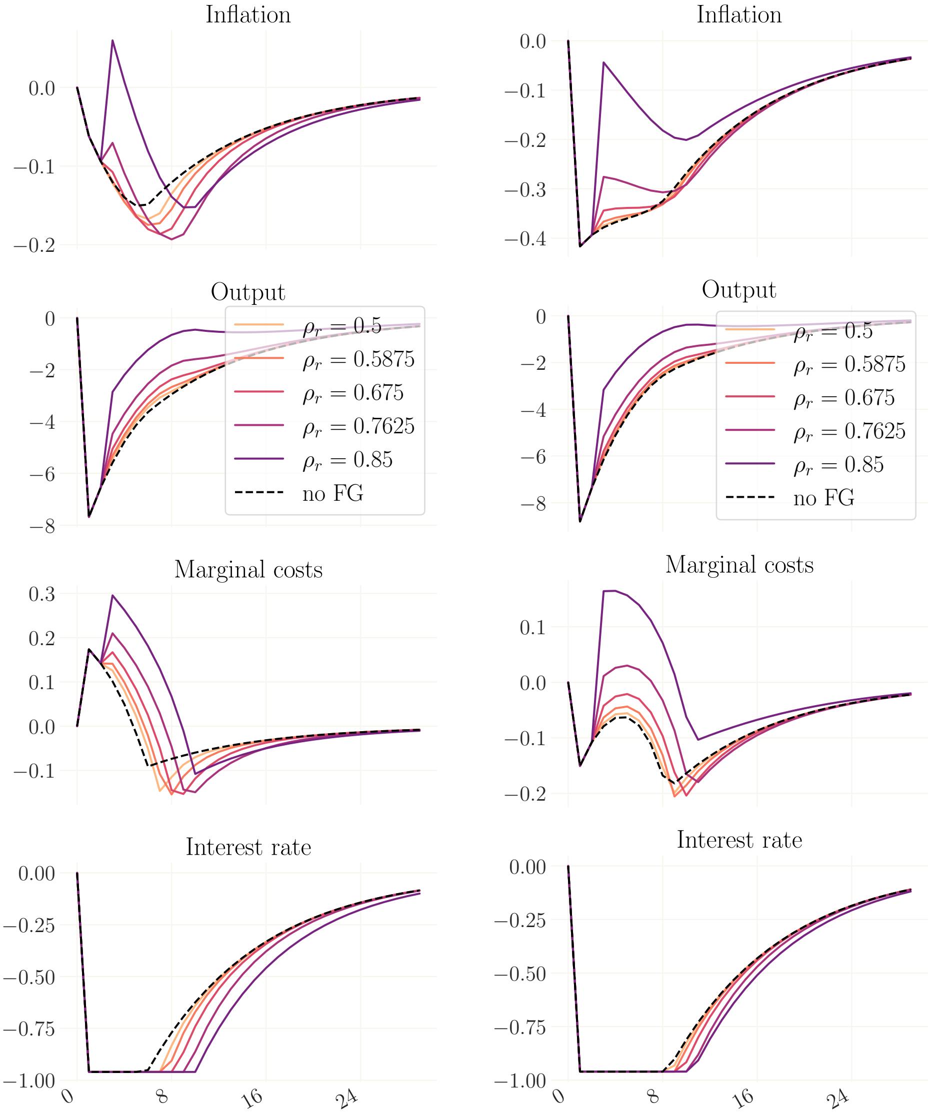


Figure 5: Impulse response functions for a 2% financial shock. Colored lines are the financial shock combined with a forward guidance shock in period 3. Different colors correspond to different persistences of the forward guidance shock. *Left:* for $\nu = 0.25$. For many values of ρ_r, the forward guidance shock is disinflationary. *Right:* for $\nu = 0.24$. For this value of ν the forward guidance shock is not disinflationary.

5.2 The systematic component of monetary policy and the Taylor principle

We now turn from monetary policy shocks towards the systematic behavior of central banks. At first glance, it may seem that monetary policy rules governing the systematic behavior are irrelevant at the ELB. However, they are crucial for macroeconomic dynamics because rational private agents take the systematic component of the monetary policy rule into account when forming expectations about future variables and the remaining ELB duration. As such, choosing an appropriate monetary policy rule is of major importance for central banks who find themselves facing the ELB. From a policy-making perspective, the minimum requirement that any appropriate rule should satisfy is that it guarantees a determinate equilibrium with non-explosive macroeconomic dynamics. The following Proposition shows the corresponding policy parameter requirements:

Proposition 3. *The policy parameters in the central bank's monetary policy rule must satisfy the following conditions to guarantee a determinate solution:*

$$\phi_\pi + \frac{1-\beta}{\kappa\gamma}\,\phi_y > 1, \tag{42}$$

$$\kappa(\sigma^{-1}\gamma - 1)\,\phi_\pi + \sigma^{-1}\phi_y > \beta - 1 - \kappa \tag{43}$$

Proof. See Appendix. ■

Equation (42) may be interpreted as a modified Taylor principle for the model economy with financial frictions. If the central bank decides to react to inflation only ($\phi_y = 0$), a necessary condition is that the associated coefficient ϕ_π needs to be larger than unity, as in Taylor (1993). If the central bank reacts to output as well ($\phi_y > 0$), determinacy requires the weighted sum of policy coefficients to be larger than unity. Compared to a standard New Keynesian framework, the key difference is that financial frictions affect the degree of substitutability between reacting to inflation and to output. Under Assumption 1, the slope of the New Keynesian Phillips curve with respect to output (the term $\kappa\gamma$) is lower due to the countercyclical credit spread. At first glance, it thus seems that policy responses to output can *substitute* more effectively for policy responses to inflation in the presence of financial frictions.

However, Equation (43) may constitute additional complications for the design of monetary policy rules. To see this, note that under Assumption 3 (i.e. under the necessary condition for a concave policy function at the ELB) the coefficient in front of ϕ_π is less than zero. Therefore, Equation (43) implies that the responses to inflation and output are *complements* for some combinations of $\{\phi_\pi, \phi_y\}$, or equivalently constitutes a lower (upper) bound restriction for the response to output (inflation). In other words, a stronger reaction to inflation must be accompanied by a corresponding stronger reaction to output. This clashes with the modified Taylor rule that exhibits the conventional substitutability.

Figure 6 displays this result graphically. As the elasticity of the credit spread with respect to entrepreneur leverage ν increases, a higher value for ϕ_y is necessary to keep the model determined for high values of ϕ_π. For example, in the case of $\nu = 0.2$, $\phi_\pi > 1.76$ requires that $\phi_y > 0$.

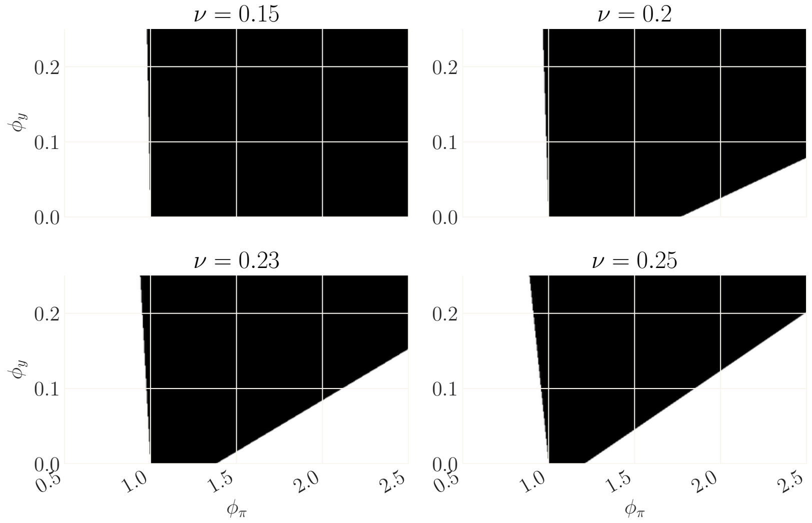


Figure 6: Determinacy regions for different values of ν.

Intuitively, abstracting from financial frictions, inflation can be stabilized by raising nominal interest rates appropriately. Higher nominal interest rates amount to higher real interest rates, decreasing consumption and output. As a consequence, real marginal costs fall, and inflation decreases. Whether the hike of nominal interest rates constitutes a reaction to (positive) deviations of inflation or output is irrelevant. In the presence of financial frictions, however, an interest rate hike as a reaction to output has the additional effect of increasing marginal costs via the costs of external financing and thus inflation. Depending on the specific characteristics of the economy, the central bank might find itself in a knife-edge scenario where the appropriate window for systematic policy responses to output deviations is quite small.

Overall, the key message emerging from this section is that the conduct of monetary policy in the presence of financial frictions and a binding ELB may prove difficult. While the hockey stick Phillips curve blurs the relationship between inflation and output at the ELB, conventional monetary policy wisdoms are abolished: short-lived forward guidance shocks may be associated with Neo-Fisherian inflation effects, and determinacy considerations may place rather tight restrictions on appropriate monetary policy rules.

6 Conclusion

This paper argues that a binding effective lower bound (ELB) on nominal interest rates may contribute to an observational disconnect between inflation and economic activity if financial shocks are prevailing. At the ELB, the costs of external financing in the form of credit spreads can dominate firms' price setting and thereby generate inflationary pressure. Via this supply-side mechanism, the Phillips curve features a considerably flatter slope when the ELB binds compared to normal times. As a consequence, the resulting observational Phillips curve is shaped like a hockey stick. These findings constitute a complementary explanation for the observed inflation puzzles during the Global Financial Crisis and its aftermath.

Our results translate into strong implications on the conduct of forward guidance and provide a potential solution to the forward guidance puzzle: similar to financial shocks, the effects of forward guidance can be decomposed in short-run disinflationary effects via the costs of external financing, and a longer-term inflationary effect via real marginal costs. For rather short-lived forward guidance impulses, the first disinflationary effect may dominate and forward guidance can in fact lower inflation. Accordingly, only forward guidance with a high expected persistence succeeds in fostering inflation and growth.

References

Abbate, A., Eickmeier, S., Prieto, E., 2021. Financial shocks and inflation dynamics. Macroeconomic Dynamics 27, 1–29.

Adam, K., Billi, R.M., 2006. Optimal monetary policy under commitment with a zero bound on nominal interest rates. Journal of Money, credit and Banking , 1877–1905.

Adam, K., Marcet, A., Beutel, J., 2017. Stock Price Booms and Expected Capital Gains. American Economic Review 107, 2352–2408. URL: `https://www.aeaweb.org/articles?id=10.1257/aer.20140205`, doi:`10.1257/aer.20140205`.

Akinci, O., Queralto, A., 2022. Credit Spreads, Financial Crises, and Macroprudential Policy. American Economic Journal: Macroeconomics 14, 469–507. URL: `https://www.aeaweb.org/articles?id=10.1257/mac.20180059`, doi:`10.1257/mac.20180059`.

Anderson, G., Cesa-Bianchi, A., forthcoming. Crossing the Credit Channel: Credit Spreads and Firm Heterogeneity. American Economic Journal: Macroeconomics .

Ball, L., Mazumder, S., 2011. Inflation Dynamics and the Great Recession. Brookings Papers on Economic Activity 42, 337–405.

Ball, L., Mazumder, S., 2018. A Phillips Curve with Anchored Expectations and Short-Term Unemployment. Journal of Money, Credit and Banking 51, 111–137.

Barth III, M.J., Ramey, V.A., 2001. The cost channel of monetary transmission. NBER Macroeconomics Annual 16, 199–240.

Bernanke, B.S., Gertler, M., Gilchrist, S., 1999. The financial accelerator in a quantitative business cycle framework. Handbook of Macroeconomics 1, 1341–1393.

Bianchi, F., Melosi, L., 2017. Escaping the Great Recession. American Economic Review 107, 1030–1058.

Blanchard, O.J., Kahn, C.M., 1980. The Solution of Linear Difference Models under Rational Expectations. Econometrica 48, 1305–1311.

Bobeica, E., Jarociński, M., 2019. Missing Disinflation and Missing Inflation: A VAR Perspective. International Journal of Central Banking 15, 199–232.

Boehl, G., 2022a. Efficient solution and computation of models with occasionally binding constraints. Journal of Economic Dynamics and Control 143, 104523. URL: `https://www.sciencedirect.com/science/article/pii/S0165188922002275`, doi:`https://doi.org/10.1016/j.jedc.2022.104523`.

Boehl, G., 2022b. Monetary policy and speculative asset markets. European Economic Review 148. URL: `https://ideas.repec.org/a/eee/eecrev/v148y2022ics0014292122001477.html`, doi:`10.1016/j.euroecorev.2022`.

Boehl, G., Goy, G., Strobel, F., 2024. A structural investigation of quantitative easing. Review of Economics and Statistics , 1–45URL: `https://doi.org/10.1162/rest_a_01205`.

Boehl, G., Strobel, F., 2024a. The empirical performance of the financial accelerator since 2008. Journal of Economic Dynamics and Control , 104927.

Boehl, G., Strobel, F., 2024b. Estimation of dsge models with the effective lower bound. Journal of Economic Dynamics and Control 158, 104784.

Budianto, F., 2023. Inflation targets and the zero lower bound. Available at SSRN 4339381 .

Budianto, F., Nakata, T., Schmidt, S., 2023. Average inflation targeting and the interest rate lower bound. European Economic Review 152, 104384.

Calvo, G.A., 1983. Staggered prices in a utility-maximizing framework. Journal of Monetary Economics 12, 383–398.

Carlstrom, C.T., Fuerst, T.S., 1997. Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis. The American Economic Review 87, 893–910. URL: `http://www.jstor.org/stable/2951331`.

Carlstrom, C.T., Fuerst, T.S., Ortiz, A., Paustian, M., 2014. Estimating contract indexation in a Financial Accelerator Model. Journal of Economic Dynamics and Control 46, 130–149. URL: `https://www.sciencedirect.com/science/article/pii/S016518891400147X`, doi:`https://doi.org/10.1016/j.jedc.2014.06.009`.

Carlstrom, C.T., Fuerst, T.S., Paustian, M., 2015. Inflation and output in New Keynesian models with a transient interest rate peg. Journal of Monetary Economics 76, 230 – 243.

Carlstrom, C.T., Fuerst, T.S., Paustian, M., 2016. Optimal Contracts, Aggregate Risk, and the Financial Accelerator. American Economic Journal: Macroeconomics 8, 119–47. URL: `https://www.aeaweb.org/articles?id=10.1257/mac.20120024`, doi:`10.1257/mac.20120024`.

Chowdhury, I., Hoffmann, M., Schabert, A., 2006. Inflation dynamics and the cost channel of monetary transmission. European Economic Review 50, 995–1016.

Christiano, L., Rostagno, M., Motto, R., 2010. Financial factors in economic fluctuations. Working Paper Series 1192. European Central Bank. URL: `https://ideas.repec.org/p/ecb/ecbwps/20101192.html`.

Christiano, L.J., Eichenbaum, M.S., Trabandt, M., 2015. Understanding the Great Recession. American Economic Journal: Macroeconomics 7, 110–67.

Cochrane, J.H., 2011. Determinacy and identification with Taylor rules. Journal of Political Economy 119, 565–615.

Cochrane, J.H., 2016. Do higher interest rates raise or lower inflation? Unpublished paper, February URL: `https://faculty.chicagobooth.edu/john.cochrane/research/papers/fisher.pdf`.

Cochrane, J.H., 2017. The new-Keynesian liquidity trap. Journal of Monetary Economics 92, 47–63.

Coibion, O., Gorodnichenko, Y., 2015. Is the Phillips Curve Alive and Well after All? Inflation Expectations and the Missing Disinflation. American Economic Journal: Macroeconomics 7, 197–232.

Daly, M.C., Hobijn, B., 2014. Downward Nominal Wage Rigidities Bend the Phillips Curve. Journal of Money, Credit and Banking 46, 51–93.

Del Negro, M., Giannoni, M., Patterson, C., 2015a. The forward guidance puzzle. Staff Reports 574. Federal Reserve Bank of New York.

Del Negro, M., Giannoni, M.P., Schorfheide, F., 2015b. Inflation in the Great Recession and New Keynesian Models. American Economic Journal: Macroeconomics 7, 168–196.

Forbes, K., 2019. Inflation Dynamics: Dead, Dormant, or Determined Abroad? Working Paper 26496. National Bureau of Economic Research.

Friedrich, C., 2016. Global inflation dynamics in the post-crisis period: What explains the puzzles? Economics Letters 142, 31–34.

Gabaix, X., 2020. A Behavioral New Keynesian Model. American Economic Review 110, 2271–2327.

Gaiotti, E., Secchi, A., 2006. Is there a cost channel of monetary policy transmission? An investigation into the pricing behavior of 2,000 firms. Journal of Money, Credit and Banking , 2013–2037.

Galí, J., 2015. Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications, 2nd Edition. Number 10495 in Economics Books, Princeton University Press.

García-Schmidt, M., Woodford, M., 2019. Are low interest rates deflationary? A paradox of perfect-foresight analysis. American Economic Review 109, 86–120.

Gerali, A., Neri, S., Sessa, L., Signoretti, F.M., 2010. Credit and Banking in a DSGE Model of the Euro Area. Journal of Money, Credit and Banking 42, 107–141.

Gertler, M., Karadi, P., 2015. Monetary Policy Surprises, Credit Costs, and Economic Activity. American Economic Journal: Macroeconomics 7, 44–76. URL: `https://www.aeaweb.org/articles?id=10.1257/mac.20130329`, doi:`10.1257/mac.20130329`.

Gilchrist, S., Schoenle, R., Sim, J., Zakrajšek, E., 2017. Inflation Dynamics during the Financial Crisis. American Economic Review 107, 785–823.

Gilchrist, S., Zakrajšek, E., 2012. Credit Spreads and Business Cycle Fluctuations. American Economic Review 102, 1692–1720. URL: `https://www.aeaweb.org/articles?id=10.1257/aer.102.4.1692`, doi:`10.1257/aer.102.4.1692`.

Gordon, R.J., 2013. The Phillips Curve is Alive and Well: Inflation and the NAIRU During the Slow Recovery. NBER Working Papers 19390. National Bureau of Economic Research, Inc.

Greenwood, R., Shleifer, A., 2014. Expectations of Returns and Expected Returns. The Review of Financial Studies 27, 714–746. URL: `https://doi.org/10.1093/rfs/hht082`, doi:`10.1093/rfs/hht082`, `arXiv:https://academic.oup.com/rfs/article-pdf/27/3/714/24449380/hht082.pdf`.

Gust, C., Herbst, E., López-Salido, D., Smith, M.E., 2017. The empirical implications of the interest-rate lower bound. American Economic Review 107, 1971–2006.

Harding, M., Lindé, J., Trabandt, M., 2022. Resolving the missing deflation puzzle. Journal of Monetary Economics 126, 15–34.

Holden, T.D., 2019. Existence and uniqueness of solutions to dynamic models with occasionally binding constraints. EconStor Preprints 144570. ZBW - Leibniz Information Centre for Economics.

Jones, C., Kulish, M., Morley, J., 2024. A Structural Measure of the Shadow Federal Funds Rate. Working Papers 2024-05. University of Sydney, School of Economics. URL: `https://ideas.repec.org/p/syd/wpaper/2024-05.html`.

Jones, C., Kulish, M., Rees, D.M., 2022. International spillovers of forward guidance shocks. Journal of Applied Econometrics 37, 131–160.

Kiley, M., 2016. Policy Paradoxes in the New-Keynesian Model. Review of Economic Dynamics 21, 1–15.

Kulish, M., Morley, J., Robinson, T., 2017. Estimating DSGE models with zero interest rate policy. Journal of Monetary Economics 88, 35 – 49.

Kulish, M., Pagan, A., 2017. Estimation and solution of models with expectations and structural changes. Journal of Applied Econometrics 32, 255–274.

Lieberknecht, P., 2019. Financial frictions, the Phillips curve and monetary policy. Discussion Papers 47/2019.

Deutsche Bundesbank.

McLeay, M., Tenreyro, S., 2020. Optimal Inflation and the Identification of the Phillips Curve. NBER Macroeconomics Annual 34, 199–255.

Meh, C.A., Moran, K., 2010. The Role of Bank Capital in the Propagation of Shocks. Journal of Economic Dynamics and Control 34, 555–576.

Ravenna, F., Walsh, C.E., 2006. Optimal monetary policy with the cost channel. Journal of Monetary Economics 53, 199–216.

Sims, E.R., Wu, J.C., 2019. The Four Equation New Keynesian Model. NBER Working Papers 26067. National Bureau of Economic Research, Inc.

Smets, F., Wouters, R., 2007. Shocks and Frictions in US business cycles: A Bayesian DSGE approach. American Economic Review 97, 586–606.

Taylor, J.B., 1993. Discretion versus policy rules in practice. Carnegie-Rochester Conference Series on Public Policy 39, 195–214.

Tillmann, P., 2008. Do interest rates drive inflation dynamics? An analysis of the cost channel of monetary transmission. Journal of Economic dynamics and Control 32, 2723–2744.

Townsend, R.M., 1979. Optimal contracts and competitive markets with costly state verification. Journal of Economic Theory 21, 265–293.

Watson, M.W., 2014. Inflation Persistence, the NAIRU, and the Great Recession. American Economic Review 104, 31–36.

Woodford, M., 2003. Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton University Press.

Appendix A Details on the model

Appendix A.1 The firm setup

The framework is largely based on Boehl (2022b), to which we refer for the full microfoundations, and closely related to the canonical financial accelerator model by Bernanke et al. (1999) – abbreviated BGG in the following. In the model, risk-neutral entrepreneurs operate wholesale firms. Entrepreneurs face idiosyncratic productivity risks and finance production by combining net worth and bank loans. The bank can only observe firm output when paying monitoring costs. The lending contract that solves this costly state verification problem is a standard debt contract: the borrower repays the loan if possible, while in the case of default the bank seizes the remaining assets. The contract terms are determined by the firm optimization problem, with the entire bargaining power accruing to the entrepreneurs. The entrepreneurs choose labor, a threshold productivity value for default, and an optimal dividend payment to equity shareholders.

This setup implies two important differences to BGG. First, the idiosyncratic risk refers to labor productivity (instead of physical capital, which does not exist in the model; labor is the only production factor). Second, entrepreneur net worth is not a state variable. In the following, we show how the entrepreneur optimization problem gives rise to the key firm equations characterizing equilibrium, which are Equations (7) and (8) in the main body.

Appendix A.2 Entrepreneurs' optimization problem

Firms' ex-post gross return on one unit of labor is determined by an idiosyncratic shock $\omega_{j,t}$. This shock is assumed to be i.i.d. across time with a continuous and at least once-differentiable CDF $F(\omega)$ over a nonnegative support and with an expected value of 1. The hazard rate $h(\omega) = \frac{dF(\omega)}{1-F(\omega)}$ is restricted to $h(\omega) = \frac{\partial(\omega h(\omega))}{\partial \omega} > 0$.

We assume that the contract between entrepreneur and financial intermediary is written in real terms with $Q_t = R_t \frac{P_t}{E_t[P_{t+1}]} U_t$. The contract is defined by a gross non-default loan rate, $Z_{j,t}$ and a threshold value $\bar{\omega}_{j,t}$ on the idiosyncratic shock. For values of the idiosyncratic shock greater or equal than the threshold, the entrepreneurs is able to repay the loan at the end of the period, otherwise they default. In the case of default, the bank pays monitoring costs μ proportional to output and seizes the remaining entrepreneur assets. The threshold value is then given by

$$\bar{\omega}_{j,t} A_t H_{j,t} = \frac{Z_{j,t}}{Q_t} B_{j,t},$$

where $B_{j,t}$ denotes the loan volume and A_t aggregate productivity (which is assumed to remain constant at unity in the main analysis). In this equation, the left-hand side shows the output produced if productivity is at the threshold, whereas the right-hand side reflects the revenue to the lender in this case (discounted by the aggregate risk-free rate).

Because the entrepreneurs have full bargaining power, they decide the contract terms and ensure that the participation constraint of the bank is binding. The optimal contract hence satisfies the following condition (in which we drop firm subscripts and the time subscript of ω_t for better readability)

$$\left([1 - F(\bar{\omega})] \, \bar{\omega} + (1 - \mu) \int_0^{\bar{\omega}} \omega dF(\omega) \right) A_t H_t / X_t = Q_t (W_t H_t - N_t),$$

where $X_t = \frac{1}{MC_t}$ such that $A_t H_t / X_t$ captures the expected gross return of production. The participation constraint of the bank equates its opportunity cost of lending (the right-hand side) to the expected gross return on the loan (the left-hand side). This equation reflects the cases where the entrepreneur can repay the loan (the first term on the left-hand side) and cases of default (the second term). Similarly, the expected return to the entrepreneur is given by

$$\left(\int_{\bar{\omega}}^{\infty} \omega dF(\omega) - (1 - F(\bar{\omega}))\bar{\omega} \right) A_t H_t / X_t,$$

which shows that the threshold $\bar{\omega}$ determines the division of expected gross profits between borrower and lender.

We now define two variables to rewrite the contract condition. First, define $\mathfrak{F}(\bar{\omega})$ as the expected gross share of profits going to the lender

$$\mathfrak{F}(\bar{\omega}) = \int_0^{\bar{\omega}} \omega f(\omega) d\omega - \bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega) d\omega$$

with $\mathfrak{F}'(\bar{\omega}) = 1 - F(\bar{\omega})$ and $\mathfrak{F}''(\bar{\omega}) = -f(\bar{\omega})$. This implies strict concavity in the cutoff value. We define similarly the expected monitoring costs as

$$\mu \mathfrak{G}(\bar{\omega}) = \mu \int_0^{\bar{\omega}} f(\omega) d\omega,$$

with $\mu \mathfrak{G}'(\bar{\omega}) = \mu \omega f(\omega)$.

The entrepreneur maximizes profit given by

$$(1-\mathfrak{F}(\bar{\omega}_t))\,A_tH_t/X_t, \tag{A.1}$$

subject to the participation constraint. Thus, each firm's optimization problem is

$$\max_{\{H_t\},\{\bar{\omega}_t\},\{\lambda_t\}} (1-\mathfrak{F}(\bar{\omega}_t))\,A_tH_t/X_t - \lambda_t\left(\left[\mathfrak{F}(\bar{\omega}_t)-\mu\mathfrak{G}(\bar{\omega}_t)\right]A_tH_t/X_t - Q_t(W_tH_t-N_t)\right). \tag{A.2}$$

The first-order conditions for this problem are:

$$\begin{aligned}
\bar{\omega} &: -\mathfrak{F}'(\bar{\omega}_t)-\lambda_t\left[\mathfrak{F}'(\bar{\omega}_t)-\mu\mathfrak{G}'(\bar{\omega}_t)\right] &&= 0 && \text{(A.3)}\\
H &: (1-\mathfrak{F}(\bar{\omega}_t))\,A_t/X_t-\lambda_t\left(\left[\mathfrak{F}(\bar{\omega}_t)-\mu\mathfrak{G}(\bar{\omega}_t)\right]A_t/X_t-Q_tW_t\right) &&= 0 && \text{(A.4)}\\
\lambda &: \left[\mathfrak{F}(\bar{\omega}_t)-\mu\mathfrak{G}(\bar{\omega}_t)\right]A_tH_t/X_t-Q_t(W_tH_t-N_t) &&= 0 && \text{(A.5)}
\end{aligned}$$

For a given $\bar{\omega}_t$, the first-order conditions imply a unique level of working capital and of leverage. Entrepreneurs thus take the same decisions in equilibrium. Based on the first-order conditions, we can now derive Equations (7) and (8). First, note that (A.3) reveals that λ_t is a function of $\bar{\omega}$ and define

$$\lambda_t = \frac{\mathfrak{F}'(\bar{\omega}_t)}{\mathfrak{F}'(\bar{\omega}_t)-\mu\mathfrak{G}'(\bar{\omega})} \equiv f_\lambda(\bar{\omega}_t). \tag{A.6}$$

Inserting (A.5) into (A.4) and using the definition of f_λ yields

$$(1-\mathfrak{F}(\bar{\omega}_t))\,A_t/X_t = f_\lambda(\bar{\omega}_t)\frac{Q_tN_t}{H_t}. \tag{A.7}$$

Dividing both sides by W_t and rearranging results in

$$X_t^{-1} = \frac{W_t}{A_t}f_\omega(\bar{\omega}_t)\frac{N_t}{W_tH_t}Q_t, \tag{A.8}$$

with $f_\omega(\bar{\omega}_t)=\frac{f_\lambda(\bar{\omega}_t)}{1-\mathfrak{F}(\bar{\omega}_t)}$. Now define $z\left(\frac{W_tH_t}{N_t}\right)=f_\omega(\bar{\omega}_t)\frac{N_t}{W_tH_t}$ with $\frac{\partial z(x)}{\partial x}>0$. This results in

$$X_t^{-1} = \frac{W_t}{A_t}z\left(\frac{W_tH_t}{N_t}\right)Q_t. \tag{A.9}$$

Setting $A_t = 1$ and using $MC_t = X_t^{-1}$ as well as the definition of Q_t yields

$$MC_t = W_t \, z \left(\frac{W_t H_t}{N_t} \right) \frac{R_t}{E_t[\Pi_{t+1}]} U_t, \tag{A.10}$$

which is Equation (8) in the main text. Equation (7) follows from collecting the overall costs of external financing in one term, i.e. all terms on the right-hand side except for W_t.

Appendix B Equilibrium Equations

This section lists the full set of the equations defining the equilibrium. On the household side, we have the inter-temporal Euler equation and the intra-temporal labor-consumption trade-off, Equations (1) and (2) in the main text:

$$C_t^{-\sigma} = \beta E_t \left[\frac{R_t}{\Pi_{t+1}} U_t C_{t+1}^{-\sigma} \right], \tag{B.1}$$

$$H_t^{\eta} = W_t C_t^{-\sigma}. \tag{B.2}$$

On the firm side, we have the aggregate production function, which is obtained by aggregating over the individual linear production functions:

$$Y_t = \frac{H_t}{v_t^p} \tag{B.3}$$

where v_t^p is a measure of price dispersion defined below. Marginal costs are given by Equation (8):

$$MC_t = W_t R_t^L \tag{B.4}$$

The price setting behavior by firms is defined by the following equations, which are standard for Calvo (1983) pricing. These specify which price is set by firms that are able to reoptimize in period t (relative to the price level, Π_t^*) and make use of two auxiliary variables f_t^1 and f_t^2.

$$f_t^1 = \frac{\varepsilon - 1}{\varepsilon} f_t^2 \tag{B.5}$$

$$f_t^1 = C_t^{-\sigma} MC_t Y_t + \beta\zeta E_t \left[\Pi_{t+1}^{\varepsilon} f_{t+1}^1\right] \tag{B.6}$$

$$f_t^2 = C_t^{-\sigma} \Pi_t^* Y_t + \beta\zeta E_t \left[\left(\frac{1}{\Pi_{t+1}}\right)^{1-\varepsilon} \left(\frac{\Pi_t^*}{\Pi_{t+1}^*}\right) f_{t+1}^2\right] \tag{B.7}$$

$$1 = \zeta \left(\frac{1}{\Pi_t}\right)^{1-\varepsilon} + (1-\zeta)\left(\Pi_t^*\right)^{1-\varepsilon} \tag{B.8}$$

$$v_t^p = \zeta \Pi_t^{\varepsilon} v_{t-1}^p + (1-\zeta)\left(\Pi_t^*\right)^{-\varepsilon} \tag{B.9}$$

The interest rate specified in the credit contract is defined by Equation (7):

$$R_t^L = z\left(\frac{W_t H_t}{N_t}\right) \frac{R_t}{E_t[\Pi_{t+1}]} U_t \tag{B.10}$$

Entrepreneur net worth evolves according to Equation (4):

$$N_t = \Psi\left(Y_t\right), \tag{B.11}$$

The central bank operates according to a monetary policy rule shown in Equation (10)

$$\frac{R_t^n}{R^n} = \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} \exp(v_t), \tag{B.12}$$

The effective lower bound (ELB) constraint is given by Equation (11):

$$R_t = \max\left\{\bar{R}, R_t^n\right\} \tag{B.13}$$

Finally, the aggregate resource constraint is

$$Y_t = C_t + \Gamma_t + (U_t - 1) D_t, \tag{B.14}$$

where Γ_t are the banks' monitoring costs from the entrepreneurs' optimization problem in Appendix A. The last term reflects the real costs (in terms of units of production) occuring through the household-side financial shock U_t. While both of these costs matter, in principle, for aggregate macroeconomic dynamics, their quantitative importance is negligible for reasonable parameterizations (see Bernanke et al., 1999). We therefore abstract from these quantitatively small terms when linearizing the model.

These 14 conditions define the equilibrium for the 14 endogenous variables

$$\left(C_t, Y_t, H_t, \Pi_t, \Pi_t^*, W_t, R_t, R_t^L, R_t^n, N_t, MC_t, f_t^1, f_t^2, v_t^p\right), \tag{B.15}$$

together with the evolution of the two exogenous shocks:

$$\ln(U_t) = \rho \ln(U_{t-1}) + \epsilon_t \tag{B.16}$$

$$v_t = \rho_r v_{t-1} + \epsilon_{r,t}. \tag{B.17}$$

The linearized equilibrium conditions are as follows:

$$\begin{aligned}
c_t &= -\sigma^{-1}\left(r_t + u_t - E_t\pi_{t+1}\right) + E_t[c_{t+1}], && \text{(B.18)}\\
w_t &= \eta h_t + \sigma c_t, && \text{(B.19)}\\
y_t &= h_t, && \text{(B.20)}\\
mc_t &= w_t + r_t^L, && \text{(B.21)}\\
\pi_t &= \kappa mc_t + \beta E_t[\pi_{t+1}], && \text{(B.22)}\\
r_t^L &= r_t - E_t[\pi_{t+1}] + \nu(w_t + h_t - n_t) + u_t, && \text{(B.23)}\\
n_t &= \psi y_t, && \text{(B.24)}\\
r_t^n &= \phi_\pi \pi_t + \phi_y y_t + v_t, && \text{(B.25)}\\
r_t &= \max\left\{\bar{r}, r_t^n\right\}, && \text{(B.26)}\\
y_t &= c_t, && \text{(B.27)}\\
u_t &= \rho\, u_{t-1} + \epsilon_t, && \text{(B.28)}\\
v_t &= \rho_r\, v_{t-1} + \epsilon_{r,t}, && \text{(B.29)}\\
& && \text{(B.30)}
\end{aligned}$$

where lower-case variables denote log-deviations from steady state.

The three-equation representation shown in Section 3.1 can be obtained by combining Equations (B.19)-(B.24) into one single New Keynesian Phillips curve and using the resource constraint Equation (B.27) to eliminate c_t.

Appendix C Proofs

Proposition 1. *The impact responses of inflation and output to a financial shock in normal times (without a binding ELB on nominal interest rates) are given by:*

$$\pi_t = \lambda_0^\pi u_t, \tag{C.1}$$

$$y_t = \lambda_0^y u_t, \tag{C.2}$$

where

$$\lambda_0^\pi = -\frac{\kappa\gamma - \kappa\sigma(1-\rho)}{(1-\beta\rho)(\sigma(1-\rho)+\phi_y) + \kappa\gamma(\phi_\pi-\rho) - \kappa\sigma(1-\rho)(\phi_\pi-\rho)}, \tag{C.3}$$

$$\lambda_0^y = -\frac{1+(\phi_\pi-\rho)\lambda_0^\pi}{\sigma(1-\rho)+\phi_y}. \tag{C.4}$$

Proof. The proof relies on the method of undetermined coefficients. We guess that the solution takes the form $\pi_t = \lambda_0^\pi u_t$ and $y_t = \lambda_0^y u_t$. Using this guess, the system of equation can be written as

$$(1-\kappa\phi_\pi - \rho(\beta-\kappa))\lambda_0^\pi u_t = \kappa u_t + \kappa(\gamma+\phi_y)\lambda_0^y u_t, \tag{C.5}$$

$$(1+\phi_y\sigma^{-1}-\rho)\lambda_0^y u_t = -\sigma^{-1}(\phi_\pi-\rho)\lambda_0^\pi u_t - \sigma^{-1}u_t, \tag{C.6}$$

where the nominal interest rate is replaced using the (unconstrained) Taylor rule. Note that expectations of future variables can be replaced by using the law of motion for the financial shocks under rational expectations. The solution is obtained by dividing both equations by u_t, substituting for λ_0^y in the first equation using the second equation and rearranging. ■

Lemma 1. *The impact responses of inflation and output to a financial shock in normal times (without a binding ELB on nominal interest rates) are negative, i.e.*

$$\lambda_0^\pi < 0, \tag{C.7}$$

$$\lambda_0^y < 0, \tag{C.8}$$

if the elasticity of the credit spread to entrepreneur leverage satisfies

$$\nu < \frac{\eta+\rho\sigma}{\psi-1-\sigma-\eta}. \tag{C.9}$$

Proof. The proof consists of three parts. First, we show that the model's determinacy conditions imply that the denominator of λ_0^π is positive. Second, the sign of λ_0^π then depends on its numerator, which is equivalent to the parameter restriction in the Lemma. Third, the sign of λ_0^y follows from λ_0^π.

First, let us consider the determinacy conditions. The forward looking components of our model can be expressed as

$$M\mathbf{x}_t = E_t[\mathbf{x}_{t+1}], \tag{C.10}$$

with $\mathbf{x}_t = (y_t, \pi_t)'$. To arrive at this formulation, we can rewrite Equations (17) and (18) (ignoring exogenous innovations and the ELB) as

$$(1+\sigma^{-1}\phi_y)y_t = -\sigma^{-1}\left(\phi_\pi\pi_t - E_t[\pi_{t+1}]\right) + E_t[y_{t+1}], \tag{C.11}$$

$$(1-\kappa\phi_\pi)\pi_t = \kappa(\gamma+\phi_y)y_t + \beta_\kappa E_t[\pi_{t+1}], \tag{C.12}$$

where we define $\beta_\kappa = \beta - \kappa$ for convenience. Then, we can rewrite

$$A\mathbf{x}_t = B\mathbf{x}_{t+1}, \tag{C.13}$$

$$\begin{bmatrix} 1+\sigma^{-1}\phi_y & \sigma^{-1}\phi_\pi \\ -\kappa(\gamma+\phi_y) & 1-\kappa\phi_\pi \end{bmatrix} \mathbf{x}_t = \begin{bmatrix} 1 & \sigma^{-1} \\ 0 & \beta_\kappa \end{bmatrix} \mathbf{x}_{t+1}. \tag{C.14}$$

It is straightforward that

$$B^{-1} = \frac{1}{\beta_\kappa}\begin{bmatrix} \beta_\kappa & -\sigma^{-1} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\beta_\kappa^{-1}\sigma^{-1} \\ 0 & \beta_\kappa^{-1} \end{bmatrix}, \tag{C.15}$$

and hence

$$M = AB^{-1} = \begin{bmatrix} 1+\sigma^{-1}\phi_y & \sigma^{-1}\phi_\pi \\ -\kappa(\gamma+\phi_y) & 1-\kappa\phi_\pi \end{bmatrix} \begin{bmatrix} 1 & -\beta_\kappa^{-1}\sigma^{-1} \\ 0 & \beta_\kappa^{-1} \end{bmatrix}, \tag{C.16}$$

$$= \begin{bmatrix} 1+\sigma^{-1}\phi_y & -\beta_\kappa^{-1}\sigma^{-1}(1+\sigma^{-1}\phi_y - \phi_\pi) \\ -\kappa(\gamma+\phi_y) & \beta_\kappa^{-1}\sigma^{-1}\kappa(\gamma+\phi_y) + \beta_\kappa^{-1}(1-\kappa\phi_\pi) \end{bmatrix}, \tag{C.17}$$

$$= \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix}. \tag{C.18}$$

The eigenvalues of the system are given by $|M-\lambda I| = \lambda^2 + p\lambda + q$, where

$$p = -(m_1 + m_4) = -\left(1 + \sigma^{-1}\phi_y + \beta_\kappa^{-1}\sigma^{-1}\kappa(\gamma + \phi_y) + \beta_\kappa^{-1}(1 - \kappa\phi_\pi)\right) \tag{C.19}$$

is the negative of the trace and

$$q = m_1 m_4 - m_2 m_3 = \beta_\kappa^{-1}(1 + \sigma^{-1}\phi_y - \kappa\phi_\pi + \sigma^{-1}\phi_\pi\kappa\gamma) \tag{C.20}$$

is the determinant. As there are no endogenous states, determinacy under the conditions by Blanchard and Kahn (1980) requires the modulus of both eigenvalues of M to be larger than zero. We can find a representation of the absolute value of these eigenvalues in terms of the elements of M as

$$|\lambda^r_{1,2}| = \begin{cases} -p/2 + \sqrt{p^2/4 - q} > 1 \\ -p/2 - \sqrt{p^2/4 - q} > 1 \end{cases} \qquad \text{if } p^2/4 \geq q, \tag{C.21}$$

$$|\lambda^i_{1,2}| = \sqrt{p^2/2 - q} > 1 \qquad \text{if } p^2/4 < q. \tag{C.22}$$

$|\lambda^r_{1,2}|$ are the real eigenvalues if the respective condition for the square root is satisfied, $|\lambda^i_{1,2}|$ are corresponding imaginary eigenvalues otherwise. Using the condition in Equation (C.21) in the second case implies that $-p/2 > 1$, or equivalently

$$p < -2. \tag{C.23}$$

Rearranging the second case in Equation (C.21) also implies

$$1 + p + q > 0. \tag{C.24}$$

Together with Equation (C.23), this implies

$$q > 1. \tag{C.25}$$

Equation (C.25) is also a necessary condition for the case of imaginary eigenvalues. Similarly, one can show that Equation (C.23) and Equation (C.24) imply that Equation (C.22) holds. Therefore, Equations (C.23)-(C.25) are jointly sufficient for both eigenvalues to be larger than one in modulus.

In our model, the three necessary condition $1+p+q>0$, $p<-2$ and $q>1$ thus read

$$\phi_\pi + \frac{1-\beta}{\kappa\gamma}\phi_y > 1, \tag{C.26}$$

$$\sigma^{-1}\phi_y + \beta_\kappa^{-1}\sigma^{-1}(\kappa\gamma + \kappa\phi_y) + \beta_\kappa^{-1}(1-\kappa\phi_\pi) > 1, \tag{C.27}$$

$$1 + \sigma^{-1}(\kappa\gamma\phi_\pi + \phi_y) - \kappa\phi_\pi > \beta_\kappa. \tag{C.28}$$

As a second step, we can use these determinacy conditions to derive a sign for the denominator of λ_0^π. Let us suppose that the denominator is positive, i.e.

$$(1-\beta\rho)(\sigma(1-\rho)+\phi_y) + \kappa\gamma(\phi_\pi - \rho) - \kappa\sigma(1-\rho)(\phi_\pi-\rho) > 0. \tag{C.29}$$

This can be rearranged to

$$\left(\phi_\pi + \frac{1-\beta}{\kappa\gamma}\phi_y - 1\right) + \frac{1-\rho}{\kappa\gamma}\left(\kappa\gamma + \beta\phi_y + \sigma(1-\beta\rho-\kappa(\phi_\pi-\rho))\right) > 0. \tag{C.30}$$

The first term in large brackets is positive, which can be seen directly from the necessary condition in Equation (C.26). After some algebraic manipulations, one can show that Equation (C.27) implies that the second term in brackets is also positive. This shows that the denominator of λ_0^π is indeed positive.

With the denominator being positive, the sign of λ_0^π depends on the numerator, including the minus in front of the fraction. The condition for $\lambda_0^\pi < 0$ is thus

$$\kappa\gamma - \kappa\sigma(1-\rho) > 0. \tag{C.31}$$

Using the definition of γ, this is equivalent to

$$\sigma + \eta - \nu(\psi - 1 - \sigma - \eta) > \sigma(1-\rho). \tag{C.32}$$

Rearranging yields the parameter restriction in terms of the elasticity of the credit spread to entrepreneur leverage.

As a last step, the sign of λ_0^y can be determined given the solution for λ_0^π. The denominator of λ_0^y is positive for conventional parameters, such that the sign is determined by the numerator, including the minus. Inserting λ_0^π, this is given by

$$-1 + (\phi_\pi - \rho)\frac{\kappa\gamma - \kappa\sigma(1-\rho)}{Z}, \tag{C.33}$$

where Z denotes the denominator of λ_0^π. After some algebraic manipulations, this is equivalent to

$$-Z^{-1}\Big((1-\beta\rho)(\sigma(1-\rho)+\phi_y) + (1-\rho)^2\kappa\sigma\Big), \tag{C.34}$$

which is unambiguously negative for $0 \leq \rho \leq 1$. ■

Lemma 2. *The impact response of inflation to a financial shock in normal times (without a binding ELB on nominal interest rates) is positive if the elasticity of the credit spread to entrepreneur leverage satisfies*

$$\nu > \frac{\eta + \rho\sigma}{\psi - 1 - \sigma - \eta}. \tag{C.35}$$

Proof. This is the converse case of Lemma 1. As argued in the corresponding proof, determinacy of the model requires the denominator of λ_0^π to be positive. The condition for $\lambda_0^\pi > 0$ is hence that the numerator (including the minus in front of the fraction) is positive. This is equivalent to

$$\sigma(1-\rho) > \gamma. \tag{C.36}$$

Using the definition of γ to obtain

$$\sigma(1-\rho) > \sigma + \eta - \nu(\psi - 1 - \sigma - \eta) \tag{C.37}$$

and rearranging yields the desired result. ■

Proposition 2. *Suppose that the ELB on nominal interest rates is expected to bind for $T > 0$ periods. The impact responses of inflation and output to a financial shock are then given by*

$$\pi_t = \lambda_T^\pi u_t + \mu_T^\pi \bar{r}, \tag{C.38}$$

$$y_t = \lambda_T^y u_t + \mu_T^y \bar{r}, \tag{C.39}$$

where

$$\lambda_T^\pi = \kappa\left(1-\gamma\sigma^{-1}\right) + \rho\left(\beta-\kappa+\kappa\gamma\sigma^{-1}\right)\,\lambda_{T-1}^\pi + \rho\kappa\gamma\,\lambda_{T-1}^y, \tag{C.40}$$

$$\mu_T^\pi = \kappa(1-\gamma\sigma^{-1}) + (\beta-\kappa+\kappa\gamma\sigma^{-1})\,\mu_{T-1}^\pi + \kappa\gamma\mu_{T-1}^y \tag{C.41}$$

$$\lambda_T^y = -\sigma^{-1} + \rho\sigma^{-1}\,\lambda_{T-1}^\pi + \rho\,\lambda_{T-1}^y, \tag{C.42}$$

$$\mu_T^y = \sigma^{-1} + \sigma^{-1}\mu_{T-1}^\pi + \mu_{T-1}^y \tag{C.43}$$

where $\{\lambda_0^\pi, \lambda_0^y\}$ *as in Proposition 1 and* $\mu_0^\pi = \mu_0^y = 0$.

Proof. Similar to Proposition 1, the proof relies on the method of undetermined coefficients. Suppose that the ELB is expected to bind for $k \geq 1$ periods. We guess that the responses of inflation and output are linear functions of the financial shock and the ELB value, as shown in Equations (C.38) and (C.39). Combining Equations (16)-(17) yields

$$\pi_t = \kappa(1-\gamma\sigma^{-1})\bar{r} + (\beta-\kappa+\kappa\gamma\sigma^{-1})E_t[\pi_{t+1}] + \kappa\gamma E_t[y_{t+1}] + \kappa(1-\gamma\sigma^{-1})u_t \tag{C.44}$$

where the interest rate is replaced by the ELB value. Expectations of future variables can be replaced by the corresponding policy functions for the case of an expected ELB duration of $k-1$ under rational expectations, using the law of motion for the financial shocks. This yields

$$\pi_t = \kappa(1-\gamma\sigma^{-1})\bar{r} + (\beta-\kappa+\kappa\gamma\sigma^{-1})(\rho\lambda_{T-1}^\pi u_t + \mu_{T-1}^\pi\bar{r}) + \kappa\gamma(\rho\lambda_{T-1}^y u_t + \mu_{T-1}^y\bar{r}) + \kappa(1-\gamma\sigma^{-1})u_t \tag{C.45}$$

Collecting terms and matching coefficients yields the solution for λ_T^π and μ_T^π. The values for λ_T^y and μ_T^y can be found similarly: start from Equation (17), replace the interest rate by the ELB value and expectations by policy functions for an ELB duration of $T-1$, match coefficients. ■

Lemma 3. *A concave inflation policy function in the expected ELB spell duration requires that the elasticity of the credit spread with respect to entrepreneur leverage satisfies*

$$\nu > \frac{\eta}{\psi - 1 - \sigma - \eta} \tag{C.46}$$

and that the size of the financial shock satisfies

$$u_t > -\bar{r} = \beta^{-\sigma} - 1. \tag{C.47}$$

Proof. If the first term $\kappa\left(1-\gamma\sigma^{-1}\right)$ in Equations (31) and (32) is negative, λ_T^{π} and μ_T^{π} are negative for all T under Assumption 2 for conventional calibrations. Conversely, the recursion can only be increasing if $\kappa\left(1-\gamma\sigma^{-1}\right)$ is larger than zero, which yields the first parameter restriction shown above. The second restriction follows from noting that the same term appears also in μ_T^{π}. Hence, the sum $u_t+\bar{r}$ must be positive. Note that $\bar{r}$ is a negative value, i.e. the negative percentage point deviation of interest rates at the ELB from their steady state. This ELB value is determined by the household preference parameters β and σ as shown in the Lemma. ■

Proposition 3. *The policy parameters in the central bank's monetary policy rule must satisfy the following conditions to guarantee a determinate solution:*

$$\phi_\pi + \frac{1-\beta}{\kappa\gamma}\,\phi_y > 1, \tag{C.48}$$

$$\kappa(\sigma^{-1}\gamma - 1)\,\phi_\pi + \sigma^{-1}\phi_y > \beta - 1 - \kappa \tag{C.49}$$

Proof. The first equation follows directly from the condition $1+p+q>0$, which is required to satisfy the Blanchard and Kahn (1980) conditions. This is Equation (C.26) in the proof for Proposition 1. The second equation can be obtained by rearranging the condition $q>1$, which is Equation (C.28) above. ■

Appendix D Numerical Solution Method

For the sake of clarity, we use a different representation of the policy functions to outline the solution procedure. The analytic solutions in Section 3 are expressed in terms recursive policy functions of u_t. A different, non-recursive way of presenting these policy functions is suggested in Boehl (2022a). The simplicity of our model allows to ease the notation therein and express our model with $\mathbf{x}_t = (\pi_t, y_t)'$ in matrix form as

$$\mathbf{x}_t + \mathbf{c} \max \{\mathbf{d}\mathbf{x}_t, \bar{r}\} = \mathbf{N} E_t \mathbf{x}_{t+1} + \mathbf{c} u_t, \tag{D.1}$$

where $\mathbf{N}$ is the system matrix of the constrained system, $\mathbf{c}$ contains the coefficients that determine how $\mathbf{x}_t$ is affected by r_t (and thereby also by u_t) and $\mathbf{d}$ contains the parameters of the monetary policy rule. $\bar{r} < 0$ is the actual model-implied lower bound of r_t.

Assume again that the economy is at the ELB for k periods. Then

$$\mathbf{x}_t + \mathbf{c}\bar{r} = \mathbf{N} E_t \mathbf{x}_{t+1} + \mathbf{c} u_t, \tag{D.2}$$

$$E_t \mathbf{x}_{t+1} + \mathbf{c}\bar{r} = \mathbf{N} E_t \mathbf{x}_{t+2} + \mathbf{c} u_{t+1}, \tag{D.3}$$

$$\dots$$

$$E_t \mathbf{x}_{t+k-1} + \mathbf{c}\bar{r} = \mathbf{N} E_t \mathbf{x}_{t+k} + \mathbf{c} u_{t+k-1}, \tag{D.4}$$

$$E_t \mathbf{x}_{t+k} = \mathbf{A}(0) u_{t+k}. \tag{D.5}$$

Recursively inserting (D.5) into (D.4) yields, acknowledging that $E_t u_{t+s} = \rho^s u_t$,

$$\mathbf{x}_t = \mathbf{N}^k \mathbf{A}(0) \rho^k u_t + \sum_{i=0}^{k-1} \mathbf{N}^i \mathbf{c} \rho^i u_t - \sum_{i=0}^{k-1} \mathbf{N}^i \mathbf{b} \bar{r}, \tag{D.6}$$

$$= \mathbf{A}(k) u_t + \mathbf{a}(k) \bar{r}. \tag{D.7}$$

Rewriting (D.6) yields

$$\pi_t = A_\pi(k) u_t + a_\pi(k) \bar{r}, \tag{D.8}$$

$$y_t = A_y(k) u_t + a_y(k) \bar{r}. \tag{D.9}$$

In verbal terms, this implies that depending on the expected number of periods at the ELB k, we can express the vector of controls $\mathbf{x}_t$ as a linear map $A_j(k)$ of u_t and the (constant) vector

$a_j(k)$. Both terms are nonlinear functions of k defined on $\mathbb{N}_0$. In other words: given k, the policy function is simply a two dimensional linear projection of the scalar u_t.

Definition 1 recapitulates the conditions for k to be an equilibrium value under the assumption that each shock causes the ELB to hold instantly without any transition period.

Definition 1 (equilibrium k)**.** *For each period t, an equilibrium value of $k \in \mathbb{N}_0$ must satisfy that the ELB binds in expectations exactly until period $t + k$. Hence,*

$$\mathbf{d}\mathbf{x}_t > \bar{r} \implies k = 0, \tag{D.10}$$

while for $k > 0$ it must hold that

$$\mathbf{d}E_t\mathbf{x}_{t+k} > \bar{r}, \tag{D.11}$$

and

$$\mathbf{d}E_t\mathbf{x}_{t+k-1} \leq \bar{r}. \tag{D.12}$$

The parsimonious nature of our model allows that, for each u_t, a k can simply be found by iterating over $k \in \mathbb{N}_0$ (where, naturally, k is likely to be small). More sophisticated iteration schemes for a general formulation of the dynamic system can be found in Boehl (2022a).

To provide some quantitative impression given our model, for $\nu = 0.2$, a 1% risk premium shock will cause the ELB to initially bind for $k = 2$ periods, a 2% shock will cause $k = 9$ and a 3% shock an endogenous duration of $k = 12$ periods.

In Figure D.1 we show the reduced-form slope of the Phillips Curve, based only on the dynamic effect in response to the risk premium shock. The figure confirms that the slope is considerably high if away from the ELB, but drops once the ELB is reached and remains consistently low as the number of expected durations at the ELB increases.

Figure D.2 plots the non-recursive policy functions for π_t. For a more moderate value of ν of 0.2, the mapping $A_\pi(k)$ from $u_t \to \pi_t$ decreases with k while the linear part $a_\pi(k)$ increases in about the same fashion. As larger shocks are necessary to cause a higher k, the dynamic effect of the shock dominates the static effect and inflation falls. For $\nu = 0.22$, $A_\pi(k)$ becomes more convex, meaning that the coefficient that translates financial shocks to inflation increases for low expected durations. This effect is not offset by the static effect of a longer anticipated ELB period, which leads to a more muted inflation response. For a value of $\nu = 0.24$, the dynamic response approaches zero while for $\nu = 0.25$, $A_\pi(k)$ turns positve for values of k larger than

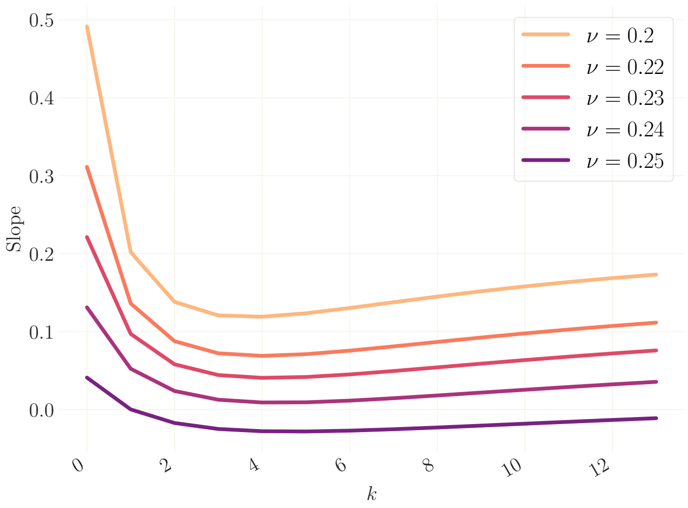


Figure D.1: Theoretical Phillips Curve slope $A_\pi(k)/A_y(k)$. This exercise ignores the static effect of the ELB, that is captured by $a_\pi(k)$ and $a_y(k)$.

two. As the static effect is again too weak to counteract, this leads to an increase of inflation on impact, as it is captured in Figure 3.

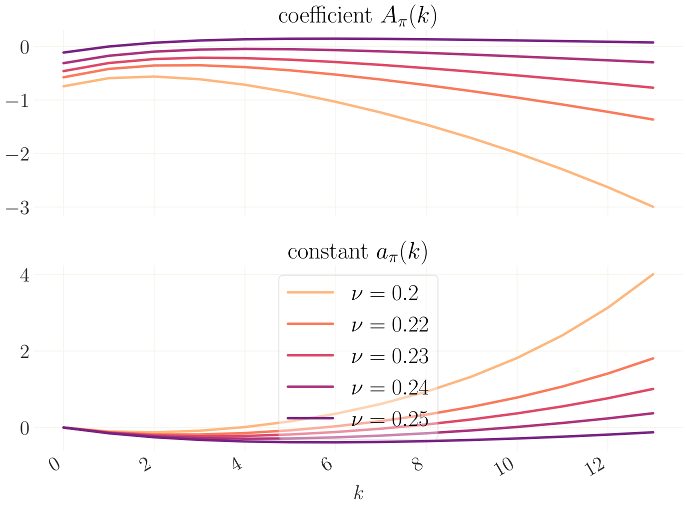


Figure D.2: Expected ELB Duration and Impact Response

Appendix E Dynamic responses to conventional shocks

This section studies the macroeconomic dynamic responses to different conventional shocks: a household preference shock, a cost-push shock and a monetary policy shock. Considering a preference shock first and abstracting from other shocks, the system is given by

$$\pi_t = \kappa\gamma\, y_t + (\beta - \kappa)E_t[\pi_{t+1}] + \kappa r_t, \tag{E.1}$$

$$y_t = -\sigma^{-1}\,(r_t - E_t[\pi_{t+1}]) - \sigma^{-1}e_t + E_t[y_{t+1}], \tag{E.2}$$

where e_t follows the same autoregressive structure as u_t and is likewise scaled by σ^{-1} to maintain direct comparability. In contrast to the model representation in Equations (16) to (18), the shock hence appears only in the Euler equation, but not in the New Keynesian Phillips curve.

Figures E.3 and E.4 show the dynamic responses to a contractionary preference shock (i.e. households having a lower preference for current-period consumption). They are fairly similar to those of a financial shock as shown in Figure 3, but feature slightly smaller responses of inflation. In contrast to the preference shock, the financial shock also induces a direct exogenous increase of the credit spread, which adds a small boost to the inflationary effect of the shock. However, the simulations reveal that overall, the endogenous spread component is able to cause a considerable flattening of the Phillips curve even without the exogenous component.

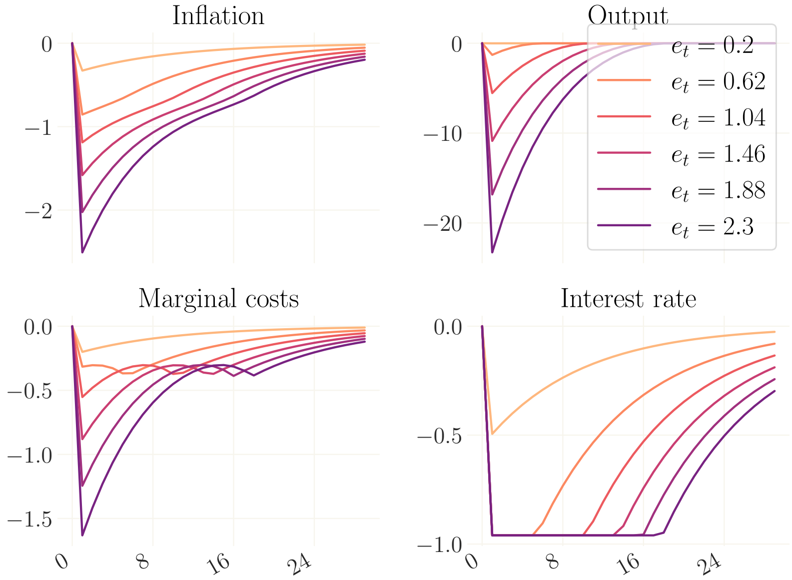


Figure E.3: Impulse response functions to different preference shocks e_t for $\nu = 0.25$.

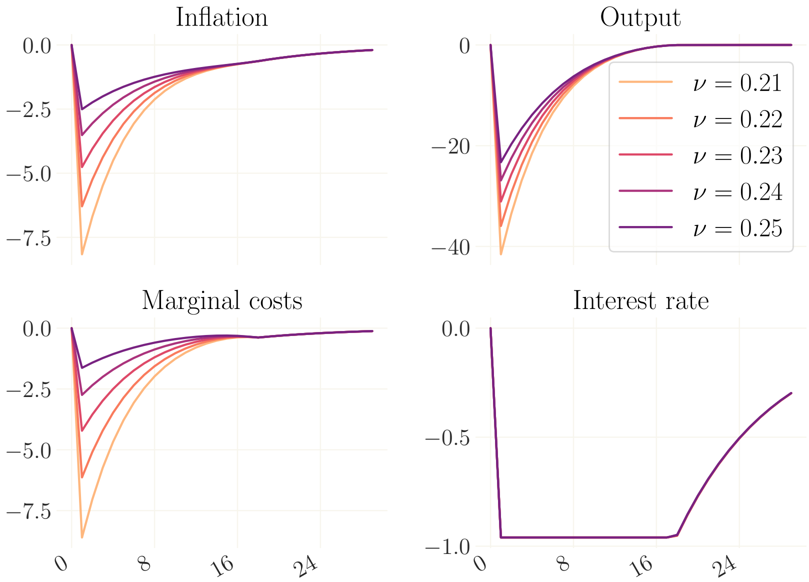


Figure E.4: Impulse response functions to a 2.3% preference shock for different values of ν causing the ELB to be binding.

We now turn to a cost-push shock, which we implement as a conventional aggregate technology shock a_t. An increase in the level of technology causes a fall in marginal costs, which shows up in the New Keynesian Phillips curve

$$\pi_t = \kappa\gamma\, y_t + (\beta - \kappa)E_t[\pi_{t+1}] + \kappa r_t - \kappa a_t, \tag{E.3}$$

where a_t is assumed to also follow the same autoregressive structure as u_t and abstracting from the financial shock.

Simulations for disinflationary technology shocks are presented in Figures E.5 and E.6. Since we are focussing on shock magnitudes where the ELB binds, output falls nevertheless and we still obtain qualitatively similar dynamics as for the financial shock. This stark contrast to normal times can be explained as follows: If inflation falls (due to lower marginal costs) and the interest rate is constraint by the ELB, the real rate increases. This gives rise to a savings motive which leads to less consumer spending, and thereby a lower level of overall output. Although quantitatively less impactful than in the case of the financial shock, the impulse responses show that the binding ELB causes a weakening of the drop in marginal costs, which causes overall less disinflationary pressure in the economy.

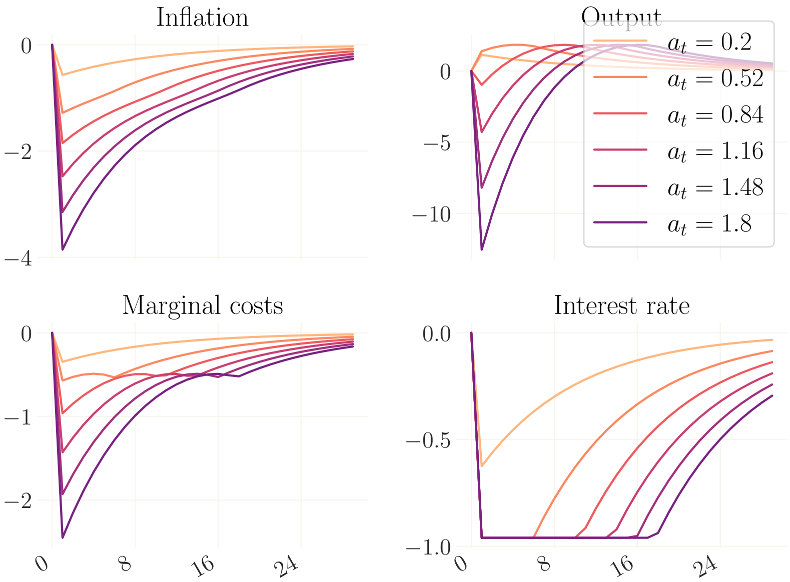


Figure E.5: Impulse response functions to to different technology shocks a_t for $\nu = 0.25$.

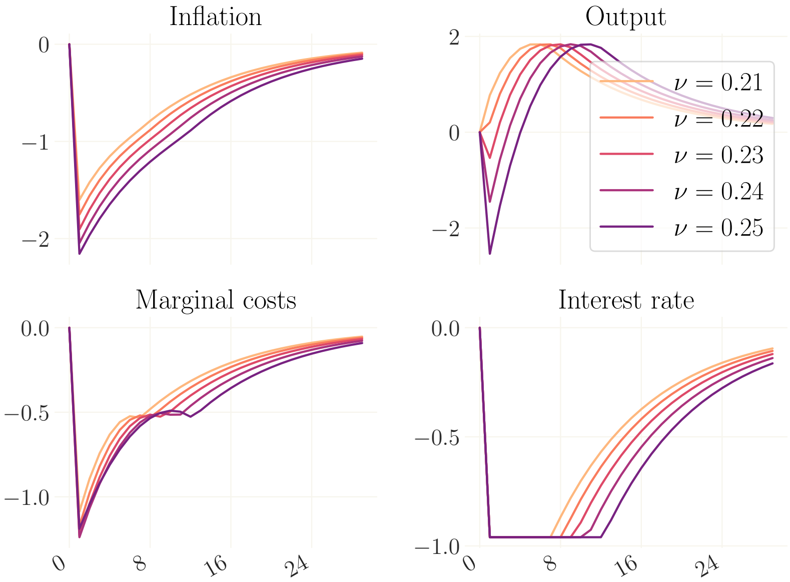


Figure E.6: Impulse response functions to a 1% technology shock for different values of ν causing the ELB to be binding.

Fig. E.7 shows impulse responses to an expansionary monetary policy shock in normal times. For relatively persistent shocks, the impulse responses are conventional: inflation and output increase. For shocks with a rather low persistence, the overall response is mildly deflationary. To understand this result, note that the shock drives a wedge between the policy rate and the rate implied by the policy rule, since our specification does not feature interest rate inertia (policy rate smoothing). Moreover, via the nominal interest rate, the shock appears in both the Euler equation as well as the Phillips curve. Because expectations are discounted with a different discount factor in these two equations (see the discussion in the main body), different shock persistences can lead to qualitatively different impulse responses. In particular, for shocks with a low persistence, the direct impact of lower interest rates on marginal costs is negative and not outweighed by the expectations feedback in the Euler equation. Formally, the role of ρ for the inflation response can be seen from Proposition 1 in the main text.

It should be noted that the output responses are fairly large compared to the empirical literature and more detailed structural models. This is a weakness of the small-scale model at hand. While the setup implies an amplification of shocks in the spirit of financial accelerator models, it lacks additional features such as e.g. habit formation, price and wage indexation, or a stronger discounting of future variables – all of which would imply a lower peak effect on output (and hump-shaped responses).

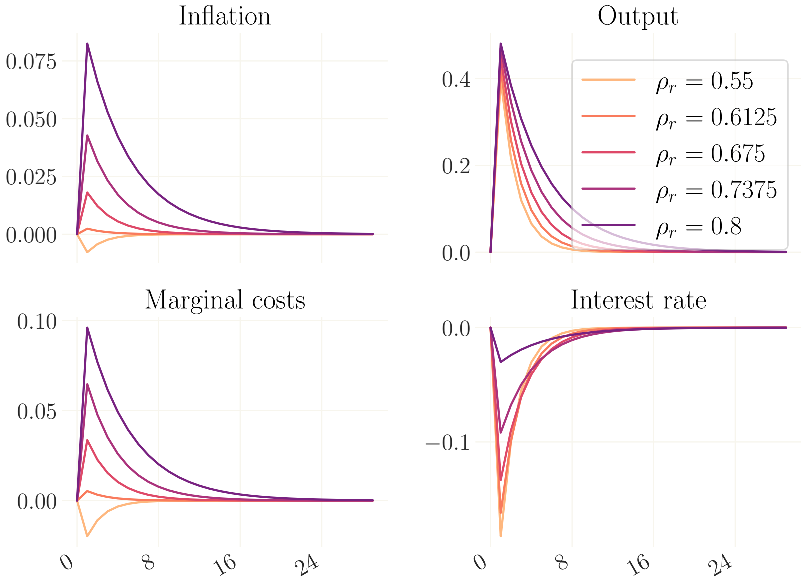


Figure E.7: Impulse response functions to an annualized 1% monetary policy shock for different values of ρ_r in normal times. ν is set to 0.20.

Appendix F The role of financial shock persistence ρ and Calvo probability ζ

As discussed in Sections 4.2 and 5.1, the persistence of financial shocks ρ is a key parameter for inflation dynamics. Figure F.8 provides an illustration: For lower values of ρ, the disinflationary impact becomes weaker and eventually changes sign.

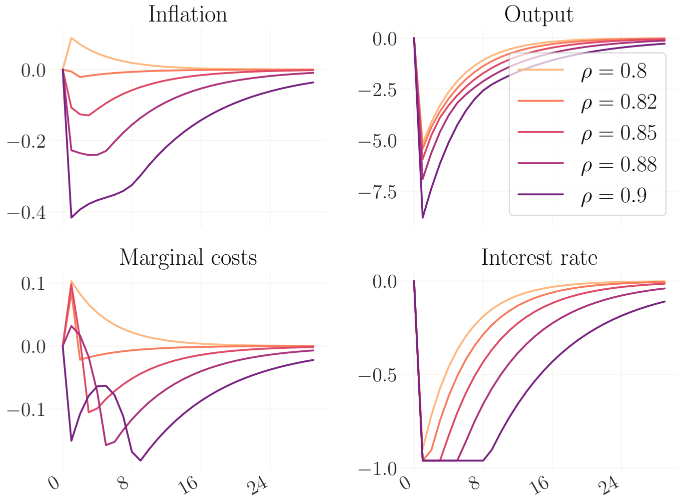


Figure F.8: Impulse responses to a 2% financial shock for different values of ρ, given $\nu = 0.24$.

In Section 4.1, we assume a standard value for the Calvo probability of $\zeta = 0.66$. Recent Bayesian estimates suggest significantly larger values. Figure F.9 shows our key figure for $\zeta = 0.9$, which is close to the estimate by Boehl and Strobel (2024b). As expected, this implies a flat Phillips curve by itself. However, the hockey stick feature remains, i.e. a flatter slope at the EL B.

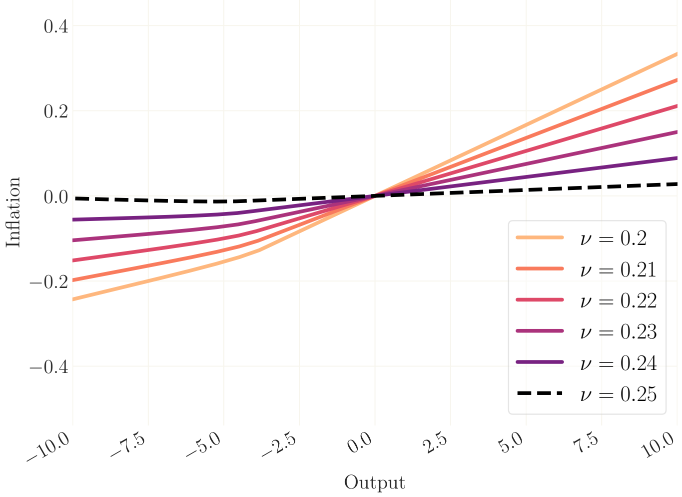


Figure F.9: Observed Phillips Curve for an economy facing financial shocks as in Figure 4 for a Calvo probability of $\zeta = 0.9$. For each value of ν, we simulate the model for u_t on the interval $[-4, 4]$ and plot the respective combination of π_t and y_t.